The discrete system of equations

$U(x) = \sum_{j=1}^{n} \xi_j \phi_j(x)$ is determined by $\{\xi_j\}_{j=1}^{n}$

Determine $\{\xi_j\}_{j=1}^{n}$ from (G) ?

(G) $\int_0^1 U' v' \, dx = \int_0^1 f v \, dx \quad \forall v \in V_h$

$$\Rightarrow \int_0^1 \Big(\sum_{j=1}^{n} \xi_j \phi_j(x) \Big)' \phi_i' \, dx = \int_0^1 f \phi_i \, dx, \quad i = 1, \dots, M$$

($\{\phi_i\}_{i=1}^{n}$ basis of V_h $\Rightarrow$ suff. to check for basis)

$$\Rightarrow \sum_{j=1}^{n} \xi_j \int_0^1 \phi_j' \phi_i' \, dx = \int_0^1 f \phi_i \, dx \qquad i = 1, \dots, M$$

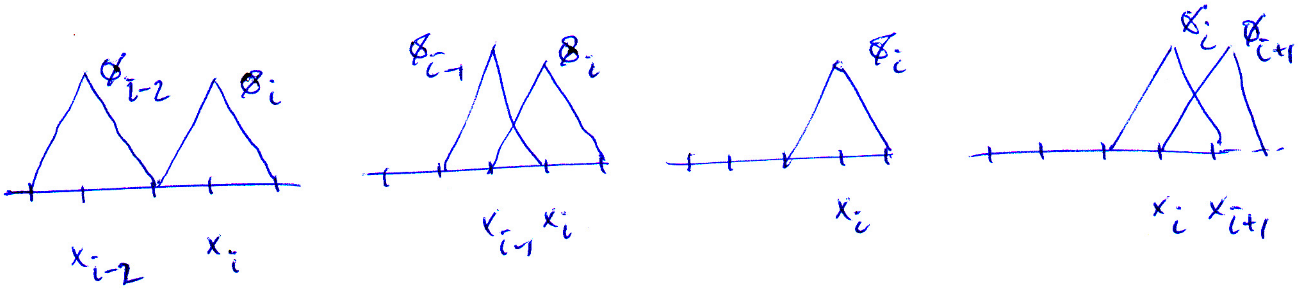
Corresponds to linear system of equations: $A \xi = b$

with matrix $A = (a_{ij})$; vectors $b = (b_i)$, $\xi = (\xi_j)$

$$a_{ij} = \int_0^1 \phi_j' \phi_i' \, dx, \quad b_i = \int_0^1 f \phi_i \, dx$$

A is sparse (most entries are zero) since

$a_{ij} = 0$ unless $i = j-1, j, j+1$:



$\phi_{i-2} \times \phi_i = 0$