Lösning till övning 2 (2011-11-10)

1 Interpolation

- **Linear interpolation**

Pressure (MPa)	Temperature (°C)
6.0	275
7.0	285

Take the ansatz $f(x) = ax + b$ and we get

$$A = \begin{bmatrix} 6 & 1 \\ 7 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 275 \\ 285 \end{bmatrix}$$

Solving this equation we get $a = 10$, $b = 215$ and $f(6.3) = 278$.

- **Polynomial of degree at most n**
The linear system which has to be solved to determine the coefficients of the polynomial is

$$A = \begin{bmatrix} 1 & -3 & 9 & -27 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

- **Newton form**
The Newton form of the interpolating polynomial is

$$P(x) = a_0 + a_1(x+1) + a_2(x+1)(x)$$

The coefficients are calculated as follows:

$$\begin{aligned} P(-1) &= a_0 = 5 \\ P(0) &= 5 + a_1 = 1 \rightarrow a_1 = -4 \\ P(1) &= 5 - 4 \cdot (2) + a_2 \cdot 2 = 3 \rightarrow a_2 = 3 \\ P(x) &= 5 - 4(x+1) + 3(x+1)x \end{aligned}$$

To find the extremum we derive P and set it to 0

$$\begin{aligned} P'(x) &= -4 + 3x + 3(x+1) \\ P'(x) &= 0 \rightarrow x = \frac{1}{6} \end{aligned}$$

The extremum is $P(\frac{1}{6}) = \frac{11}{12}$.

- **Error in piecewise linear interpolation** (Bradie p.392 exercise 5.15.11, similar to ex. 5.15) To determine the largest error we use the following theorem (Bradie p.390)
  Theorem
  Let f be continuous, with two continuous derivatives, on the interval $[a, b]$ and let s be the piecewise linear interpolant of f relative to the partition

  $$a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$$

  Then

  $$max_{x \in [a,b]} |f(x) - s(x)| \leq \frac{1}{8} h^2 max_{x \in [a,b]} |f''(x)|$$

  where $h = max_{0 \leq i \leq n-1}(x_{i+1} - x_i)$.
  In our case $h = 0.01$ and $f = tan(x)$. Deriving the tangent functions twice gives

  $$tan''(x) = 2 \cdot tan(x)(1 + tan(x)^2)$$

  The max is achieved in $x = \pi/4$, $max_{x \in [0,\pi/4]} |tan''(x)| = 4$. Using all these information we can determine the upper bound of the error to be $5.0 \cdot 10^{-5}$.

2 Least square method (minstakvadratmetoden)

The coefficients c_1 and c_2 are found by calculating $A^T Ax = A^T b$ where

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad \text{and} b = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$$

We get $c_1 = \frac{47}{6}$ and $c_2 = \frac{-9}{6}$.