

2, Uppskatta skilnaderna efter att
lösley med olika initialvår : $u(n) - \tilde{u}(n)$

$$\begin{aligned} |u(n+1)k - \tilde{u}(n+1)k| &= |u(n) + k f(u(n)) - (\tilde{u}(n) + k f(\tilde{u}(n)))| \\ &\leq |u(n) - \tilde{u}(n)| + k |f(u(n)) - f(\tilde{u}(n))| \\ &\leq (1 + kL) |u(n) - \tilde{u}(n)| \end{aligned}$$

1. + 2. $\Rightarrow \dots \Rightarrow$ Total error $\leq \boxed{kM \exp(Lt)}$
at time t
($\max_u |f(u)| \leq M$)

$\Rightarrow$ An approximate solution to the ODE can be
computed to ~~the~~ ^a precision proportional to
 $k \exp(Lt)$, by time stepping: $u^{n+1} = u^n + k f(u^n)$
(Generalized Fundamental Theorem of Calculus)

Compare the errors $|u(t) - f(t)|$ vs $|\tilde{u}(t) - f(t)|$
Error $\leq C_0 L t k$ vs $\leq C_1 \exp(Lt) \cdot k$
($C_0 \sim C_1 \sim 1$)

Ex: $L = 10, t = 30 \Rightarrow$ Error $\leq C_0 300 \cdot k$
vs Error $\leq C_1 \exp(300) \cdot k$
 $\gg 10^{100}$ google!