L1

Construct approx. solution $U(x) = \sum_{j=1}^{M} \xi_j \phi_j(x) \in V_h$

<u>Residual</u> $R(U) = -U'' - f$

(How well $U(x)$ satisfies (*); exact sol. $u(x)$: $R(u)=0$)

<u>Galerkin's method</u>: Find $U \in V_h$: $(R(U), v) = 0 \ \forall v \in V_h$

(Choose $U \in V_h$ s.t. $R(U)$ orthogonal to all $v \in V_h$)

L_2-scalar product in $(0,1)$: $(f,g) = \int_0^1 f(x) g(x)\, dx$

$(R(U), v) = \int_0^1 R(U) v\, dx = \int_0^1 (-U'' - f) v\, dx = 0 \Leftrightarrow -\int_0^1 U''v\, dx = \int_0^1 fv\, dx$

<u>Weak form / Variational formulation</u>:

$-\int_0^1 u''v\, dx = \int_0^1 u'v'\, dx - u'(1)v(1) + u'(0)v(0) = \int_0^1 u'v'\, dx$
$= \int_0^1 fv\, dx$

(Since $v \in V_h \Rightarrow v(0) = v(1) = 0$; use part. integration)

<u>Galerkin FEM</u>: Find $U \in V_h$ such that

(G) $\int_0^1 U'v'\, dx = \int_0^1 fv\, dx \quad \forall v \in V_h$

<u>Weak form of DE (*)</u>: Find $u \in V$ such that

(W) $\int_0^1 u'v'\, dx = \int_0^1 fv\, dx \quad \forall v \in V$

$V = \{$All functions $v(x)$ ~~such that~~ $v(0) = v(1) = 0$ & integrals are well defined$\}$