PARTIAL DIFFERENTIAL EQUATIONS

SIMULATION TECHNOLOGY MODULE

1. Background

- Elastic string - equivalence of mass-spring discretization and wave equation (43-46)
- PDE, FEM and boundary conditions (ch. 149-159)

2. Goals

2.1. **Understanding.**

- Wave equation as limit of discrete mass-spring system.
- Basic properties of linear PDEs for waves and diffusion with boundary/initial conditions.
- Basic properties of FEM approximation of linear PDEs.
- General components (data structures and algorithms) of a FEM program.

2.2. **Skills.**

- Construct simple mass-spring simulator for the wave equation.
- Construct simple FEM program for linear PDE problems.

3. Software Interfaces

You are given a number of prototype FEM solvers in the "game/" directory using the DOLFIN general FEM system for the following PDE (initial and boundary value problems):

- $\dot{u} - \epsilon u'' = f$ (Heat/Diffusion equation, 1D and 2D)
- $\ddot{u} - \epsilon u'' = f$ (Wave equation, 1D and 2D)
- $\dot{u} + \beta u' - \epsilon u'' = f$ (Convection-Diffusion equation, 1D and 2D)

See the course home page for references to DOLFIN documentation. Also, use the Python `help()` function, it should provide the necessary informaton in most situations.

You are also given a minimal FEM program in just 81 lines of Python code, in "python/myassemble.py". This can be used to study the components of a FEM program, and the data structures, methods and algorithms used.

3.1. **Visualization.** The prototype solvers output the solution function as VTK files (`.vtu/.pvd`). These files can be visualized with the `paraview` program which is installed in the Ubuntu computer lab environment. This is a user-friendly graphical visualization environment, with many powerful featues. The output files can become quite large in 2D/3D, make sure you have enough space when you run the simulations.

For instructions, see the ParaView web site. A simple introduction video can be seen on YouTube, illustrating how to open and view an animation (which the prototype solvers output).

4. Examination

The examination of this module consists of the questions below, to be answered and evaluated as part of your project report.

5. Questions

Answer these questions as part of your project report. When a question mentions "your model", it means the equation/model you have chosen as your project.

- Derive the 1D wave equation from a particle-spring model (or another continuum model from a particle model, see ch. 150.4 for a transition from the wave equation to the heat/diffusion equation).
- What boundary and initial conditions need to be specified for your equation/model?
- Derive the FEM formulation for your equation/model.
- Describe the structure of your solver, how do you compute the solution for one timestep?
- Describe the components (basic data structures and algorithms) of a FEM program.