Now choose $v = \pi_h u \in V_h$
(the error estimate is true for all $v \in V_h$)

$$\Rightarrow \quad \|(u-U)'\|_a^2 \le \|(u-\pi_h u)'\|_a$$

Interpolation error estimates then gives
that $\|(u-\pi_h u)'\|_a \le C_i \|h u''\|_a$

$$\Rightarrow \quad \|u-U\|_E = \|u'-U'\|_a \le C_i \|h u''\|_a$$

(Theorem 8.1) A priori error estimate
since it depend on solution $u(x)$.

A posteriori error estimates:

$$e = u - U \Rightarrow \|e'\|_a^2 = \int_0^1 a e' e' \, dx$$
$$= \int_0^1 a u' e' \, dx - \int_0^1 a U' e' \, dx = \int_0^1 f e \, dx - \int_0^1 a U' e' \, dx$$

FEM (*) with $v = \pi_h e \in V_h \Rightarrow$

$$\|e'\|_a^2 = \int_0^1 f(e-\pi_h e)\,dx - \int_0^1 a U' (e-\pi_h e)'\,dx$$
$$= \int_0^1 f(e-\pi_h e)\,dx - \sum_{j=1}^{M+1} \int_{I_j} a U' (e-\pi_h e)'\,dx$$