A priori error estimate (Thm 21.3):

$$\|u-U\|_V \leq \frac{\varkappa_2}{\varkappa_1}\|u-v\|_V \quad \forall v \in V_h$$

If $\|\cdot\|_V = \|\cdot\|_a$ then $\|u-U\|_a \leq \|u-v\|_a$
(Galerkin solution U optimal in energy norm)

Proof: For all $v \in V_h$:

$$\varkappa_1 \|u-U\|_V^2 \leq a(u-U, u-U) = a(u-U, u-U) + a(u-U, U-v)$$
$$= a(u-U, u-v) \leq \varkappa_2 \|u-U\|_V \|u-v\|_V \quad \square$$

Sobolev spaces $H^1(\Omega)$ & $H^1_0(\Omega)$

$$H^1(\Omega) = \left\{ v : \int_\Omega (|\nabla v|^2 + v^2)\,dx < \infty \right\}$$

$$(v,w)_{H^1(\Omega)} = \int_\Omega (\nabla u \cdot \nabla w + vw)\,dx$$

$$\|v\|_{H^1(\Omega)} = \sqrt{(v,w)_{H^1(\Omega)}} = \left(\int_\Omega (|\nabla v|^2 + v^2)\,dx \right)^{1/2}$$

$H^1_0(\Omega)$ is a subspace of $H^1(\Omega)$ with the same norm and scaler product such that

$$H^1_0(\Omega) = \{ v \in H^1(\Omega) : v = 0 \text{ on } \Gamma \}$$