## EXERCISES

In Exercises 1–9, compute the indicated matrices given

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 0 \\ -3 & -1 & 5 \\ 1 & 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 2 \\ 3 & -1 \\ 2 & -4 \end{bmatrix},$$

$$\text{and} \quad D = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 2 & -2 \\ 0 & 0 & 3 \end{bmatrix}.$$

If an operation cannot be performed, indicate why not.

**1.** **(a)** $2A + C^T$ **(b)** $C - 3B$

**2.** **(a)** $AB$ **(b)** $AD$

**3.** **(a)** $CA$ **(b)** $AC$

**4.** **(a)** $BD$ **(b)** $DB$

**5.** **(a)** $BC$ **(b)** $CB$

**6.** **(a)** $3B - 2D$ **(b)** $2D^T + B$

**7.** **(a)** $\det(D)$ **(b)** $\det(A)$

**8.** **(a)** $C^T D$ **(b)** $BA^T$

**9.** **(a)** $-2A^T + 5C$ **(b)** $B^T + D$

**10.** Let $A$ be a nonsingular matrix.
**(a)** Show that $A^{-1}$ is unique.
**(b)** Show that $A^{-1}$ is nonsingular and $(A^{-1})^{-1} = A$.
**(c)** Show that $A^T$ is nonsingular and $(A^T)^{-1} = (A^{-1})^T$.
**(d)** If $B$ is nonsingular, show that $AB$ is nonsingular and $(AB)^{-1} = B^{-1}A^{-1}$.

**11.** Can an $n \times m$ matrix with $n \neq m$ be symmetric? Explain.

**12.** Recalculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 4 & 1 \\ -2 & 1 & -3 & 2 \\ 0 & 0 & 0 & 2 \\ 3 & 2 & 1 & -1 \end{bmatrix}$$

by first expanding along the second column.

**13.** Show that

$$\det\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) = a_{11}a_{22} - a_{12}a_{21}.$$

**14.** Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

**(a)** Show that $A$ is nonsingular provided $a_{11}a_{22} - a_{12}a_{21} \neq 0$.