PARTIAL DIFFERENTIAL EQUATIONS

SIMULATION TECHNOLOGY MODULE

1. BACKGROUND

- Elastic string - equivalence of mass-spring discretization and wave equation (43-46)
- PDE, boundary conditions (123,124)
- FEM (142)
- FEM data structures (NEW)
- Heat/Diffusion equations with FEM solution (126,144,145,149,150)
- Wave equation with FEM solution (135,148,151)
- Extra course (1.5hp): FEM error estimation (146,147)

2. GOALS

2.1. **Understanding.**

- Wave equation as limit of discrete mass-spring system.
- Basic properties of linear PDEs for waves and diffusion with boundary/initial conditions.
- Basic properties of FEM approximation of linear PDEs.
- General data structures and algorithms of a FEM program.

2.2. **Skills.**

- Construct simple mass-spring simulator for the wave equation.
- Construct simple FEM program for linear PDE problems.
- Redesign general FEM software to model 2D/3D PDE problems.

3. SOFTWARE INTERFACES

Python: mass-spring simulator, simple FEM solver

Python (C++): FFC/Dolfin as general PDE solver

4. GAME/INTERACTIVE SIMULATION

Use your ODE solvers developed in previous modules:

- Implement the 1D mass-spring model of ch. 43.
- Implement the 1D viscoelastic mass-spring model of ch. 44.

- Implement the 2D mass-spring model of ch. 45.
- Implement the 3D mass-spring model of ch. 46.
- Implement a FEM method for the wave equation of ch.148.
- Implement a FEM method for the heat equation of ch.144.

Construct FEM simulators for linear PDE from Puffin template:

- Implement a FEM method for the heat equation of ch.149.
- (Implement a FEM method for the convection-diffusion equation)
- Implement a FEM method for the Poisson equation of ch.150.
- Implement a FEM method for the Wave equation of ch.151.

5. QUESTIONS

- Derive a 1D mass-spring model for an elastic string
- Derive the 1D wave equation from a mass-spring model.
- What boundary and initial conditions need to be specified for a wave and a heat equation?
- Derive weak formulations for a wave and a heat equation.
- Derive FEM formulations for a wave and a heat equation.
- Derive ODE system for FEM space discretization for a wave and a heat equation.
- Describe the basic data structures and algorithms of a FEM program.
- (Estimate the error for a FEM approximation of the Poisson equation.)