Lecture 8 :

Convection - Diffusion - Reaction

$$
\begin{cases}
\dot{u} + \nabla\cdot(\beta u) + \alpha u - \nabla\cdot(\varepsilon \nabla u) = f & \text{in } \Omega\times I \\
u = g_- & \text{on } (\Gamma\times I)_- \\
u = g_+ \text{ or } \varepsilon\,\partial_n u = g_+ & \text{on } (\Gamma\times I)_+ \\
u(\cdot,0) = u_0 & \text{on } \Omega
\end{cases}
$$

$\beta = (\beta_1, \beta_2, \beta_3)$ convection field

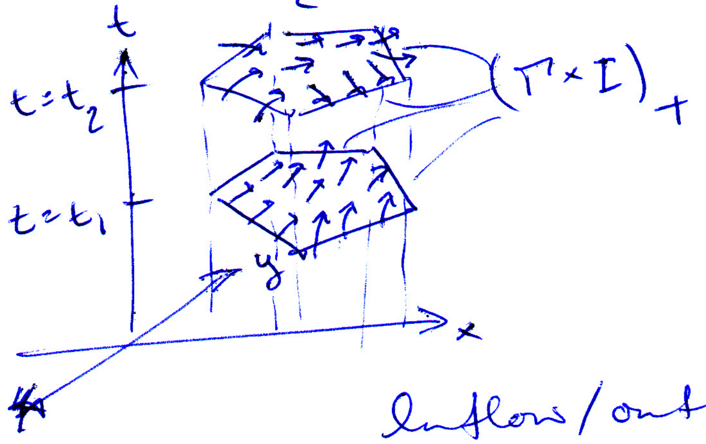
α = absorption coefficient

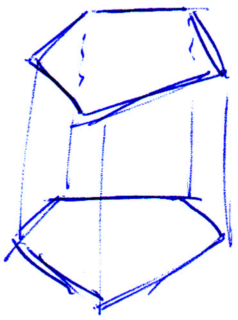
ε = diffusion coefficient

$(\Omega \subset \mathbb{R}^3 ,\ \Gamma = \partial\Omega ,\ I = (0,T])$

$(\Gamma\times I)_- = \{ (x,t) \in \Gamma\times I : \beta\cdot n < 0 \}$ inflow boundary

$(\Gamma\times I)_+ = \{ (x,t) \in \Gamma\times I : \beta\cdot n > 0 \}$ outflow boundary





$\Omega \times I$

Inflow/outflow boundary may change with time !

$\nabla\cdot(\beta u) = \beta\cdot\nabla u + (\nabla\cdot\beta) u$

We may use $\beta\cdot\nabla u \Rightarrow \alpha \to (\nabla\cdot\beta) + \alpha$

$\nabla\cdot(\beta u) \,/\, \beta\cdot\nabla u$: divergence/non-divergence form.