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Now choose $v = \pi_h u \in V_h$
(the error estimate is true for all $v \in V_h$)

$$\Rightarrow \| (u - U)' \|_a^2 \leq \| (u - \pi_h u)' \|_a^2$$

Interpolation error estimates then gives

$$\text{that } \| (u - \pi_h u)' \|_a \leq C_i \| h u'' \|_a$$

$$\Rightarrow \| u - U \|_E = \| u' - U' \|_a \leq C_i \| h u'' \|_a$$

(Theorem 8.1) A priori error estimate

since it depend on solution $u(x)$.

A posteriori error estimates :

$$\begin{aligned} e = u - U &\Rightarrow \| e' \|_a^2 = \int_0^1 a e' e' dx \\ &= \int_0^1 a u' e' dx - \int_0^1 a U' e' dx = \int_0^1 f e dx - \int_0^1 a U' e' dx \end{aligned}$$

~~At this point we can use the following identity:~~

FEM (*) with $v = \pi_h e \in V_h \Rightarrow$

$$\begin{aligned} \| e' \|_a^2 &= \int_0^1 f (e - \pi_h e) dx - \int_0^1 a U' (e - \pi_h e)' dx \\ &= \int_0^1 f (e - \pi_h e) dx - \sum_{j=1}^{N+1} \int_{I_j} a U' (e - \pi_h e)' dx \end{aligned}$$