Lecture 3

Poisson's equation in $\mathbb{R}^2$ with non-homogeneous Dirichlet boundary conditions:

$$(D) \quad \begin{cases} -\Delta u = f & \text{in } \Omega \subset \mathbb{R}^2 \\ u = g & \text{on } \Gamma \end{cases}$$

g is given boundary data.

Variational formulation: Find $u \in V_g$ s.t.

$$(V) \qquad (\nabla u, \nabla v) = (f, v) \quad \forall v \in V_0$$

where $V_g = \{ v : v = g \text{ on } \Gamma \ \& \int_\Omega (|\nabla v|^2 + v^2)\,dx < \infty \}$

$V_0 = \{ v : v = 0 \text{ on } \Gamma \ \& \int_\Omega (|\nabla v|^2 + v^2)\,dx < \infty \}$

V_g trial space, V_0 test space

The test space is chosen to be V_0 (and not V_g) so that the boundary integral disappears in the integration by parts:

$$(-\Delta u, v) = -\int_\Gamma \cancel{(\nabla u \cdot n) v}^{\,\to 0}\,ds + (\nabla u, \nabla v)$$