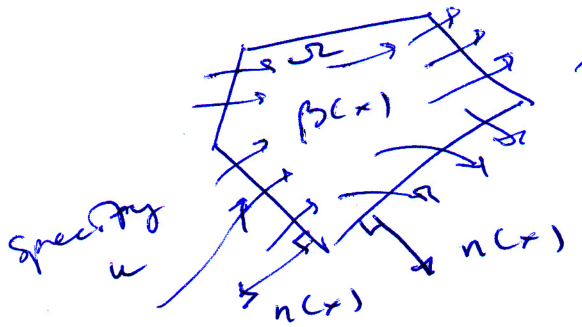
Stationary problem

$$\begin{cases} \beta\cdot\nabla u + \alpha u - \nabla\cdot(\varepsilon\nabla u) = f & \text{in } \Omega \\ u = g_- & \text{on } \Gamma_- \\ u = g_+ \text{ or } \varepsilon\partial_n u = g_+ & \text{on } \Gamma_+ \end{cases}$$

$\Gamma_- = \{x\in\Gamma : \beta\cdot n < 0\}$ inflow boundary

$\Gamma_+ = \{x\in\Gamma : \beta\cdot n > 0\}$ outflow boundary



specify u or $\varepsilon\partial_n u$

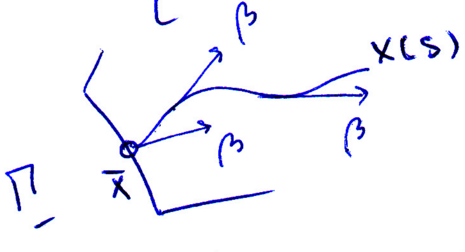
(if $\varepsilon>0$)

$\frac{\varepsilon}{|\beta|}$ small – convection dominated (hyperbolic)

$\frac{\varepsilon}{|\beta|}$ large – diffusion dominated (elliptic – see Ch. 15)

$\varepsilon\to 0 \Rightarrow$ reduced problem:

$$\begin{cases} \beta\cdot\nabla u + \alpha u = f & \text{in } \Omega \\ u = g_- & \text{on } \Gamma_- \end{cases}$$



$$\begin{cases} \frac{dx}{ds} = \beta(x(s)) & s>0 \\ x(0) = \bar{x} \end{cases}$$

Assume non-closed streamlines $x(s)$: $x(s)\neq\bar{x}$ for $s>0$.

Corresponds to ODE along streamlines $x(s)$:

$$\frac{d}{ds}\big(u(x(s))\big) + \alpha(x(s))\,u(x(s)) = (\beta\cdot\nabla u + \alpha u)(x(s)) = f(x(s)) \qquad s>0$$

$\left(\text{chain rule: } \frac{d}{ds}u(x(s)) = \frac{dx}{ds}\cdot\nabla u\right)$

Initial data for ODE: $u(x(0)) = g_-(\bar{x})$

Information propagates sharply along streamlines!