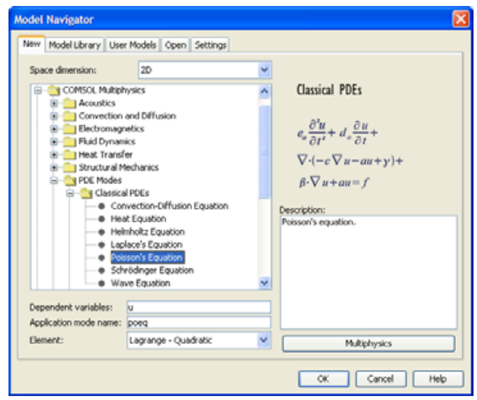
Numerical Solution of PDE: Comsol Multiphysics 3.5

This is an introduction to get you acquainted with Comsol Multiphysics for numerical solution of PDE by finite elements. The program has many facilities and we will use only a small fraction. The documentation is available (PDF and HTML) under the help button, **Help**, rightmost on the menu bar (top). The simplest is *Quick Start*, but even that is quite comprehensive. The GUI-functions are hopefully intuitive.

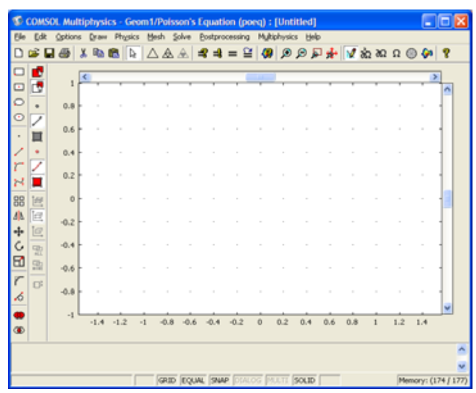
Note added 101030:
The new version COMSOL Multiphysics v. 4.0 has a completely re-designed GUI. These notes are <u>useless</u> for COMSOL 4. We may change to COMSOL 4 only in the not-so-near future.

Example 1, The Poisson equation on an ellipse
Start comsol. In the first screen **`ModelNavigator`** we choose 2D and `ApplicationModes>ComsolMultiphysics>PDEModes>ClassicalPDEs>Poisson's Equation.`
We see that the element type is **`Lagrange P2-`** triangles. The solution is approximated with second degree polynomials, one over each triangle, a globally C^0 function. The elements can be changed later, if you wish. But the name of the solution (here **`u`**) must be changed here.

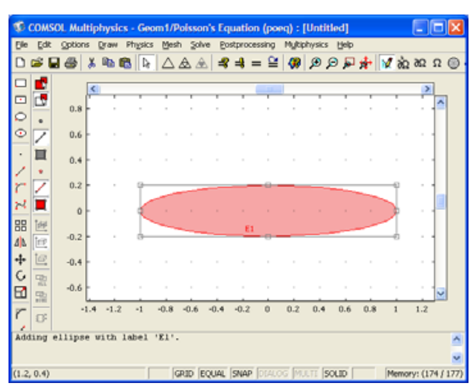


The next screen shows the work plane (2D …) and one usually works from left to right in the menu bar:
1. Create geometry (**`Draw`**)
2. Define the PDE (**`Physics`**)
3. Set boundary conditions (**`Physics`**)
4. Make the mesh (**`Mesh`**)
5. Solve (**`Solve`**)
6. Plot (**`Postprocessing`**)

Under **`File`** we find operations like **`open, save, save as`**, etc. **`Options`** gives facilities like defining constants and expressions. It is practical to have names for important model parameters like heat transfer coefficients, specific heat, etc.



Geometry
The geometry is built in a drawing program, from elementary bodies (2D Rectangles and ellipses), or surfaces bounded by Bezier-curves. The bodies are put together by set operations (union, intersection, difference). We make an ellipse, centered at the origin and half axes 1 and 0.2. Note that the objects are snapped to the grid, change the grid under **`Options>GridSettings`** if needed. Our domain is only the single ellipse so we move on to

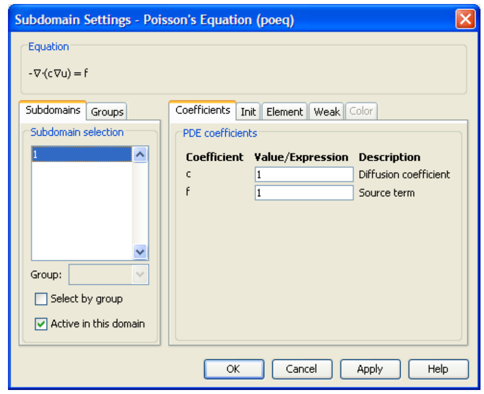


Define PDE

`Physics>SubdomainSettings` (there is `boundary` too, in a minute…)
The PDE is in the Equation pane,

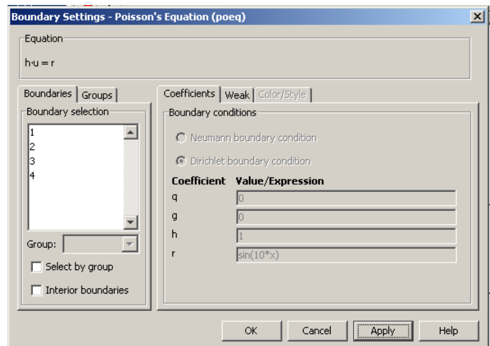
$$-\nabla \cdot (c \nabla u) = f$$

and we need to define the coefficient c and the source term f. One can give functions of x,y, u, ux, uy and, if time-dependent, t. There is only one subdomain, the ellipse, so click it and accept a $c = f = 1$ by OK.



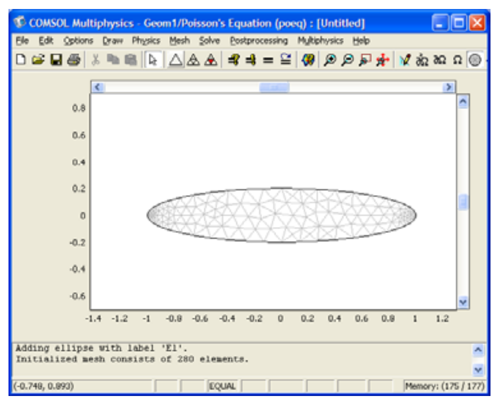
`Physics>BoundarySettings`

Each ellipse gives four boundaries. Choose them all by **`ctrl-a`**; They can be selected also by clicking (shift-clicking, ctrl-clicking) on the work pane. The default boundary condition is apparently Dirichlet: $h\ u = r$ with $h = 1$ and $r = 0$ and we change to $r = \sin(10x)$ for some variation, OK.



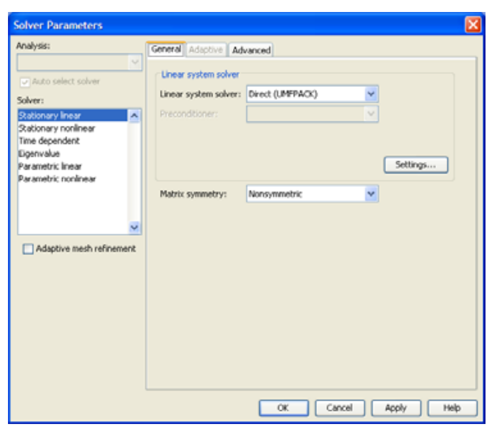
Element mesh

The triangle icon gives a mesh. It looks good, OK. The mesh can be controlled in some detail by parameters under
`Mesh>MeshParameters` .



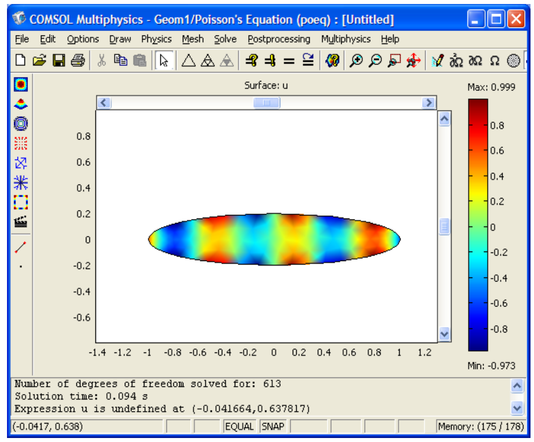
Solution

The generated equations are solved when you click the = icon. Scroll down to **`Solve>SolverParameters`** and check the possibilities offered. Our choice of c, f and boundary conditions makes the problem linear so the default selection **`stationary linear`** is correct, OK. Details that can be very important for large (and non-linear) problems, like choice of solver for linear systems, are controllable here. It says **`Unsymmetric`** about the matrix – whereas it is in fact symmetric – but never mind, OK, and solve by =.



After a fraction of a second, the system with 613 unknowns has been constructed and solved and painted as colored iso-surfaces on the work pane, and the log-line (bottom of screen) says

```
Number of degrees of freedom
solved for: 613
Solution time: 0.093 s
```



Convergence study

To see how the solution converges with refinement of the mesh, we compute *u* in the center. Choose

`Postprocessing>DataDisplay>Subdomain`

where we give the coordinates for the center – (0,0) and see

```
Value: 0.020201, Expression: u, Position: (0,0)
```

in the log-line.

Make a regular refinement (each triangle split into four) by the four triangle icon, solve again, and print u(center) again:

```
Number of degrees of freedom solved for: 2345
Solution time: 0.188 s
Value: 0.019069, Expression: u, Position: (0,0)
```

and again, and again …, ... try up to, say, a million degrees of freedom when the solver runs out of memory. Note: the numbers may be different for your ellipse because the initial mesh may be different.

Convergence:

u(0,0)	**diff.**
1201	
69	-1132
231	162
234	3
231	-3

The order of convergence should be 2 for regular solutions in energy norm and 3 in max-norm. The exponent is hard to see from the data, but the convergence is rapid. The convergence is *not* as regular as you have seen for e,g, the trapezoidal rule for quadrature, because the mesh is so irregular.

Now try piecewise linear basis functions instead. **`Lagrange P1`** elements are chosen under **`Physics>SubdomainSettings>Element`**

Repeat the experiment: Slower convergence.

#dof	time	u(0.0)	diff.
613	0.094	0.019103	
2345	0.156	0.019115	12
9169	0.547	0.019205	90
36257	2.172	0.019224	19
144193	11.204	0.019230	6

Example 2 – Poisson's equation on a four-leaf clover

Start a new model by choosing **`File>New,`** Choose PDE like above: we want to solve the same equation but on a four-leaf clover.

Geometry

Create a circle. Copy it with **`ctrl-c`** (like in Windows), paste with **`ctrl-v`**, give the desired translation in the dialog box, paste twice more with suitable translations.
Create the union of the four disks in **`Draw>CreateCompositeObject, e.g.`** "**`select all`**" (union is the default-operation, as you see in the formula window), OK. The new object was named **`CO1`** (Composite Object 1).

Define PDE

`Physics>SubdomainSettings`

Again, only one subdomain, the unionen; we could have kept all the interior boundary curves from the four circles, but we chose (unconsciously) (default!) to remove them, so they are now gone. Select the only domain and accept $c = f = 1$ by OK.

Boundary conditions

`Physics>BoundarySettings`

Many boundaries, 8 in all; each circle gives two and the two inner were erased.
Select all by ctrl-a; The default condition is - obviously - Dirichlet: $r = \sin(10x)$ like above, OK.

Element Mesh

The Triangle-icon provides an initial mesh which looks good so OK.

Solution

Solve by = and after a second or so the color picture appears. We carry out the convergence study like above – possibly you have different coordinates, no matter,

#dof	tid	u(0,0)	diff.
5057	0.422	-0.253078	
20033	1.469	-0.253353	275
79745	7.218	-0.253463	110
318209	39.953	-0.253507	44

Order of convergende somewhere between 1 and 2, DIFFERENT from the single ellipse. Why? Could it be that the solution is not as regular (smooth) as is required ? The sharp intruding corners are suspects. Plot |gradu| and it will show,
`PlotParameters>Surface>SurfaceData`, select **`|gradu|`**, OK maybe even more clearly in a 3D-graph**,**
`PlotParameters>Surface>HeightData,` select **`|gradu|`**, click the check box **`HeightData`**, OK.

One can show (by separation of variables) that the singularity is of the type $r^{\frac{\alpha}{\pi}}$ when the exterior angle is α (here $\pi/2$). That is enough to reduce the rate of convergence noticeably.