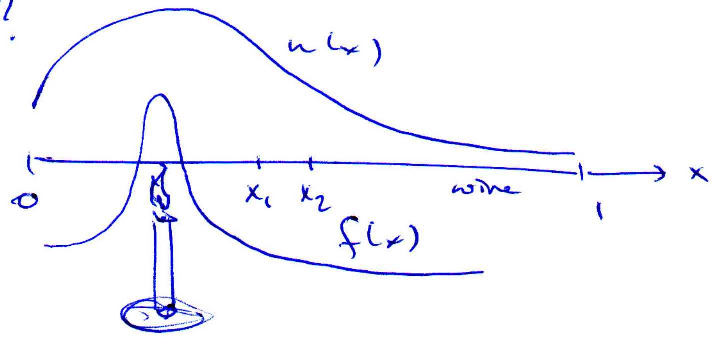
Lecture 1

Heat conduction in a thin heat-conducting wire in $[0,1]$, heated by heat source $f(x)$.

What is the stationary distribution of the temperature $u(x)$?



$q(x)$ - heat flux in the direction of positive x-axis

Energy conservation: for arbitrary sub-interval $(x_1,x_2) \subset (0,1)$: net heat flux through end points = produced heat in (x_1,x_2) per unit time.

$$q(x_2)-q(x_1)=\int_{x_1}^{x_2} f(x)\,dx$$

Fundamental Theorem of Calculus: $q(x_2)-q(x_1)=\int_{x_1}^{x_2} q'(x)\,dx$

$$\Rightarrow \int_{x_1}^{x_2} q'(x)\,dx = \int_{x_1}^{x_2} f(x)\,dx$$

(x_1,x_2) arbitrary + assuming $q'(x), f(x)$ cont.

$$\Rightarrow \quad q'(x)=f(x) \quad 0<x<1 \quad \text{(Differential eqn.)}$$

Constitutive relation: Fourier's law: $q(x)=-a(x)u'(x)$

(Heat flows from warm to cold, proportional to $a(x)>0$ (coeff. of heat conduct.)

$\Rightarrow$ Stationary heat eqn. $-(a(x)u'(x))'=f(x) \quad 0<x<1$