Putting (2) into (1) & using that

$$\lambda_0(x)+\lambda_1(x)=1 \qquad \& \qquad (\xi_0-x)\lambda_0(x)+(\xi_1-x)\lambda_1(x)=0$$

$$\Rightarrow \pi_1 f(x) = \left(f(x)+f'(x)(\xi_0-x)+\tfrac{1}{2}f''(\eta_0)(\xi_0-x)^2\right)\lambda_0(x)$$
$$+\left(f(x)+f'(x)(\xi_1-x)+\tfrac{1}{2}f''(\eta_1)(\xi_1-x)^2\right)\lambda_1(x)$$
$$= f(x)\underbrace{(\lambda_0+\lambda_1)}_{=1} + f'(x)\underbrace{\left((\xi_0-x)\lambda_0+(\xi_1-x)\lambda_1\right)}_{=0}$$
$$+\tfrac{1}{2}f''(\eta_0)(\xi_0-x)^2\lambda_0(x)+\tfrac{1}{2}f''(\eta_1)(\xi_1-x)^2\lambda_1(x)$$

$$\Rightarrow |f(x)-\pi_1 f(x)| = \left|\tfrac{1}{2}\left(f''(\eta_0)(\xi_0-x)^2\lambda_0(x)+f''(\eta_1)(\xi_1-x)^2\lambda_1(x)\right)\right|$$

$$\le \frac{1}{2}\left(\frac{|\xi_0-x|^2|x-\xi_1|}{|\xi_0-\xi_1|}|f''(\eta_0)|+\frac{|\xi_1-x|^2|x-\xi_0|}{|\xi_1-\xi_0|}|f''(\eta_1)|\right)$$

$$\le \frac{1}{2}\left(\frac{|\xi_0-x|^2|x-\xi_1|}{|\xi_0-\xi_1|}+\frac{|\xi_1-x|^2|x-\xi_0|}{|\xi_1-\xi_0|}\right)\max_{[a,b]}|f''|$$

$$= \frac{1}{2}\left(\frac{(x-\xi_0)^2(\xi_1-x)}{(\xi_1-\xi_0)}+\frac{(\xi_1-x)^2(x-\xi_0)}{(\xi_1-\xi_0)}\right)\max_{[a,b]}|f''|$$

$$= \frac{1}{2}|x-\xi_0||x-\xi_1|\max_{[a,b]}|f''|$$

$\square$

Theorem 5.2: For $\xi_0\le x\le \xi_1$

$$|f'(x)-(\pi_1 f)'(x)| \le \frac{(x-\xi_0)^2+(x-\xi_1)^2}{2(\xi_1-\xi_0)}\max_{[a,b]}|f''|$$