Comsol Laboration: Heat Conduction in a Chip

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1 Physical configuration

A chip on a circuit board is heated inside and cooled by convection by the surrounding fluid. We consider two cases

1. The stand-alone chip, with dimensions $H \times W \times L$ as indicated in Figure 1.
2. The same chip but with aluminum cooling flanges added to improve cooling, see Figure 2.

We wish to investigate

- What is the steady state temperature distribution, and how long does it take to reach steady state?
- How do the chip dimensions influence the max. temperature?
- How does the temperature change when cooling flanges are added?

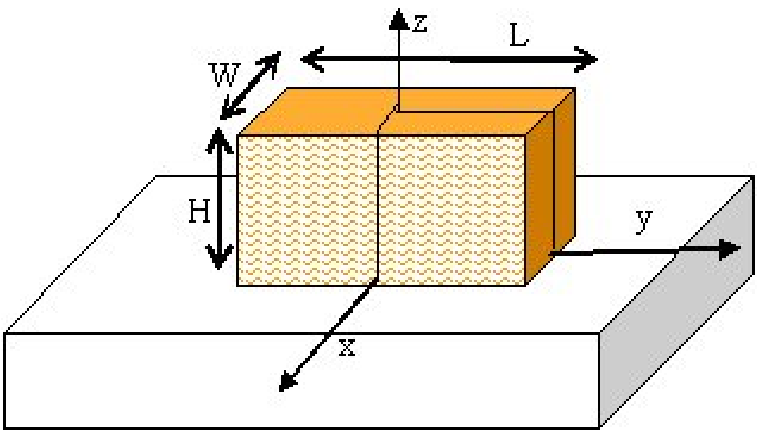

Figure 1: Chip geometry

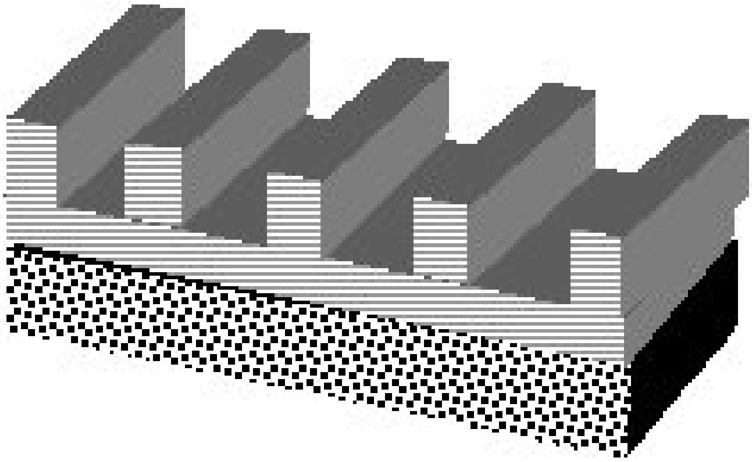

Figure 2: Chip with cooling fins

2 Model

The mathematical model is an initial-boundary value problem for the heat equation, to determine $T(x, y, z, t)$ for $t > 0$ when

$$\begin{aligned} \rho C \frac{\partial T}{\partial t} &= \nabla \cdot (k \nabla T) + Q, && x \in \Omega, \\ k \frac{\partial T}{\partial n} &= h\left(T_{ext} - T\right) && x \in \partial\Omega, \end{aligned} \tag{1}$$

with initial data $T(x, y, z, 0) = f(x, y, z)$. In particular, the steady state temperature obtained after long time is of interest,

$$\begin{aligned} 0 &= \nabla \cdot (k \nabla T) + Q, && x \in \Omega, \\ k \frac{\partial T}{\partial n} &= h\left(T_{ext} - T\right), && x \in \partial\Omega. \end{aligned}$$

The parameters here are the mass density ρ [kg/m^3], the specific heat C [J/(kg K)], the thermal conductivity k [W/(mK)] and the heat source Q [W/m^3]. Moreover, T_{ext} is the ambient temperature and h [W/(m^2K)] the convection heat transfer coefficient between the chip surface and surroundings.

We make some simplifying assumptions:

1. No heat transfer to the card ($h = 0$).
2. Constant heat transfer coefficient h on all the other sides.
3. Uniform heat source Q in the whole chip.
4. The chip is thin, $H \ll W$, $H \ll L$, and we shall study the impact of thinness. We use symmetry to compute on a quarter chip, $H \times (W/2) \times (L/2)$ part of the chip.

Effective material data for the silicon chip is $\rho_{\mathrm{Si}} = 2300$ [kg/m^3], $C_{\mathrm{Si}} = 750$ [J/(Kg K)], k_{Si} = 3.6 [W/mK], and for aluminum ρ_{Al} = 1600 [kg/m^3], C_{Al} = 900 [J/(kg K)], k_{Al} = 60 [W/mK].

Total power is P [W], ie. $Q = \frac{P}{V}$ [W/m^3] where V is the volume of the chip. The heat transfer coefficient h when the chip is cooled by a weak fan is h_{coef} (e.g. 100) [W/m^{2}K] and $T_{ext} = 20[^o\mathrm{C}]$.

3 Preparatory analysis

An important characteristic quantity is the *Biot number* $Bi = \frac{h}{k/d}$, the ratio of thermal resistance $1/h$ at the surface and d/k inside the chip itself. Here d is a characteristic length for heat transport through the chip which in our case would be H, the smallest dimension. When Bi is large, the surface temperature is close to T_{ext}, and when Bi is small, the temperature will be nearly uniform in the body.

1. What is the steady temperature when Bi (ie h) is small? Give a formula. Hint: All the power produced must be removed by the cooling, and the temperature is almost uniform.

2. Derive for large Bi the approximate value $T_{max} = H^2Q/(2k) + T_{ext}$. Is this perhaps an upper or lower limit?
   Hint: When $H/L \ll 1$ and $L \approx W$ the differential equation can be simplified to

$$0 = k\frac{d^2T}{dz^2} + Q, \qquad \frac{dT}{dz}|_{z=0} = 0, \qquad T(H) = T_{ext}$$

4 Lab session work – Task A

4.1 Chip

Model: Use 3D,
`COMSOL Multiphysics/PDE Modes/PDE, Coefficient Form / Time-dependent analysis`.
This will also do the steady calculation. It is worthwhile to define the material parameters as `Constants` under `Options`. Then you can easily change values etc, and the equations get to be in understandable terms.
Geometry: A brick! Give the right dimensions under `Properties`
PDE: Set the value of k and Q (named `c` and `f`) in the menu.
Boundary conditions: Neumann conditions – insulation on the bottom and symmetry-boundaries is obtained by `q = g = 0`. On the cooled surfaces, with `q = hcoef` and source term `g = hcoef*Text`, where `hcoef` and `Text` may first be defined as `Constant` under `Options`.
Solver: For steady case use `Stationary linear`, for transient runs : `Time Dependent`. Choose a finish time based on the calculated time-scale (see below).
Tasks:

1. Calculate the maximum temperature when
   P = 30 [W], h_{coef} = 20 [W/(m^{2}K)], $W = L = 20$ mm and $H = 2$ mm

2. Check your result by choosing `hcoef` to make Bi = 10 and 100 and comparing to the formula derived in the preparatory analysis (2.) above.

3. Do the mesh refinement study for the temperature of an upper corner. Try linear and quadratic elements.

4. Plot the temperature profiles and find maximum temperatures for $L = 20$ mm and $W =$ 10, 1, and 0.2 mm. Keep the power per unit volume Q constant. Explain why the max. temperature drops when W decreases.

4.2 Chip with cooling flanges

The temperature can be reduced by better cooling. Let the chip-octant be $10 \times 10 \times 2$ mm^3. Place a 1 mm thick aluminum cap on the chip, then 5 cooling fins 1 mm thick and 1 cm high, essentially as shown in Figure 2 (but higher fins!).
Geometry: First one creates the top as a brick and places it by the filling the right geometry data under `Draw/Object Properties`. Then make an outer flange, place it, then copy it and give the right translation in the dialog box (four times). Create the new `Composite Object` from the union of all the flanges and the lid without `Interior borders`. Finally, make an object composed of top, flanges, and chip, with retained `Interior borders`. That's it!
PDE: Fill in coefficients `c` and `f` in the two `Subdomains` (Al and Si).
Boundary conditions: Select `Neumann` for *all* boundaries, with heat transfer coefficient `q` and source term `g` like above. Then change the (few) symmetry-boundaries to insulation.
Tasks:

1. How much is the maximum temperature reduced by the cooling fins?

2. Is it worthwhile to make the flanges higher and / or thinner and more numerous?

A thought: Total surface area exposed to air is an important parameter in the model (see 1. in the preparatory analysis above). Then, maybe we could do with just a little Al, by making many densely packed thin flanges. But that is not how the actual chip cooler looks. Why? What is wrong with our model?

5 Lab session work – Task B

Now we return to the chip without cooling flanges and investigate transient warm-up and cool-down so you must use the time-dependent solver. Assume that the chip, after being on for a long time, is turned off at $t = 0$ so $Q = 0$ for $t > 0$. How does it cool off?

5.1 Preparatory analysis

The Newton cooling law, often used in crime fiction to determine time of death, is

$$\tau \frac{dT}{dt} = T_{ext} - T,$$

when Bi is small.

- What is the time-scale τ for the chip? Give a formula. Hint: Start from the PDE (1), integrate over the entire chip and use Gauss's theorem. Use that T is almost constant through the chip.

- Derive a similar model for heating when the chip is turned back on. Hint: It needs a source term.

5.2 Lab tasks

The question "Is it as fast to heat up as to cool down?" is imprecise. Please specify, observe (by computing) and justify/motivate.

Select `Time dependent` under `Solver/Solver Parameters`. Modify the coefficient `da` in the `Subdomain` window and add initial data under the tab `Init` in the same window. It is easiest to run a simulation starting with $T = T_{ext}$, turn on Q at $t = 0$ and shut off after so long time that the temperature has become (almost) stationary: $Q = (t < t_{end}) \times Q_0$. Compare your observed warm-up / cool-down times to the time-scale τ determined above.