Heat Conduction in a Chip

Comsol Laboration

DN1240, HT12/VT13

1 Physical configuration

A chip on a circuit board is heated inside and cooled by convection by the surrounding air. The stand-alone chip has dimensions $H \times W \times L$ as indicated in Figure 1. We wish to investigate:

- What is the steady state temperature distribution, and how long does it take to reach steady state?
- How do the chip dimensions influence the maximum temperature?
- How does the temperature change when cooling flanges are added, as in Figure 2?

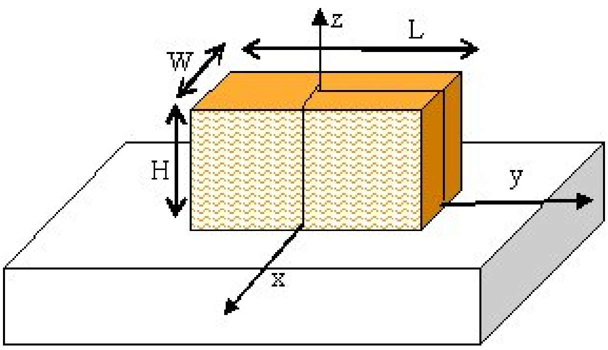

Figure 1: Chip geometry

2 Model

The mathematical model is an initial-boundary value problem for the heat equation, to determine $T(x, y, z, t)$ for $t > 0$ and (x, y, z) inside the chip, denoted Ω, when

$$\begin{aligned} \rho C \frac{\partial T}{\partial t} &= \nabla \cdot (k \nabla T) + Q, & (x, y, z) &\in \Omega, \\ k \frac{\partial T}{\partial n} &= h\,(T_{ext} - T) & (x, y, z) &\in \partial\Omega, \end{aligned} \tag{1}$$

with initial data $T(x, y, z, 0) = f(x, y, z)$. In particular, the steady state temperature obtained after long time is of interest,

$$\begin{aligned} 0 &= \nabla \cdot (k \nabla T) + Q, & (x, y, z) &\in \Omega, \\ k \frac{\partial T}{\partial n} &= h\,(T_{ext} - T)\,, & (x, y, z) &\in \partial\Omega. \end{aligned}$$

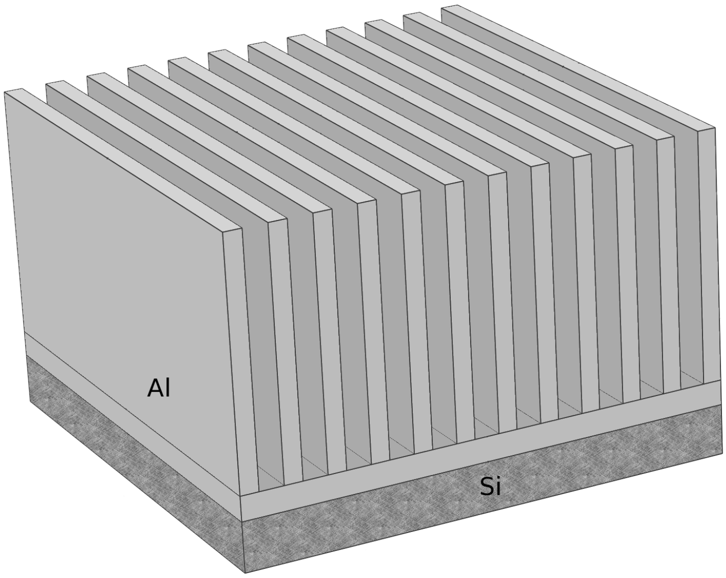

Figure 2: Chip with cooling fins

The parameters here are the mass density ρ [kg/m^3], the specific heat C [J/(kg K)], the thermal conductivity k [W/(mK)] and the heat source Q [W/m^3]. Moreover, T_{ext} is the ambient temperature and h [W/(m^2K)] the convection heat transfer coefficient between the chip surface and surroundings.

We make some simplifying assumptions:

1. No heat transfer to the card ($h = 0$).
2. Constant heat transfer coefficient h on all the other sides.
3. Uniform heat source Q in the whole chip.

Effective material data for the silicon chip is $\rho_{\mathrm{Si}} = 2300$ [kg/m^3], $C_{\mathrm{Si}} = 750$ [J/(Kg K)], k_{Si} = 3.6 [W/mK], and for aluminum ρ_{Al} = 1600 [kg/m^3], C_{Al} = 900 [J/(kg K)], k_{Al} = 60 [W/mK]. Total power is P [W], i.e. $Q = P/V$ [W/m^3] where V is the volume of the chip. The heat transfer coefficient is $h = 100$ [W/m^{2}K] when the chip is cooled by a fan and $T_{ext} = 20$ [°C].

3 Preparatory analysis

An important characteristic quantity is the *Biot number* $Bi = dh/k$, the ratio of thermal resistance inside the chip (d/k) and thermal resistance at the surface ($1/h$). Here d is a characteristic length for heat transport through the chip which in our case would be H, the smallest dimension. When Bi is large, the surface temperature is close to T_{ext}, and when Bi is small, the temperature will be nearly uniform in the body.

1. What is the steady temperature when Bi (ie h) is small? Give a formula.
   Hint: All the power produced must be removed by the cooling, and the temperature is almost uniform.
2. Assume that the chip is thin, $H \ll W$, $H \ll L$, and derive for large Bi the approximate value $T_{max} = H^2Q/(2k) + T_{ext}$. Is this perhaps an upper or lower limit?

Hint: When the chip is thin the temperature changes slowly in the x- and y-directions so the differential equation can be simplified to

$$0 = k\frac{d^2T}{dz^2} + Q, \qquad \left.\frac{dT}{dz}\right|_{z=0} = 0, \qquad T(H) = T_{ext}.$$

4 Lab session work – Task 1

4.1 Chip

Model: Use 3D,
`COMSOL Multiphysics/PDE Modes/PDE, Coefficient Form / Time-dependent analysis`.

This will also do the steady calculation. It is worthwhile to define the material parameters as `Constants` under `Options`. Then you can easily change values etc, and the equations get to be in understandable terms.
Geometry: A brick! Give the right dimensions under `Properties` Note that to reduce the problem further you can use symmetry and compute only on a quarter chip, $H \times (W/2) \times (L/2)$. Symmetry boundaries should then have Neumann boundary conditions $\partial T/\partial n = 0$.
PDE: Set the value of k and Q (named `c` and `f`) in the menu.
Boundary conditions: Neumann conditions – insulation on the bottom and symmetry-boundaries is obtained by `q = g = 0`. On the cooled surfaces, with `q = hcoef` and source term `g = hcoef*Text`, where `hcoef` and `Text` may first be defined as `Constant` under `Options`.
Solver: For steady case use `Stationary linear`, for transient runs : `Time Dependent`. Choose a finish time based on the calculated time-scale (see below).
Tasks:

1. Calculate the maximum temperature when
   P = 30 [W], h_{coef} = 100 [W/(m^{2}K)], $W = L =$ 20 mm and $H =$ 2 mm

2. Check your result by choosing `hcoef` to make Bi = 10 and 100 and comparing to the formula derived in the preparatory analysis (2) above.

3. Do a mesh refinement study for the temperature of an upper corner. Try linear and quadratic elements.

4. Plot the temperature profiles and find maximum temperatures for $L =$ 20 mm and $W =$ 10, 1, and 0.2 mm. Keep the power per unit volume Q constant. Explain why the maximum temperature drops when W decreases.

4.2 Chip with cooling flanges

The temperature can be reduced by better cooling. Use the same dimensions as above but now place a 1 mm thick aluminum cap on the chip, and then 12 cooling fins 0.75 mm thick and 1 cm high, with a 1 mm spacing between them, as shown in Figure 2. You can still use the symmetry trick as in Section 4.1; the quarter chip will be $10 \times 10 \times 2$ mm^3 and include 6 fins.
Geometry: First one creates the top as a brick and places it by the filling the right geometry data under `Draw/Object Properties`. Then make an outer flange, place it, then copy it and give the right translation in the dialog box (five times). Create the new `Composite Object` from the union of all the flanges and the lid without `Interior borders`. Finally, make an object composed of top, flanges, and chip, with retained `Interior borders`. That's it!

PDE: Fill in coefficients `c` and `f` in the two `Subdomains` (Al and Si).
Boundary conditions: Select `Neumann` for *all* boundaries, with heat transfer coefficient `q` and source term `g` like above. Then change the (few) symmetry-boundaries to insulation.
Tasks:

1. How much is the maximum temperature reduced by the cooling fins?

2. Is it worthwhile to make the flanges higher and / or thinner and more numerous?

A thought: Total surface area exposed to air is an important parameter in the model (see (1) in the preparatory analysis above). Then, maybe we could do with just a little Al, by making many densely packed thin flanges. But that is not how the actual chip cooler looks. Why? What is wrong with our model?

5 Lab session work – Task 2

Now we return to the chip without cooling flanges and investigate transient warm-up and cool-down so you must use the time-dependent solver. Assume that the chip, after being on for a long time, is turned off at $t = 0$ so $Q = 0$ for $t > 0$. How does it cool off?

5.1 Preparatory analysis

The Newton cooling law, often used in crime fiction to determine time of death, is

$$\tau \frac{dT}{dt} = T_{ext} - T, \tag{2}$$

when Bi is small.

- What is the time-scale τ for the chip? Give a formula.
  Hint: Start from the PDE (1), integrate over the entire chip and use Gauss's theorem. Exploit the fact that T is almost constant through the chip.

- Derive a similar model for heating when the chip is turned back on.
  Hint: It needs a source term.

5.2 Lab tasks

The question "Is it as fast to heat up as to cool down?" is imprecise. Please specify, observe (by computing) and justify/motivate.

Select `Time dependent` under `Solver/Solver Parameters`. Modify the coefficient `da` in the `Subdomain` window and add initial data under the tab `Init` in the same window. It is easiest to run a simulation starting with $T = T_{ext}$, turn on Q at $t = 0$ and shut off after so long time that the temperature has become (almost) stationary: $Q = (t < t_{end}) \times Q_0$. Plot the temperature as a function of time. Solve analytically the ODE in (2) and the similar one for heating. Compare those solutions with your observed warm-up / cool-down curves.

Optional question: Suppose there is a control system for the chip fan that increases the fan power when the chip temperature rises, and vice versa. Model this by letting h depend on T, for instance linearly: $h = \alpha + \beta T$. How should one pick α, β to obtain a steady state temperature of ca 50 [°C]?