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Numerical treatment of Differential Equations II, 2D1255, 2006

Problem set 1

Hand in a solution no later then Feb 3.

Consider

$$q_t = (0.25q_x)_x + ((1+0.1x)q_y)_y + F(x,y,t), \quad 0 \le x \le 1,\, 0 \le y \le 1,$$

together with boundary conditions prescribing no flux at the boundaries, and zero initial condition. The source function consists of a point source at $(x,y) = (1/2, 1/2)$ with strength

$$g(t) = \begin{cases} 2 & \text{if } 0 \le t \le 0.25 \\ 0 & \text{otherwise.} \end{cases}$$

The source function can be described with the help of delta functions:

$$F(x,y,t) = \delta(x-0.5)\delta(y-0.5)g(t).$$

1. **Conservation** Determine

$$\int_0^1 \int_0^1 q(x,y,t)dxdy \tag{1}$$

as a function of t. Discretize in space using a conservative finite volume discretization. Use an implicit method for the time discretization. (why not explicit?) Show that your method is conservative.

2. **Efficiency** Assume $\Delta x = \Delta y = \Delta t = h = 1/(N+1)$. Theoretically, how fast does the number of operations grow with decreasing h i a straight forward implementation? What is the corresponding operation count for the fast technique from chapter 6.7 in *Demmel: Applied Numerical Linear Algebra*. Implement this method and do calculations for different values of h, and compare how fast the work grows now. Do your results agree with theoretical predictions? Is it true that $N = 2^p$ is much faster than $N = 2^p + 1$?

   Perform calculations until $t = 1$ with $\Delta x = \Delta y = \Delta t = 0.05$, and make plots of the solution at t=0.1,0.2,0.3,0.5, 1.0. Use either surface or contour. Check conservation by computing and plotting

$$\Sigma \Delta x \, \Delta y \, Q^n_{i,j},$$

   at the discrete time levels t_n, together with the corresponding exact quantity (1).

Hints: 1. First consider the problem with coefficients equal to unity (1), and use notation as in Demmel: chapter 6. 2. To get the fast sine transform to workit may be useful to first apply it to a vector where you know the answer.