Proof: $(\pi_1 f)'(x) = f(\xi_0)\lambda_0'(x) + f(\xi_1)\lambda_1'(x)$

Using Taylors formula &

$\lambda_0'(x) + \lambda_1'(x) = 0$ & $(\xi_0 - x)\lambda_0'(x) + (\xi_1 - x)\lambda_1'(x) = 1$

$$\Rightarrow f'(x) - (\pi_1 f)'(x) = -\frac{1}{2}\left(f''(\eta_0)(\xi_0 - x)^2\lambda_0'(x) + f''(\eta_1)(\xi_1 - x)^2\lambda_1'(x)\right)$$

$$\Rightarrow |f'(x) - (\pi_1 f)'(x)| \le \frac{1}{2}\,\frac{(x-\xi_0)^2 + (x-\xi_1)^2}{(\xi_1 - \xi_0)}\max_{[a,b]}|f''|$$

since $|\lambda_i'(x)| = \dfrac{1}{\xi_1 - \xi_0}$ $\square$

Maximum norm $\|f\|_{L_\infty(a,b)} = \max_{x\in[a,b]} |f(x)|$

L_1 - norm $\|f\|_{L_1(a,b)} = \int_a^b |f(x)|\,dx$

L_2 - norm $\|f\|_{L_2(a,b)} = \left(\int_a^b f^2(x)\,dx\right)^{1/2}$

Theorem 5.1 & 5.2 $\Rightarrow$

$$\|f - \pi_1 f\|_{L_\infty(a,b)} \le \frac{1}{8}(b-a)^2\|f''\|_{L_\infty(a,b)}$$

$$\|f' - (\pi_1 f)'\|_{L_\infty(a,b)} \le \frac{1}{2}(b-a)\|f''\|_{L_\infty(a,b)}$$

($\frac{1}{8}$, $\frac{1}{2}$ "Interpolation constants")