Lösning till övning 3 (2011-11-29)

1 ODE

- **Error for Euler's method** (Bradie p.558, 7.2.17)
  The global error associated with Euler's method is $O(h)$ i.e. $w_h(b) - y(b) = c \cdot h + O(h^2)$ where c is a constant.

$$w_h(b) - y(b) = c \cdot h + O(h^2) \quad (1)$$

$$w_{h/2}(b) - y(b) = c \cdot \frac{h}{2} + O(h^2) \quad (2)$$

  Using 1 and 2 we get

$$\frac{w_h(b) - w_{h/2}(b)}{w_{h/2}(b) - w_{h/4}(b)} = \frac{c\frac{h}{2} + O(h^2)}{c\frac{h}{4} + O(h^2)} \xrightarrow{h \to 0} 2$$

- **Code sketch** (Bradie p.558, 7.2.20)
  First write a program to solve the ODE with Euler's method as done in Lab 3, exercise 3.2. To show that the global error associated with Euler's method is $O(h)$ one has several options. For example:
  - Plot the difference between the approximation in the last point and the exact solution with *loglog* as done in Lab 3, exercise 3.2. The gradient should be one.
  - One could use the preceding exercise and look at $\frac{w_h(b)-w_{h/2}(b)}{w_{h/2}(b)-w_{h/4}(b)}$.
  - One could bisect h and compare the error of two consecutive calculations (one with h and one with $h/2$). The error should be reduced by $\frac{1}{2}$ as shown in 1 and 2.
- **High order equation** (Bradie p.633, 7.8.8)

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$$

$$\vec{u}' = \begin{pmatrix} u_1' \\ u_2' \\ u_3' \\ u_4' \end{pmatrix} = \begin{pmatrix} u_2 \\ u_2^2 + u_1 - sin(t) \\ u_4 \\ \sqrt{u_3} - tu_4 \end{pmatrix}$$

- **Exam example 08-12-17**