A skeleton for reports in the course 2D1255

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1 Introduction

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If you do not know anything about latex you should consult
http://www.sgr.nada.kth.se/misc/tex/
before you start to write your report.
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In this assignment we study the solution of hyperbolic equations.

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Discribe the aim of the assignment.
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2 Statement of problem

The problem we consider is our favorite equation

$$\begin{aligned} &u_t + u_x = 0, \quad -\infty < x < \infty, t > 0, \\ &u(x,0) = f(x) \end{aligned} \tag{1}$$

where f is smooth and with compact support.

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If the problem can stated in very few lines you may omitt this
section, and instead state the problem in the introduction.
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Next describe how you solve the problem, and the results. An
alternativ to the  2 sections below would be to have a section
with name  Methods, with possibly subsections Analysis and
Numerical Methods, and a section with name Results, which also
can have subsections.
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3 Analytical solution

The solution to (1) can be found via characteristics. If

$$\frac{dX(t)}{dt} = 1, \quad X(0) = x_0,$$

then $u((X(t)), t) = f(x_0)$ is the solution of (1) along the characteristic $X(t) = t + x_0$. I.e we can write the solution $u((X(t)), t) = f(X(t) - t)$. Our solution may look like the the picture in figure 1.

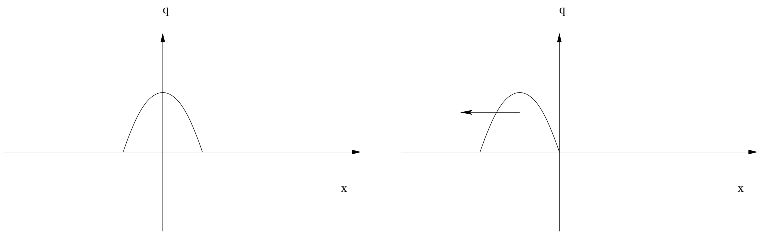

Figure 1: An initial hump moving to the right.

4 Numerical experiments

If we want to solve (1) numerically we may consider the semi-discrete approximation

$$\frac{\partial v_j}{\partial t} = -a\big(\frac{4}{3}D_0(h) - \frac{1}{3}D_0(2h)\big)v_j,\ j = 1, 2, \ldots, n-2, n-1. \tag{2}$$

Which can be integrated in time using the fourth order Runge-Kutta method.

5 Conclusions

Exact solution are propagated without damping, while numerical solutions are damped.