DN2222
Applied Numerical Methods
- part 2:
Numerical Linear Algebra

Lecture 8
Wrap up

2011-12-09

Laboration 1

- Most of those handing in Lab1 on Nov 14 or earlier got 5 bonus points.
- Erroneous results resulted in not full bonus points.
- Often, you had not tried the different methods on all the examples. If the instruction was unclear, no credits were deducted.

Laboration 1, cont.

- No-one of those handing in Lab1 on Nov 14 or earlier got less than 3.
- Handing in a report later than Nov 14 that is passed makes Lab 1 cleared but without bonus points.
- To pass the course, all three laborations must be passed.
- Lab-reports #1 are at Student Expedition from Monday (again).

Laboration 2

- Deadline for bonus points was Nov 28.
- I'm still assessing them, sorry...
- To get credit for an exercise EXPLAIN what you have found! Don't make the reader draw the conclusions.
- Bad example: "This is the original grid numbering and this is the new one." - so HOW did it change? WHAT do you (the author) see? WHY is the new one better? WHAT have you (the author) understood?

Laboration 2, good examples:

- With standard numbering Cholesky factorization resulted in 1416 fill-ins. With RCM renumbering Cholesky resulted in 910 fill-ins and with AMD only 464 fill-ins.
- I wrote my code so that it would stop if the total time exceeded 3 minutes. However, this never happened. I ran out of memory for N=768 for sparse matrices and N=192 for full.
- A figure showing the order of node numbering with hand written arrows. (can be done by Matlab as well).

Laboration 3

- Deadline for bonus points is today (or Dec 11?)
- The first two exercises should be reported equally "sloppy" as before.
- The third (and last) exercise should be reported in a complete report showing: what is the task, how did you solve it, what are your findings, what are your conclusions, etc. Include your Matlab code as an attachment.

Laboration 3, cont.

- What is the advantage of using only k columns?

Laboration 3, cont.

- What is the advantage of using only k columns?

$||(I - U_k U_k^T)z||$ is the norm of the residual.

If is is calculated as $B = I - U_k U_k^T$ and then $r = Bz$ then the number of operations is roughly $(k+1)N^2$.

But if you calculate it by multiplying together $U_k^T z$ first, and then multiply with U_k and finally subtract z the number of operations is only $(2k+1)N$.

The Written Exam

- The exam is on Friday Dec 16 between 10-13 in D31,32,35.
- The online application closed on Dec 2.
- Something went wrong with the on-line application for the exam during my visit to China so please register for the exam on the paper form I'm sending round in the classroom. (Or apply at StudExp!)
- The written exam will focus on theorems, proofs, concepts and general ideas. Maybe a little hand calculations. No need for Matlab coding.

The Aim of the Course

- The goal is to provide you with more efficient, reliable (really working) methods to solve common problems, like systems of linear equations or least square curve fittings.
- The methods should be fast and reliable.
- To judge reliablilty we must understand how the computer works - ie working with finite arithmetics and thus round-off.
- We must also have common ways of measurement, like norms and flops.

Some review questions

- **Q2.** When is a floating point number normalised?

- **Q4.** What is the overflow threshhold? What is its approximate value in IEEE double precision?
- **Q7.** How do you bound the rounding error when computing a sum of three numbers, $S = a + b + c$?
- **Qx.** Define forward and backward error.
- **Q9.** What are well-posed and ill-posed problems?
- **Q12.** What are forward and backward stability?

—– Lecture 2 Errors & Gaussian Elimination—

- **Q14.** Show that $||x||_\infty = \max_i x_i$ is a matrix norm.
- **Q17.** Show that for any operator norm $||I|| = 1$.
- **Q21.** Define the condition number of a matrix.
- **Q22.** Describe what can happen with the solution x of a linear system $Ax = b$ if the right hand side b is perturbed into $b + \delta b$.
- **Q23.** Describe what can happen with the solution x of a linear system $Ax = b$ if the matrix A is perturbed into $A + \delta A$.
- **Qx.** Does $||x||_2 < ||y||_2$ imply $||x||_\infty < ||y||_\infty$?
- **Q26.** Show that partial pivoting is enough to guarantee that Gaussian elimination can be performed on a square non-singular matrix.
- **Q28.** How many operations are needed to compute the Gaussian elimination of an $n \times n$ matrix A?

—– Lecture 3 Sparse Matrices —–

- **Q38.** Define how a graph (V, E) defines a matrix, and how a matrix A gives a graph $G(A)$.
- **Q40.** What is pivoting for sparsity? Why is it done? Describe in graph and matrix terms.
- **Q41.** Describe the Symmetric Minimum Degree algorithm.
- **Q42.** Describe the Reverse Cuthill McKee algorithm.

—– Lecture 4-5 Singular Value Decomposition —–

- **Qx.** Describe the basic ideas of a linear least squares problem.
- **Q44.** Formulate the SVD theorem.
- **Q50.** What is meant by a rank deficient matrix?
- **Q52.** How can you get the solution x of a least squares problem, $\min_x ||Ax - b||_2$ using SVD?

—– Lecture 5-6 Eigenvalues —–

- **Q56.** What is meant by two matrices being similar?

- **Qx.** Show that two similar matrices have the same set of eigenvalues.
- **Qx.** Do two similar matrices have the same set of eigenvectors?
- **Q60.** Formulate the Schur Normal Form.
- **Qx.** What is the major difference between the Jordan form and the Schur form.

—– Lecture 6-7 Large problems —–

- **Q72.** Describe the power method for computing eigenvalues. When does it converge?
- **Q73.** What is a Krylov series?
- **Q74.** What is the purpose of the Arnoldi algorithm? Describe the algorithm.

Good Luck

with your future use of

efficient and reliable

numerical methods!

Ninni