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CSC NA
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DN2251/2
Computational Algebra

KTH Computer Science and Communication

Examination January 14, 2006, 10-13 in D33-D34:

- Read all questions first, and start with the easy ones. They are not ordered.
- No books except standard dictionaries allowed!

Q 1. Describe a floating point number in terms of sign, fraction, base and exponent. When is a floating point number normalized?

Q 2. Describe what happens with the solution x of the linear system $Ax = b$ when the right hand side b is perturbed into $b + \delta b$!

Q 3. What is pivoting for sparsity in Gaussian elimination? Why is it done? Describe in matrix and graph terms!

Q 4. Show that the diagonal elements of a symmetric positive definite matrix are positive!

Q 5. Show that for any real matrix X the product $A = XX^T$ is positive semidefinite. What is the condition on X that makes A positive definite?

Q 6. How can one determine $A^{(k)}$, the matrix of rank k closest to a given matrix A, using the SVD.

Q 7. The Schur theorem is the basis of transformation algorithms for computing eigenvalues.

1. State the Schur theorem without proof!
2. Assume that you have computed the Schur normal form of the matrix A. How can you use it to find the eigenvalues? How do you compute the eigenvectors?

Q 8. The Rayleigh quotient, $\rho(A, x) = \frac{x^H A x}{x^H x}$ is defined for any complex square matrix A and any nonzero vector x!

1. What is the value of the Rayleigh quotient $\rho(A, x)$ when x is an eigenvector of A?
2. Show that if the complex matrix A is Hermitian, the Rayleigh quotient is real for any nonzero complex vector x!

Good Luck!