Cables and Reels

Comsol Laboration

DN1240, HT12/VT13

1 Physical configuration

We consider a cable that connects a power outlet to a heating element. We want to find out how much warmer the cable is than the ambient temperature, and where it is warmest. The cable is a RVOE-twin conductor-cable with a cross section as shown in Figure 1, top. There are two layers of plastic; the inner one is t (= 0.85) mm thick. The area of the copper conductors A is 1.5 mm^2 each and the outer diameter of the cable is D_y (= 9 mm).

In particular we want to compare the cable temperatures in two cases:

1. The cable hangs in free air.
2. The cable is very carefully wound into a reel as in Figure 1, bottom, with $D_d = 100$ mm.

The specifications for the cable says it can support a higher electrical load in the first case than in the second case. The difference in temperatures should explain why.

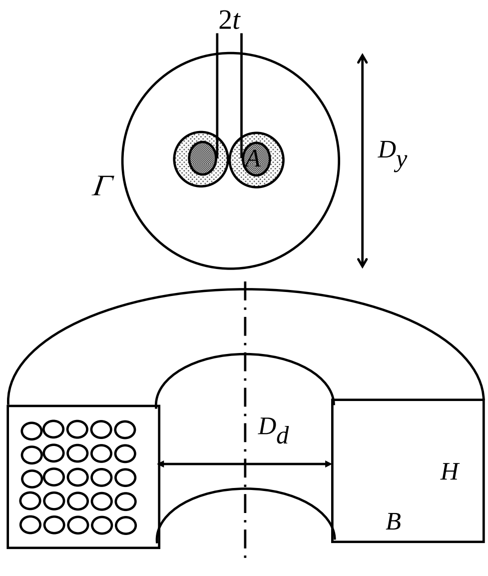

Figure 1: Top, cable cross section, Bottom, reel geometry

2 Model

The mathematical model is a boundary value problem for the steady heat equation,

$$0 = \nabla \cdot (k\nabla T) + Q \text{ in } \Omega,$$
$$k\frac{\partial T}{\partial n} = h\,(T_{\text{ext}} - T) \text{ on } \Gamma. \tag{1}$$

The domain Ω with boundary Γ is the interior of the cable in the first case, and the whole reel in the second case. The parameters are the heat conductivity k [W/(mK)], the heating power per volume Q [W/m^3], the heat transfer coefficient h [W/m^2/K] (determines the cooling rate) and the ambient temperature T_{ext} [°C]. Note that k and Q are constant with respect to time and temperature, but varies in space.
For this example we take $h = 10$ (free air cooling) and $T_{\text{ext}} = 20$. As for the remaining parameters, the following data may be useful: *Copper* has electric conductivity σ_{Cu} of 5.8 10^7 $[\Omega\text{m}]^{-1}$ and heat conductivity $k_{\text{Cu}} = 380$ [W/(mK)]. *Plastic*: no electric conductivity (so $Q = 0$ in the plastic) and heat conductivity $k_{\text{Pl}} = 0.2$ [W/(mK)]. The two kinds of plastic are assumed identical (except for color . . .), so no boundary between them is needed.

3 Preparatory analysis

1. Show that the heating power per volume Q in the copper (to leading order) is $Q = \left(\frac{P}{UA}\right)^2 \frac{1}{\sigma_{\text{Cu}}}$ where U is the voltage $[V]$ of the power outlet and P is the power $[W]$ of the heating element. You may follow the steps
   - What is the total current to give total power P?
   - What is the resistance in l m of the Cu-conductor?
   - What is the potential drop? And the power?

   (We assume a purely resistive load, and that U^2/P is large compared to the cable resistance.)

2. The boundary Γ has arithmetic mean temperature
$$T_{\text{mean}} = \frac{1}{|\Gamma|}\int_\Gamma T\,d\Gamma = T_{\text{ext}} + \frac{2AQ}{hp} \tag{2}$$
   where $|\Gamma| = p$ is the length of the perimeter of the cable cross section. Why is it so?

3. The temperature in the copper has very small gradients compared to the plastic. Quantify this statement, close to the Cu-Pl interface.
   Hint: Which quantity is continuous across the interface?

4. Suppose that the two Cu-regions can be replaced by one central circular disk Cu with area $2A$, radius R_i. Then, the equation for T in the plastic becomes
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0, \quad T(D_y/2) = T_{\text{mean}} \tag{3}$$
   Show (3) and explain why
$$T(r) = C_1 \ln r + C_2.$$
   and
$$2\pi R_i k_{Pl}\left.\frac{dT}{dr}\right|_{r=R_i} = 2AQ.$$
   Determine C_1, C_2 and the temperature at the Cu-Pl interface.

4 Lab session work – Task 1

4.1 Free cable

We consider the 2D case and assume all variables are constant in the z-direction along the cable. The domain Ω is then a cross section of the cable as in Figure 1.
Model: Use 2D, mode
`COMSOL Multiphysics/PDE Modes/PDE, Coefficient form/Stationary analysis`,
and the pre-selected quadratic polynomial elements.
Geometry: The union of one large and two small circles gives the desired computational domain: three sub-domains, two for Cu and one for Pl.
PDE: Define `P, U, A, sigCu, ..., Q` as constants. It is possible to write expressions, like for Q, enter the expressions under `Options/Constants`.
Boundary conditions: Neumann (Robin) type along the outer boundary, with specified heat transfer coefficient.
Tasks:

1. Compute and plot the temperature field when the voltage is $U = 230$ [V] and the power of the heating element is $P = 2000$ [W]. Record the maximum temperature in the cable.

2. Make an initial mesh, refine the triangulation (at least) twice and note the max-temperatures for the three calculations. What is the order of accuracy/convergence? Is the use of max-temperature a good way to assess order of accuracy? Are there singularities in the solution (temperature field)?

3. Change to linear elements. What is the order now?

4. Check the mean temperature formula (2) above by `Boundary integration`.

4.2 Cable on a reel

For the reel case we assume radial symmetry and let Ω be a 2D cross-section of the reel. We use the axisymmetric version of the PDE. Moreover, for simplicity we let the plastic fill up the whole space between the copper conductors inside the reel.
Model: Use `Axisymmetric (2D)`, mode and then choose
`COMSOL Multiphysics/Heat Transfer/Conduction/Steady-state analysis`.
We model one half (symmetry) of a cross section of the reel with $5 \times 3 \times$(2 Cu-circles) packed so the distance between the 2-groups becomes 10 mm. The whole pattern is placed in a rectangle whose smallest distance to the circles is 2 mm everywhere. Then fill the space between the Cu-circles with plastic, and cool the outer boundaries like above.
Geometry: There is an "array"-constructor (icon with four small squares) which makes $n \times m$ copies in an $n \times m$ pattern. Or manually: First make one Cu-circle, then its neighbor in the two-group. Form from them a composite object, say G. Copy and translate to make a row. Make a composite object from these 5 G, say R. Copy and translate R to make three rows. Make a composite object, say Cu, from the three R. Create the rectangle R1, form the union R1+Cu - Note: Keep interior borders!
PDE: There are 31 subdomains. But 30 of them have the same data, so select all 31 and give them Cu-properties and heat source. Then change the data in the (single) plastic domain.
Boundary conditions: The three outer boundaries are as in 4.1, with the same transfer coefficients. The symmetry plane is insulated (no heat flux).

Tasks:

1. Compute and plot the temperature field with the same voltage and power as above. Record the maximum temperature in the cable reel. Also try a color 3D plot and visualize the heat flux with streamlines (with many start points). Where are the temperature gradients strongest? Plot $|\nabla T|$ to show.

2. Suppose the cable melts at 100 °C. What is then (approximately) the maximum electrical load P allowed according to our models for the free hanging cable and the cable on the reel, respectively?

5 Lab session work – Task 2

5.1 Free cable

As we saw above, the temperature gradient in the copper is very small compared to the one in the plastic. A model where the copper has constant (but unknown) temperature should be quite accurate. This temperature would then come in as a Dirichlet boundary condition in a model for only the plastic. We denote this approximation by $\tilde{T}$. The model would look like:

$$\begin{aligned} \nabla \cdot \left(k\nabla\tilde{T}\right) &= 0 \text{ in } \tilde{\Omega}, \\ k\frac{\partial\tilde{T}}{\partial n} &= h\left(T_{\text{ext}} - \tilde{T}\right) \text{ on } \Gamma, \\ \tilde{T} &= T_{\text{Cu}} \text{ on } \tilde{\Gamma}, \end{aligned}$$

where $\tilde{\Omega}$ is the plastic subset of Ω and $\tilde{\Gamma}$ is the boundary between the plastic and the copper. The value T_{Cu} corresponds to the "constant" temperature in the copper. It must be determined so the correct total heat flows through the plastic,

$$\int_\Gamma k\frac{\partial\tilde{T}}{\partial n}d\Gamma = \int_\Gamma k\frac{\partial T}{\partial n}d\Gamma.$$

How is this done?
Hint: The solution $\tilde{T}$ is an affine function of $T_{\text{Cu}} - T_{\text{ext}}$,

$$\tilde{T}(x;T_{\text{Cu}}) = T_{\text{ext}} + [T_{\text{Cu}} - T_{\text{ext}}]\tilde{T}_1(x), \tag{4}$$

for some function $\tilde{T}_1(x)$ independent of T_{Cu} and T_{ext}. From the requirement on the heat flow through the plastic it then follows that T_{Cu} should be an affine function of the external temperature and the power per volume,

$$T_{\text{Cu}} = T_{\text{ext}} + KQ. \tag{5}$$

Tasks:

1. Verify (4) and (5). Derive a theoretical expression for K in terms of $\tilde{T}_1(x)$ and parameters.

2. Compute K using COMSOL.

3. The relationship (5) should hold also (approximately) for the full model (1) with T_{Cu} taken as a temperature value inside the copper. Verify this empirically by using the COMSOL parametric solver (under `Solve/Solver Parameters`) with `Q` as parameter. Determine K this way and compare with 2 above.

5.2 Reel cross section

Where should the element mesh be refined? Try adaptive refinement (select under solver parameters) and see if you agree with the algorithm.