Homework 1, DN2230

Due November 9, 2010

If a correct solution is handed in before the deadline (i.e. November 9) two bonus points will be awarded to the final written exam. If a solution that is handed in before that date is not correct, it has to be redone, but the second time without yielding bonus points for the exam. Question 5 is optional, but if not solved before the deadline, it is not possible to get two bonus points on this homework.

Solutions must be clearly written, and easy to follow. If not, they will not generate bonus points, and must be redone.

1. Adapt the Power Iteration method so that it is possible to find the second largest (in magnitude) eigenvalue of a symmetric matrix, and the corresponding eigenvector. Give a Matlab program that uses your algorithm to compute these for the matrix
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 9 \end{pmatrix}.$$
2. Exercise 7.3 in "Numerical Linear Algebra" (NLA).
3. What is the computational work in flops to perform a QR factorization of a symmetric tridiagonal matrix $A \in \mathbb{R}^{m\times m}$? We are primarily interested in the order of the method; hence, the task here is to determine an optimal number α, such that the computational work is less than or equal to a constant times m^α.
4. Exercise 28.4 (b) in NLA. Use your result from question 2 here. We are again interested in the orders of the methods, i.e. to get exponents α as in question 3.
5. (Optional, but generating bonus) Exercise 28.4 (a) in NLA. For the bonus point you must present an algorithm that gives the optimal computational work order.
6. Exercise 29.1 in NLA. In addition to the task in part (e), try your program to see how large a matrix $A \in \mathbb{R}^{m\times m}$ could be such that your program is able to find all the eigenvalues in a reasonable amount

of time (say 30 seconds). Is it different when you use the unshifted QR algorithm and when you use the Wilkinson shift? Use matrices A similar to the one in the exercise (i.e. `A=diag(m:-1:1)+ones(m,m)`).

Good luck!