Övning 3 (2011-11-29)

1 ODE

- **Error for Euler's method** (Bradie p.558, 7.2.17)
  Suppose we use Euler's method to approximate the solution of the initial value problem $y' = f(t, y)$, $y(a) = \alpha$ over the interval $a \leq x \leq b$. Let $w_h(b)$ denote the approximation to $y(b)$ obtained with a step size of h. Since the global error associated with Euler's method is $O(h)$, toward what value do we expect the expression

$$\frac{w_h(b) - w_{h/2}(b)}{w_{h/2}(b) - w_{h/4}(b)}$$

  to converge as h is reduced?

- **Code sketch** (Bradie p.558, 7.2.20)
  Sketch a program i MATLAB which confirms that the global error associated with Euler's method is $O(h)$.

$$\begin{aligned} x' &= 1 - x + e^{2t}x^2 \quad 0 \leq t \leq 0.9 \\ x(0) &= 0 \end{aligned}$$

  The exact function is $x(t) = e^{-t}tan(e^t - 1)$.

- **High order equation** (Bradie p.633, 7.8.8)
  Convert the following to a system of first-order differential equations.

$$x'' = (x')^2 + x - sin(t), \quad y'' = \sqrt{y} - ty' \tag{1}$$
$$\tag{2}$$

- **Exam example 08-12-17**
  The functions $u(t)$ and $v(t)$ satisfy

$$u'' + v^2 u = sin(t), \quad v'' + u^2 v = sin(2t) \tag{3}$$
$$u(0) = 1, \quad u'(0) = 0.2 \quad , v(0) = -0.2, \quad v(0)' = 0.5 \tag{4}$$

  First, write a MATLAB program that calculates the solution on $0 \leq t \leq 2$. The program should plot $u(t)$ and $v(t)$ for $0 \leq t \leq 2$ and write out $u(2)$ and $v(2)$.
  Complete the program so that

$$\int_0^2 u(t)^2 + v(t)^2 dt$$

  is calculated as well.