Fö5

A posteriori feluppskattning

$$\boxed{|u(T)-U(T)| \le S_c(T)\ \max_n k R_n(U) + S_d(T)|u(0)-U(0)|}$$

$$\begin{cases} R_n(U) = \max_{I_n} |\dot{U} + AU - F| & \text{(residual)} \\ S_c(T) = \int_0^T |\dot{\varphi}(s)|\,ds & \text{(stabilitetsfaktor map tidsdiskretisering)} \\ S_d(T) = |\varphi(0)| & \text{(stabilitetsfaktor map fel i initialdata)} \end{cases}$$

1. $A < 0$: $S_d(T) \le \exp(|A|T)$, $S_c(T) \le \exp(|A|T)$
2. $A \ge 0$: $S_d(T) \le 1$, $S_c(T) \le 1$
3. $A = i$: $S_d(T) \le 1$, $S_c(T) \le T$
4. $A = \sin(t)$: $S_d(T) \le \exp(1)$, $S_c(T) \le \exp(1)T$

(För generella olinjära system $\dot{u} + f(u) = 0$, motsvarar A Jacobianen $f'(u(t))$ vad gäller stabilitet)

Fö5

Felrepresentation :

$$0 = \int_0^T e(-\dot{\varphi} + A\varphi)\,dt$$

Partiell integration :

$$0 = \int_0^T e(-\dot{\varphi} + A\varphi)\,dt = \int_0^T (\dot{e} + Ae)\varphi\,dt - e(T)\varphi(T) + e(0)\varphi(0)$$

$$\Rightarrow |e(T)| = \int_0^T (\dot{e} + Ae)\varphi\,dt + e(0)\varphi(0)$$

- $e(0) = u(0) - U(0) = u^0 - U^0$ fel i initialdata.

$$\dot{e} + Ae = \dot{u} + Au - (\dot{U} + AU) = F - \dot{U} - AU = -R(U)$$

$$\Rightarrow \boxed{|e(T)| = -\int_0^T R(U)\varphi\,dt + e(0)\varphi(0)}$$

Felrepresentation

1/ lokala fel : residual $R(U)$

2/ fel i initialdata : $e(0) = u(0) - U(0)$

3/ känslighet / stabilitet : $\varphi(t)$ duallösning