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Numerical treatment of Differential Equations II, 2D1255, 2005

Problem set 5

Hand in a solution no later then April 18.

The relevant background for this problem is the material by Ken Mattsson on SBP operators and the SAT method.

Consider solving the linear advection equation,

$$q_t + q_x = 0, \quad 0 \leq x \leq 1,$$

with periodic boundary condition

$$q(0,t) = \sin^4(2\pi t).$$

Use the following initial data:

$$q(x,0) = \sin(30\pi x)e^{-100(x-0.5)^2}.$$

until $t = 4$. Use central approximation of order 4 in space and the classical Runge-Kutta method of order 4 in time. Investigate, by analysis and numerical experiments, accuracy and stability for different ways of treating the boundary. Determine the CFL limit experimentally. Comment on the performance. Do you see any reflections from the boundary at $x = 1$? Plot $\|Q(t)\|$ as a function of time.

1. Extend the simple ghost cell procedure $Q_0(t) = g(t)$ and $Q_N(t) = Q_{N-1}$ by using $Q_{-1}(t) = g(t)$ and $Q_{N+1}(t) = Q_N(t)$.

2. Improve the accuracy by using $Q_0(t) = g(t)$, and one sided approximations of order 4 to approximate q_x close to the boundary.

3. Use the SBP operator D_1 of section 1.2 in the appendix to approximate $\partial/\partial x$. Continue using RK4 in time. Impose the boundary condition using the SAT method. What should the coefficient of the penalty term be in order to guarantee stability? Note that the last 4 rows of D_1 are obtained by

$$D_{n+1-m,n+1-j} = -D_{mj},\ m = 1, \ldots 4,\ j = 1, \ldots 6.$$