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DN2251/2 Computational Algebra

KTH Computer Science and Communication

Examination October 22, 2007, 9-12 in M37,M38, M21:

- Read all questions first, and start with the easy ones. They are not ordered.
- No books except standard dictionaries allowed!

Q 1. What is machine epsilon? What is its approximate value in IEEE double precision?

Q 2. You compute a value of the second degree polynomial $p(x) = ax^2+bx+c$ by the Horner algorithm given by

$$p(x) = (a * x + b) * x + c$$

with IEEE floating point arithmetic. Show that the computed value $fl(p(x))$ is the exact value of a polynomial with perturbed coefficients $\hat{a}, \hat{b}$ and $\hat{c}$, and show how they are related to the original coefficients a, b and c!

Q 3. Show that $\|x\|_1 = \sum_i |x_i|$ is a vector norm! (Show that it satisfies the 3 conditions for a vector norm!)

Q 4. Show that for any matrix X the product $C = XX^T$ is positive semidefinite. What is the condition on X that makes C positive definite?

Q 5. Describe how a graph (V, E) defines a matrix, and how a matrix A gives a graph $G(A)$. Consider only the case of nondirected graphs.

Q 6. How does fill in occur when one does Gaussian elimination on a sparse matrix. Describe it in matrix and graph terms!

Q 7. Formulate the least squares problem of fitting a sum of rational functions $1/(t-\tau_k)$, where the poles, τ_k, $k = 1,\ldots,p$, are known, to a series of measurements, (y_i, t_i), $i = 1,\ldots,n$. Why do we call this a *linear* least squares problem?

Q 8. The Schur theorem is the basis of transformation algorithms for computing eigenvalues.

1. State the Schur theorem without proof!
2. Show that the Schur form of a Hermitian matrix is real and diagonal!

Q 9. You perform the shifted QR algorithm. If you, by chance, have chosen the shift $\sigma_k = \lambda_j$ an eigenvalue of A, and compute $A_k - \sigma_k I = Q_k R_k, \quad A_{k+1} = R_k Q_k + \sigma_k I$, show that the last row of the transformed matrix A_{k+1} will be zero except for its last element that will be λ_j.

Good Luck!