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Numerical treatment of Differential Equations II, 2D1255, 2005

Problem set 3

Hand in no later then February 24.

Well-posedness, stability and other properties of numerical schemes

Read the distributed material on well-posedness, chapter 8 in LeVeque.

1. Consider the 2π periodic Cauchyproblem for

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + u,$$

in the cases $\alpha = 1$ and $\alpha = -1$. In both cases the initial condition is

$$u(x, 0) = \sin(x).$$

Investigate the well-posedness of both cases by numerical experiments and theoretical considerations. Use a finite difference approximation with central differences in space and forward difference in time. In the wellposed case apply von Neuman analysis to determine how Δt and Δx should be related. Is there any stable discretization of the ill-posed case?

2. Consider the Lax-Friedrichs method applied to the advection equation. Show that the method is Lax-Richtmeyer stable in the 1-norm if the CFL condition is satisfied. Derive the modified equation. Discuss how to choose Δt and Δx to avoid strong damping effects.

3. Consider the same problem as in problem set 2.

   a) Run your program for the problem in problem set 2 with different values $\Delta t/\Delta x$. How large can Δt be without violating the CFL condition? Plot solutions for different $\Delta t/\Delta x$. When is the damping of waves largest/smallest? Compare with your prediction in the previous question.

   b) Write a similar program that solves the problem using the Lax-Wendroff method on page 72 in LeVeque. According to section 8.6.2 the method is dispersive. Hand in plots of the solution where effects of dispersion errors can be seen. Don't use $\Delta t/\Delta x = 1$. How is this effect changed by higher resolution? Do you see any damping? Compare with the solution obtained with the Lax-Friedrichs method.