# Comsol Laboration: Resistors and Diodes

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## 1 Physical configuration

We shall model the potential field of a thin conductive layer of copper foil (thickness $d$) in a plane, its resistance and the electric current flowing through it. The configuration is composed of three bricks put together as shown in Figure 1.

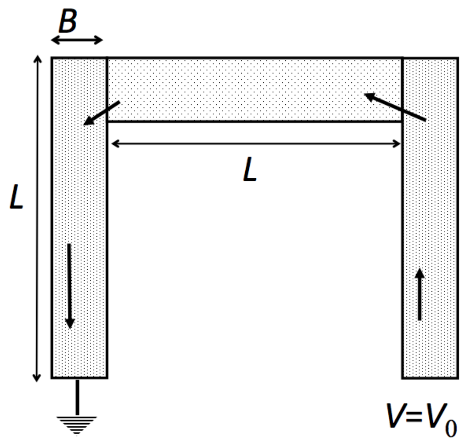



**Figure 1:** Geometry

We consider two cases:

- Same (isotropic) material in all three bricks.
- An orthotropic material in the middle brick. This turns the configuration into a rectifier.

## 2 Model

We consider a 2D model[1]. Ohm's law says that the current density $\mathbf{J}$, a vector with units [A/m] (in 2D), is related to the electrical potential $V(x, y)$ by

$$\mathbf{J} = -\mathbf{S} \cdot \nabla V = -\begin{pmatrix} S_x & 0 \\ 0 & S_y \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} \quad (1)$$

[1]All quantities constant in the $z$-direction.

where **S** is the conductivity-tensor for orthotropic materials with conductivity $S_x$ along the $x$-axis and $S_y$ along the $y$-axis. The conservation law for electric charge is then

$$\nabla \cdot \mathbf{J} = \frac{\partial}{\partial x}\left(S_x \frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(S_y \frac{\partial V}{\partial y}\right) = 0. \quad (2)$$

Associated boundary conditions are that the potential is zero ("earth"), or any given potential at the boundaries where we connect the configuration (i.e. the bottom two). On the other boundaries the current through the edges vanishes: $\mathbf{J} \cdot \mathbf{n} = 0$. Note that an isotropic material has $S_x = S_y$ and if the conductivity is constant in space, (2) becomes Laplace's equation.

For the computations below we use $L = 5$ [cm], $B = 1$ [cm], $d = 35$ [$\mu$m] and apply $V_0 = 5$ [V]. The (isotropic) conductivity of copper is $S = 6 \cdot 10^7$ [$(\Omega\text{m})^{-1}$].

# 3 Preparatory analysis

1. Make an estimate of the resistance of the configuration if the layer thickness is $d$ and the material has isotropic conductivity $S$. First calculate the resistance of a $B \times L \times d$ "brick" with the potential difference $V$ between the two $B \times d$ end surfaces. Then consider the three blocks as a series circuit to estimate the total resistance.

2. At the corners where the bricks meet (lower side) the solution to the Laplace equation has a singularity; the gradient, i.e. the current density is infinite. Let $(r, \theta)$ be polar coordinates centered at one of those corners and consider the infinite problem, when $L, B \to \infty$. Show that for certain values of $\alpha$ and $\beta$

$$V(r, \theta) = c_0 + c_1 r^\alpha \cos(\beta\theta) \quad (3)$$

is a solution to this Laplace equation (for all constants $c_0$, $c_1$) if we disregard boundary conditions on edges that do not touch the corner. Determine the smallest positive such $\alpha$ and the corresponding $\beta$.

# 4 Lab session work – Task A

## 4.1 Resistance in isotropic material

**Model:** In the `Model Navigator`, select 2D and
`COMSOL Multiphysics/PDE Modes/PDE, Coefficient Form/Stationary Analysis`

**Geometry:** Create three rectangles of appropriate size and then take the union of them under `Draw/Create Composite Object`
**Constants and expressions:** Define for instance $S_{\text{Cu}}$, $d$ and $V_0$ as constants under `Options/Constants`.
**PDE:** Set the conduction coefficient, named `c`, to $S_{\text{Cu}}$ and the right hand side, named `f`, to zero in all subdomains under `Physics/Subdomain Settings`.
**Boundary conditions:** Set all edges to `Neumann` with `q = g = 0` (the defaults) under `Physics/Boundary Settings`. Then change the bottom two boundaries to `Dirichlet` with `h`=1 and `r` equal to zero and $V_0$ to the left and right respectively.
**Tasks:**

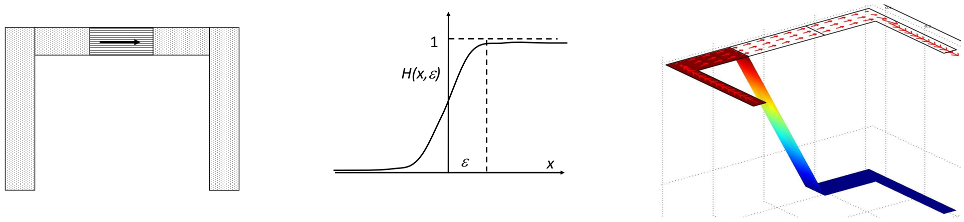
1. Plot the potential and the current density (as Arrow plot). Also plot $|\nabla V|$ and check that the solution is not smooth at the internal corners. (Zoom in to see this better.)

2. Use `Boundary integration` to calculate the total current $I$ as a line integral over the grounded edge, scaled with the thickness $d$,
$$I = d \int_D \mathbf{J} \cdot \mathbf{n} ds.$$
Then the resistance is $R = V_0/I$. Compare with your estimate in the preparatory analysis.

3. Refine the mesh repeatedly and show the convergence. How many elements are needed for $1\%$ error?

4. Try then repeated local refinement of the problem-corners; Experiment! How many elements are needed to obtain $1\%$ error?

5. Close to the corner (3) is a good approximation of the solution. It follows that the current density blows up as
$$|\mathbf{J}| = S|\nabla V| \sim r^{\alpha-1},$$
when we approach the corner. Use a solution with high accuracy and plot $|\nabla V|$ along a line shooting out from the corner. (With `Line plot` under `Cross-section Plot Parameters`.) Verify that the blow-up rate $\alpha - 1$ is correct.




(a) (b) (c)

**Figure 2:** a) Geometry, b) Smooth Heaviside function, c) Computed potential and current

## 4.2 Half wave rectifier

We now replace the middle part of the horizontal strip by an orthotropic material that conducts electricity only in the $x$-direction, and whose conductivity depends on field strength:
$$S_y = 0, \qquad S_x = s_{\mathrm{Lo}} + H(V_x, \epsilon)(s_{\mathrm{Hi}} - s_{\mathrm{Lo}}),$$
where $H$ is a smoothed Heaviside function and the notation $V_x$ is used for $\frac{\partial V}{\partial x}$, etc. See Figure 2. Obviously, if $s_{\mathrm{Lo}} < s_{\mathrm{Hi}}$ the material conducts current less easily when $\frac{dV}{dx} < 0$. When we take $s_{\mathrm{Lo}} \ll s_{\mathrm{Hi}}$ it becomes a diode that conducts left. We take $s_{\mathrm{Hi}} = S_{\mathrm{Cu}}$ and $s_{\mathrm{Lo}} = S_{\mathrm{Cu}}/100$.

**Geometry:** Replace the middle brick by three equally sized bricks. Create a composite object as before and make sure to keep the interior boundaries between the mid section and the rest.
**Constants and expressions:** First define the constants `sigLo` and `sigHi` for the low an high conductivity of the diode. Then define a `Global Expression` (under `Options/Expressions`) for the nonlinear diode conductivity as
`sigDiod = sigLo + flsmhs(ux, sc) * (sigHi-sigLo).`
Here `flsmhs` is a Comsol predefined function representing a smoothed Heaviside function of width `sc`. You can take `sc`=0.01 for instance.

**PDE:** Modify the coefficient `c` in the middle subdomain (under `Physics/Subdomain Settings`). Change from `Isotropic` to `Anisotropic diagonal` in the drop-down menu. Enter zero for $S_y$ and `sigDiod` for $S_x$.
**Boundary conditions:** As before.
**Tasks:**

1. Solve the problem and plot the potential and current density with Arrow as before. Note that it is now a non-linear problem! Try both $V_0 = 5$ and $V_0 = -5$. Verify that the diode (essentially) only conducts current in one of the cases.

2. Use the parametric solver to solve the problem for $V_0 = \sin(2\pi t)$ in the interval $0 < t < 3$. Plot the total current $I$ as a function of $t$. Animate the potential and current density!

   In Comsol:

   **Solver:** Select `Parameteric` under `Solve/Solver Parameters`). Choose `Parameter name` as e.g. `tt` (not `t` since it is predefined by Comsol!) and set the `Parameter values` to e.g. `range(0,0.01,3)`
   **Constants and expressions:** Remove `V0` from the list of constants and define it instead as a `Global Expression` with `V0=sin(2*pi*tt)`. Furthermore, define the variable `Itot` under `Options/Integration Coupling Variables/Boundary variable` such that it calculates the current passing through selected boundaries. (Use the default order 4 for the integration.) Then `Itot` can be plotted under `Postprocessing/Global Variables Plot`.

# 5 Lab session work – Task B

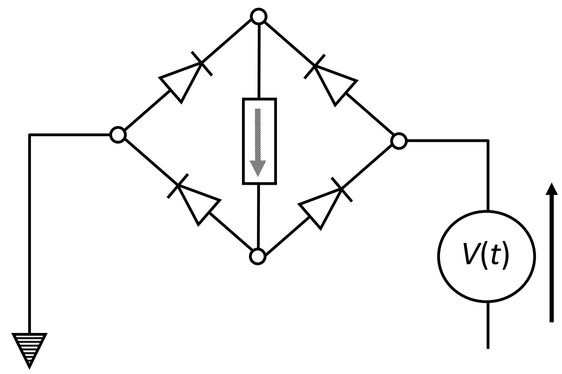



**Figure 3:** Full-wave rectifier

Build a full-wave rectified (four diodes) with a load as shown schematically in Figure 3. Calculate the current $I$ through the load, and plot $I$ as a function of $t$ so as to show that it works. It is customary to draw as shown in Figure 3, but perhaps easier to orient the diodes horizontally in the COMSOL model. But think about how the S-tensor looks for $45^o$ orthotropic diode material!