F10

- Partiella differentialekvationer

Exempel

Black-Scholes-Merton för prissättning av optioner $V(x,t)$:

x: pris på underliggande papper (aktie, …)

σ: volatilitet

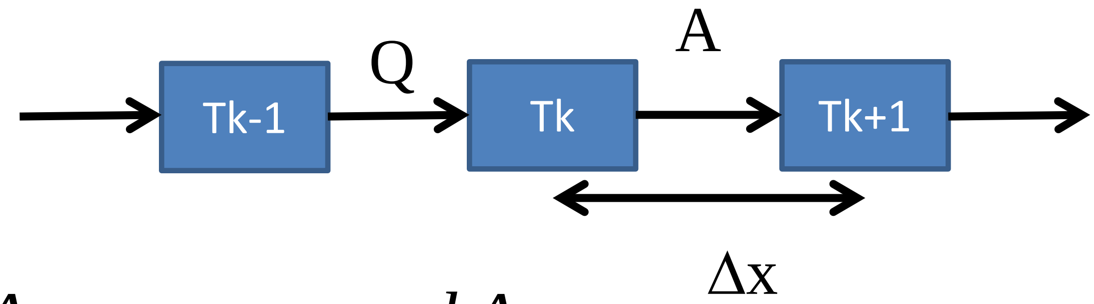
r: riskfri ränta

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V}{\partial x^2} + xr\frac{\partial V}{\partial x} - rV = 0$$

PDE, I: Begynnelsevärdesproblem

Värmeledning enligt

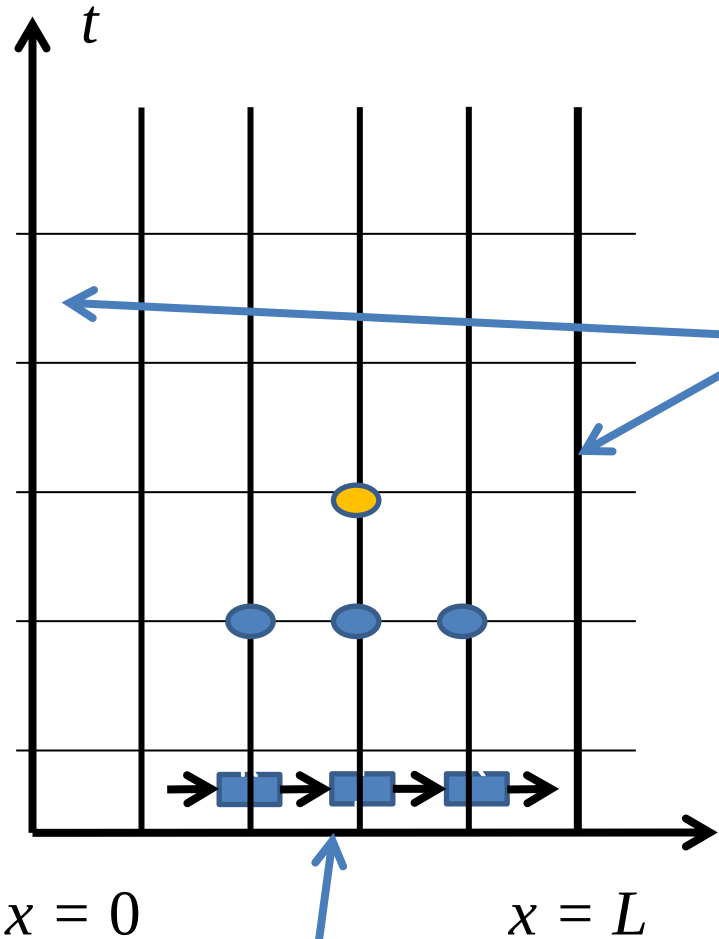
Fourier: Q = - k dT/dx



$$\underbrace{\rho A \Delta x C}_{m} \frac{dT_k}{dt} = -\frac{kA}{\Delta x}(T_k - T_{k-1}) + \frac{kA}{\Delta x}(T_{k+1} - T_k)$$

$$\rho C \frac{dT_k}{dt} = k \frac{T_{k+1} - 2T_k + T_{k-1}}{\Delta x^2} \Rightarrow \frac{\partial T(x,t)}{\partial t} = \frac{k}{\rho C} \frac{\partial^2 T(x,t)}{\partial x^2}$$

PDE, II



Rand-villkor

$T(0,t) = T\mathrm{L},\ T(L,t) = \underline{T}\mathrm{R}$

$$\frac{\partial T(x,t)}{\partial t} = \frac{k}{\rho C}\frac{\partial^2 T(x,t)}{\partial x^2} \Rightarrow$$

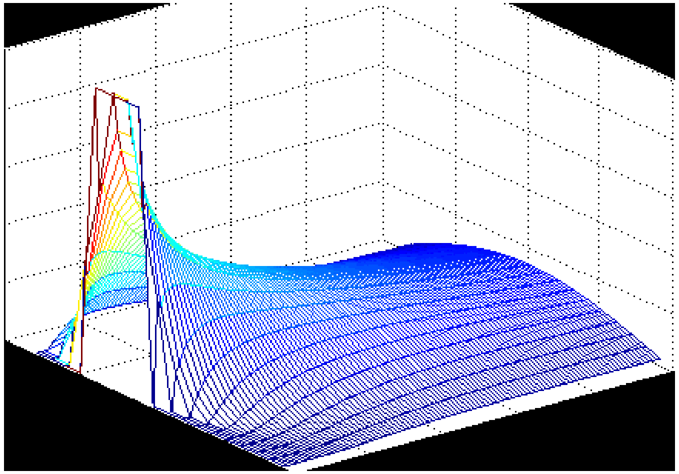
$$T_k^{n+1} = T_k^n + \frac{k\Delta t}{\rho C \Delta x^2}(T_{k-1}^n - 2T_k^n + T_{k+1}^n)$$

Begynnelse-villkor:

$T(x,0) = f(x)$

PDE, III

```
n = 20;
x = linspace(0,1,n);
dx = x(2)-x(1); dx2 = dx*dx;
TL = 0; TR = 0;
T  = zeros(n-2,1);
T(x > 0.5 & x < 0.7) = 1;
dt = 0.001;
Ttab = zeros(n-2,100); Ttab(:,1)=T;
for n = 1:99
Ttab(:,n+1) = Ttab(:,n) + …
dt*diff(diff([TL;Ttab(:,n);TR]))/dx2;
end
mesh(Ttab)
```



Prova större dt

PDE, IV

Taylorutveckling: Lokalt fel $0(\Delta t^2 + \Delta x^2 \Delta t)$,

Globalt … $0(\Delta t + \Delta x^2)$, om metoden är ***stabil:***

Begränsad felfortplantning (”ränta på gamla fel”)

$$T_k^{n+1} = (1-2\lambda)T_k^n + \lambda(T_{k-1}^n + T_{k+1}^n), \lambda = \frac{k\Delta t}{\rho C \Delta x^2}$$

$$|T_k^{n+1}| \le (1-2\lambda)|T_k^n| + \lambda(|T_{k-1}^n| + |T_{k+1}^n|)$$

$$\le (1-2\lambda+\lambda+\lambda)\max_j |T_j^n| = \max_j |T_j^n|$$

om $\boldsymbol{\lambda}$ **< ½:** **Kräver Δt prop. mot Δx^2**

PDE, V

Baklänges Euler istället

$$(1+2\lambda)T_k^{n+1} - \lambda(T_{k-1}^{n+1} + T_{k+1}^{n+1}) = T_k^n$$

Stabil (i L^2) för alla $\lambda > 0$ men kräver lösning av stort ekvationssystem i varje steg

MATLAB: **`ode15s`** utvecklad just för sådana problem.

Formulera som $\frac{d}{dt}\mathbf{T} = \mathbf{AT} + \mathbf{b}$

$$\mathbf{A} = \frac{k}{\rho C \Delta x^2}\begin{pmatrix} -2 & 1 & 0 & \dots \\ 1 & -2 & 1 & \dots \\ 0 & 1 & -2 & \dots \\ 0 & 0 & 1 & \dots \end{pmatrix}, \mathbf{b} = \frac{k}{\rho C \Delta x^2}\begin{pmatrix} TL \\ 0 \\ \dots \\ TR \end{pmatrix}$$

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Gamla tentor

- Q&A