Gunilla Kreiss,
NADA
2005-01-18

Numerical treatment of Differential Equations II, 2D1255, 2005

Homework problem 1

Hand in a report no later then January 27.

You may want to review material from Edsbergs course 2D1225

Explicit methods for initial value problems

As an example of a method in this class we consider the Euler forward method. We will study three problems, two linear and one non-linear, in order to understand the accuracy and stability properties of the method.

1. **Accuracy** We expect the method to be first order accurate. Why? To investigate this experimentally we consider a problem that can be solved exactly,

$$\frac{du}{dt} = \sin(t)u, \quad u(0) = 4.$$

   Compute numerical solutions until $t = 10$ using different time-steps, and plot together with the exact solution as functions of time. Also, make a plot (use loglog) of the error at $t = 10$ as a function of the time-step. Deduce the order of accuracy from your plot.

2. **Linear Stability** Derive and plot the stability region for the Euler method. We want to use method to solve

$$\frac{dv}{dt} = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} v, \quad v(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

   Determine the theoretical stability limit on the time-step for this problem. Plot two typical solutions, one stable and one unstable, in phase space, together with the exact solution.

3. **Stability for a non-linear problem** Consider

$$\frac{du}{dt} = \begin{pmatrix} -u_1 - u_2^3 + 6\cos(t) \\ u_1^3 - u_2 + \frac{3}{2}\sin\frac{3t}{2} \end{pmatrix}, \quad u(0) = \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$$

   using the forward Euler method. For non-linear problems the stability properties depend on the solution, and the method is in generally stable only in parts of the phase space, (u_1, u_2), when the time-step h is constant. Where in phase space is the method stable when $h = 0.05$? Solve the problem for $\alpha = 1$ until $t = 10$, using the forward Euler method with $h = 0.05$, and plot the solution in phase space. Does the solution pass through unstable regions? Discuss the correctness of the rule of thumb : h must be restricted so that the method applied to the linearized problem is always stable.