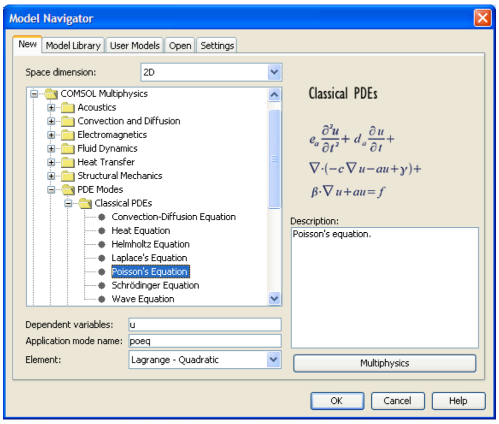
Numerical Solution of PDE: Comsol Multiphysics 3.5

This is an introduction to get you acquainted with Comsol Multiphysics for numerical solution of PDE by finite elements. The program has many facilities and we will use only a small fraction. The documentation is available (PDF and HTML) under the help button, **Help**, rightmost on the menu bar (top). The simplest is *Quick Start*, but even that is quite comprehensive. The GUI-functions are hopefully intuitive.
(Note: The new version COMSOL Multiphysics v. 4.0 has a completely re-designed GUI. These notes are <u>useless</u> for COMSOL 4.)

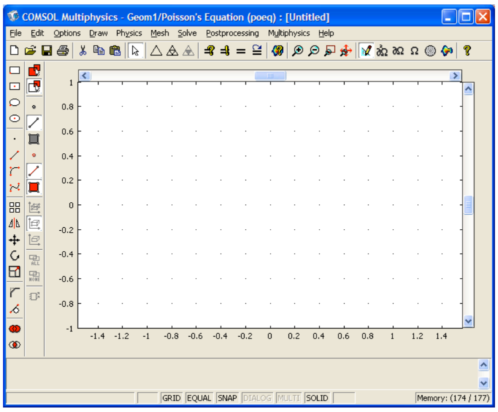
Example 1, The Poisson equation on an ellipse
Start comsol. In the first screen **`ModelNavigator`** we choose 2D and `ApplicationModes>ComsolMultiphysics>PDEModes>ClassicalPDEs>Poisson’s Equation.`
We see that the element type is **`Lagrange Quadratic`**-triangles. The solution is approximated with second degree polynomials, one over each triangle, a globally C^0 function. The elements can be changed later, if you wish. But the name of the solution (here **`u`**) must be changed here.



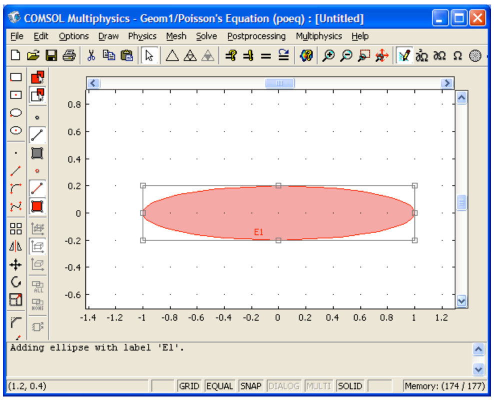
The next screen shows the work plane (2D …) and one usually works from left to right in the menu bar:

1. Create geometry (**`Draw`**)
2. Define the PDE (**`Physics`**)
3. Set boundary conditions (**`Physics`**)
4. Make the mesh (**`Mesh`**)
5. Solve (**`Solve`**)
6. Plot (**`Postprocessing`**)

Under **`File`** we find operations like **`open`**, **`save`**, **`save as`**, etc. **`Options`** gives facilities like defining constants and expressions. It is practical to have names for important model parameters like heat transfer coefficients, specific heat, etc.



Geometry
The geometry is built in a drawing program, from elementary bodies (2D Rectangles and ellipses), or surfaces bounded by Bezier-curves. The bodies are put together by set operations (union, intersection, difference). We make an ellipse, centered at the origin and half axes 1 and 0.2. Note that the objects are snapped to the grid, change the grid under **`Options>Axis/Grid Settings`** if needed. Our domain is only the single ellipse so we move on to

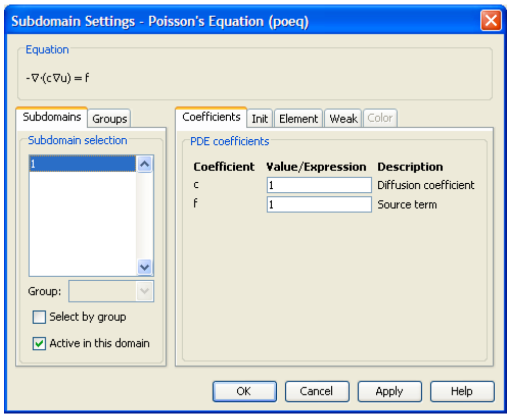


Define PDE

`Physics>SubdomainSettings` (there is `boundary` too, in a minute…)
The PDE is in the Equation pane,

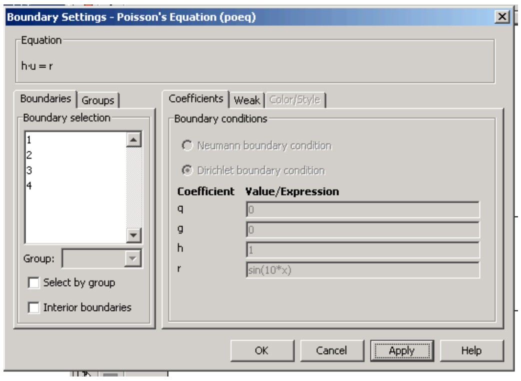
$$-\nabla \cdot (c\nabla u) = f$$

and we need to define the coefficient *c* and the source term *f*. One can give functions of *x,y, u, ux, uy* and, if time-dependent, *t*. There is only one subdomain, the ellipse, so click it and accept a $c = f = 1$ by OK.



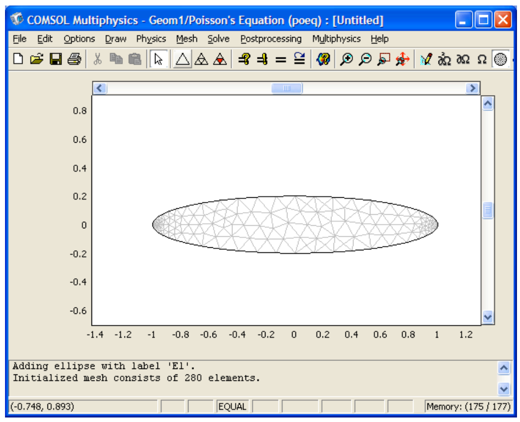
`Physics>BoundarySettings`

Each ellipse gives four boundaries. Choose them all by **`ctrl-a`**; They can be selected also by clicking (shift-clicking, ctrl-clicking) on the work pane. The default boundary condition is apparently Dirichlet: $h\ u = r$ with $h = 1$ and $r = 0$ and we change to $r = \sin(10x)$ for some variation, OK.



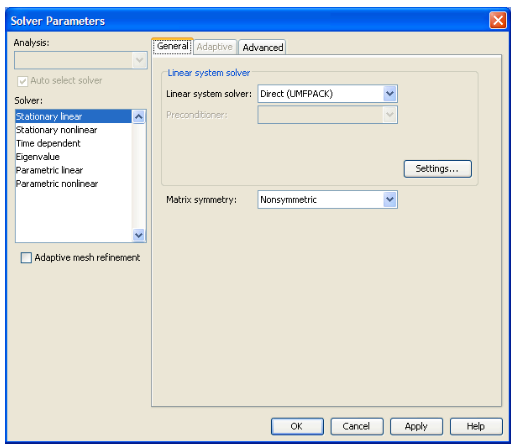
Element mesh

The triangle icon gives a mesh. It looks good, OK. The mesh can be controlled in some detail by parameters under the menu **`Mesh`**.



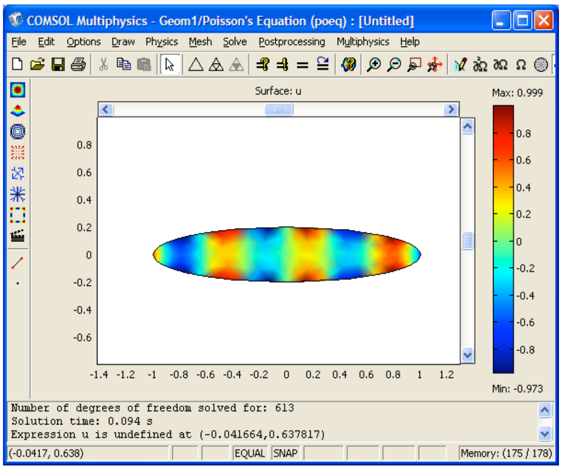
Solution

The generated equations are solved when you click the = icon. Under the menu choice **`Solve>SolverParameters`** there are various options offered. Details that can be very important for large (and non-linear) problems, like choice of solver for linear systems, are controllable here.



After a fraction of a second, the system with 485 unknowns has been constructed and solved and painted as colored iso-surfaces on the work pane, and the log-line (bottom of screen) says

```
Number of degrees of freedom
solved for: 485
Solution time: 0.019 s
```



Convergence study

To see how the solution converges with refinement of the mesh, we compute u in the center. Choose

`Postprocessing>DataDisplay>Subdomain`

where we give the coordinates for the center – (0,0) and see

```
Value: 0.019326, Expression: u, Position: (0,0)
```

in the log-line.

Make a regular refinement (each triangle split into four) by the four triangle icon, solve again, and print *u(center)* again:

```
Number of degrees of freedom solved for: 1833
Solution time: 0.029 s
Value: 0.019251, Expression: u, Position: (0,0)
```

and again, and again …, ... try up to, say, a million degrees of freedom when the solver runs out of memory. Note: the numbers may be different for your ellipse because the initial mesh may be different.

Convergence:

#dof	time	u(0.0)	diff.
485	0.031	0.019326	
1833	0.029	0.019251	0.000075
7121	0.072	0.019235	0.000016
28065	0.279	0.019231	0.000004

The order of convergence should be 2 for regular solutions in energy norm and 3 in max-norm. The data suggest just second order convergence pointwise, however. In general, the convergence with FEM in Comsol is *not* as regular as you have seen for e.g., the trapezoidal rule for quadrature, because the mesh is so irregular.

Now try piecewise linear basis functions instead. **`Lagrange Linear`** elements are chosen under **`Physics>SubdomainSettings>Element`**

Repeat the experiment: Larger errors but essentially the same second order convergence.

#dof	time	u(0.0)	diff.
135	0.023	0.016747	
485	0.026	0.018632	0.001885
1833	0.031	0.019079	0.000447
7121	0.069	0.019191	0.000112
28065	0.277	0.019220	0.000029
111425	1.305	0.019228	0.000008

Example 2 – Poisson's equation on a four-leaf clover
Start a new model by choosing **`File>New,`**Choose PDE like above: we want to solve the same equation but on a four-leaf clover.

Geometry
Create a circle. Copy it with **`ctrl-c`** (like in Windows), paste with **`ctrl-v`**, give the desired translation in the dialog box, paste twice more with suitable translations.
Create the union of the four disks in **`Draw>CreateCompositeObject, e.g.`** "**`select all`**" (union is the default-operation, as you see in the formula window), OK. Untick the option "**`Keep interior boundaries`**". The new object was named **`CO1`** (Composite Object 1).

Define PDE
`Physics>SubdomainSettings`
Again, only one subdomain, the unionen; we could have kept all the interior boundary curves from the four circles, but we chose (unconsciously) (default!) to remove them, so they are now gone. Select the only domain and accept $c = f = 1$ by OK.

Boundary conditions
`Physics>BoundarySettings`
Many boundaries, 8 in all; each circle gives two and the two inner were erased.
Select all by ctrl-a; The default condition is - obviously - Dirichlet: $r = \sin(10x)$ like above, OK.

Element Mesh
The Triangle-icon provides an initial mesh which looks good so OK.

Solution
Solve by = and after a second or so the color picture appears. We carry out the convergence study like above – possibly you have different coordinates, no matter,

```
#dof     tid     u(0,0)    diff.
  5057  0.422   -0.253078
 20033  1.469   -0.253353 0.000275
 79745  7.218   -0.253463 0.000110
318209 39.953   -0.253507 0.000044
```

Order of convergende somewhere between 1 and 2, DIFFERENT from the single ellipse. Why? Could it be that the solution is not as regular (smooth) as is required ? The sharp intruding corners are suspects. Plot |gradu| and it will show,
`PlotParameters>Surface>SurfaceData`, select **`|gradu|`**, OK maybe even more clearly in a 3D-graph,
`PlotParameters>Surface>HeightData,` select **`|gradu|`**, click the check box **`HeightData`**, OK.

One can show (by separation of variables) that the singularity is of the type $r^{\frac{\pi}{\alpha}}$ when the interior angle is α (here $3\pi/2$). That is enough to reduce the rate of convergence noticeably.