DN2222 - Autumn 2011

Programming assignment 1

Study the relevant topics before preparing solutions to the assignments. In the reading advice **D** means the book *Applied Numerical Linear Algebra* by *JW Demmel* and **L** means the lecture notes *Topics in Numerical Linear Algebra* by *A Ruhe*.

You are encouraged to work in groups of two and you may work at any time. The course assistant Ashraful Kadir will answer questions concerning these assignments at the scheduled lab sessions.

Hand in a report by November 11 at the Student expedition. The report should be short and show your conlusions. It may be hand written and should preferably include MATLAB-plots, etc.

Do not include all of your MATLAB code or printout. Only include a few lines showing the most interesting parts/results. I do not like to see long MATLAB programs or diary files!

Exercise 1. Properties of floating point arithmetic (D: 1.4-1.6)

There are several ways to compute a polynomial. Three of these are:

Sum: $p(x) = \sum_{k=0}^{n} c_k x^k$ given a vector of coefficients c_k

Product: $p(x) = \prod_{i=1}^{n} (x - r_i)$ given a set of roots r_i

Eigenvalue problem: $p(x) = \det(A - xI)$ for A, a $n \times n$-matrix

These representations behave differently numerically in floating point arithmetic. The product can be computed with a small relative forward error while the other two can only ba computed with a bounded backward error in the coefficients or matrix elements. The difference is prominent in the neighborhood of a zero of the polynomial, especially if the zero is multiple.

Start with the polynomial

$$p(x) = (x - 6)^5 = x^5 - 30x^4 + 360x^3 - 2160x^2 + 6480x - 7776$$

a) Plot the polynomial around the root. Use 200 points ditributed evenly or randomly over the inteval $[5.992, 6.008]$. Use the product and sum version. Does the plot give a good clue to where the roots of the polynomial are?

b) Compute the roots of the polynomial using MATLABS *roots* and plot the found roots as points in the complex plane. How does the error in the root correspond to your stopping criteria?

c) Find a root using bisection method with different stopping criteria. How does the error in the root correspond to your stopping criteria?

d) Repeat exercise a-b-c for $p(x) = (x - 6)^N$ with $N = 7$ and $N = 9$.

Exercise 2. Gaussian Elimination (**D:** 2.3, **L:** 1.1)

Your task is to write a Matlab routine for Gaussian elimination *without* pivoting.

Formulate your routine in terms of operation on vectors and matrices.

Show by a few simple examples that your routine is correct.

Exercise 3. The need for pivoting (**D:** 2.4.1)

Use the routine you just wrote to solve the simple 2×2-system $Ax = b$ with

$$A = \begin{pmatrix} \epsilon & 1 \\ 2 & -3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Choose smaller and smaller values of ϵ like $10^{-k}, k = 1 : 17$. Compare your computed solution x to what you get with Matlab's built-in backslash-routine, that uses pivoting).

Plot the norm of the error $e = x - \hat{x}$ and the norm of the residual $r = Ax - b$ as a function of the value of ϵ. (Use in the plots the Matlab solution as "exact solution" $\hat{x}$. The Matlab solution can be assumed to be very close to the exact solution.)

(Hint: use Matlab's `loglog`).

Exercise 4. Pivoting, singular submatrices (**L:** 1.3)

Try your routine on the following non-singular matrix

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 2 \\ 21 & 22 & 20 & 21 & 20 \\ 17 & 17 & 18 & 20 & 20 \end{pmatrix}$$

Your routine will get "Inf" and "NaN". Why?

(Hint: Look at the upper left 3×3-submatrix)

Exercise 5.

Modify the upper left element B_{11} to $1 + \epsilon$ for some small values. Now the "infinites" will be large numbers. How large will they be compared to ϵ? What happens with the "not-a-numbers"?

Good luck! /Ninni