Comsol Laboration: Cables and Reels

JO, CSC

January 11, 2012

1 Physical configuration

This is a cross section of a RVOE-twin conductor-cable: Two layers of plastic, the inner t (= 0.85 mm) thick; The area of the copper conductors A is 1.5 mm^2 each and the outer diameter of the cable is D_y (= 9 mm). How much warmer than the ambient, and where is it warmest, if it connects a ($U =$) 230V-power outlet to a P (= 2000W) heating element ? We assume a purely resistive load.

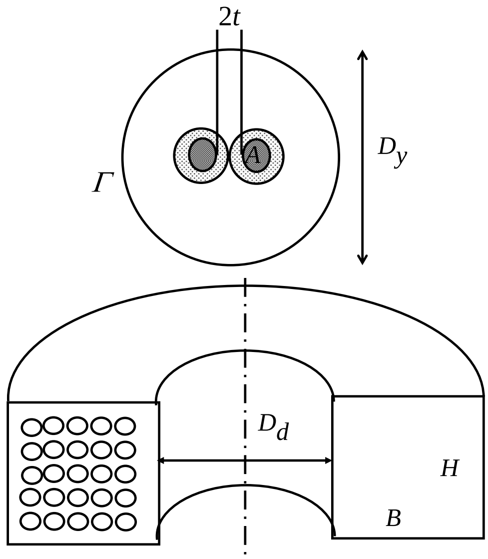

Figure 1: Top, cable cross section, Bottom, reel geometry

Two cases:

1. The cable hangs in free air and is cooled by it with a heat transfer coefficient of h [W/m^2/K]

2. The cable is very carefully wound into a reel, picture bottom with $D_d = 100$ mm.

Data that may be useful: *Copper* has electric conductivity σ_{Cu} of 5.8 10^7 $[\Omega m]^{-1}$ and heat conductivity $k_{Cu} = 380$ [W/(mK)]. *Plastic*: no electric conductivity and heat conductivity $k_{Pl} = 0.2$ [W/(mK)]. The two kinds of plastic are assumed identical (except for color ...), so no boundary between them is needed. Values for h, and T_{ext} should be taken as

h = 10 [W/(m^{2}K)], $\quad T_{ext} = 20[^oC]$

Determine the steady temperature distribution $T(x, y, z)$ for the two cases with Comsol Multiphysics.

2 Model

The mathematical model is a boundary value problem for the steady heat equation,

$$0 = \nabla \cdot (k\nabla T) + Q \text{ in } \Omega \tag{1}$$

$$k\frac{\partial T}{\partial n} = h\left(T_{ext} - T\right) \text{ on } \partial\Omega \tag{2}$$

where all material data are constant with respect to time and temperature, but varies in space. In particular $Q = 0$ in the plastic.

3 Preparatory analysis

1. Show that the heating power Q $[W/m^3]$ in the copper (to leading order) is $Q = \left(\frac{P}{UA}\right)^2 \frac{1}{\sigma_{Cu}}$, in the steps
   - what is the total current to give total power P?
   - what is the resistance in l m of the Cu-conductor?
   - what is the potential drop? and the power?
2. The boundary Γ has arithmetic mean temperature

$$T_{mean} = \frac{1}{|\Gamma|}\int_\Gamma T d\Gamma = T_{ext} + \frac{2AQ}{hp} \tag{3}$$

   where $|\Gamma| = p$ is the length of the perimeter of the cable cross section. Why is it so?

3. The temperature in the copper has very small gradients compared to the plastic. Quantify this statement, close to the Cu-Pl interface, by considering the heat flux through the interface.
4. Suppose that the two Cu-regions can be replaced by one central circular disk Cu with area $2A$, radius R_i. Then, the equation for T in the plastic becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0, \quad T(D_y/2) = T_{mean} \tag{4}$$

   Show eqn. (4) and explain why

$$T(r) = C_1 \ln r + C_2. \tag{5}$$

   and

$$2\pi R_i k_{Pl}\frac{dT}{dr}|_{r=R_i} = 2AQ. \tag{6}$$

   Determine C_1, C_2 and the temperature at the Cu-Pl interface.

4 Lab session work – Task A

4.1 Free cable

Model: Use 2D, mode
`COMSOL Multiphysics/PDE Modes/PDE, Coefficient form/Stationary analysis`,
and the pre-selected quadratic polynomial elements.
Geometry: The union of one large and two small circles gives the desired computational domain: three sub-domains, two for Cu and one for Pl.
PDE: Define `P, U, A, sigCu, ..., Q` as constants. It is possible to write expressions, like for Q, enter the expressions under `Options/Constants`.
Boundary conditions: Neumann (Robin) type along the outer boundary, with specified heat transfer coefficient.
Tasks:

1. Make an initial mesh, refine the triangulation (at least) twice and note the max-temperatures for the three calculations. What is the order of accuracy/convergence? Is the use of max-temp. a good way to assess order of accuracy? Are there singularities in the solution (temperature field)?

2. Change to linear elements. What is the order now?

3. Check the mean temperature formula (3) above by `Boundary integration`.

4.2 Cable on a reel

Model: Use `Axisymmetric (2D)`, mode and then choose
`COMSOL Multiphysics/Heat Transfer/Conduction/Steady-state analysis`.
We model one half (symmetry) of a cross section of the reel with $5 \times 3 \times$(2 Cu-circles) packed so the distance between the 2-groups becomes 10 mm. The whole pattern is placed in a rectangle whose smallest distance to the circles is 2 mm everywhere. Then fill the space between the Cu-circles with plastic, and cool the outer boundaries like above.
Geometry: There is an "array"-constructor (icon with four small squares) which makes $n \times m$ copies in an $n \times m$ pattern. Or manually: First make one Cu-circle, then its neighbor in the two-group. Form from them a composite object, say G. Copy and translate to make a row. Make a composite object from these 5 G, say R. Copy and translate R to make three rows. Make a composite object, say Cu, from the three R. Create the rectangle R1, form the union R1+Cu - Note: Keep interior borders!
PDE: There are 31 subdomains. But 30 of them have the same data, so select all 31 and give them Cu-properties and heat source. Then change the data in the (single) plastic domain.
Boundary conditions: The three outer boundaries are as in 4.1, with the same transfer coefficients. The symmetry plane is insulated (no heat flux).
Tasks: Compute the temperature field. Show as 3D plot and color with streamlines (with many start points) to visualize the heat flux. Where are the temperature gradients strongest? Plot $|\mathrm{grad}\, T|$ to show.

5 Lab session work – Task B

5.1 Free cable

As we saw above, the temperature gradient in the copper is very small compared to the one in the plastic. A model where the Cu has constant (but unknown) temperature should be quite accurate. This temperature would then come in as a boundary condition in a model for only the plastic. Make this model! The "constant" temperature in Cu must be determined so the correct total heat flows through the plastic. How is that done? Hint: The Cu-temperature is an affine function of external temperature and the power,

$$T_{Cu} = T_{ext} + KQ \tag{7}$$

You can now determine K by a solution with some assumed value for T_{Cu}. Verify eqn. (7).

1. empirically by using the COMSOL parametric solver (under `Solve/Solver Parameters`) with `TCu` as parameter, and
2. theoretically by using the differential equation.

5.2 Reel cross section

Where should the element mesh be refined? Try adaptive refinement (select under solver parameters) and see if you agree with the algorithm.