Galerkin method: Find $U \in V_h \subset V$ s.t.

$$a(U,v) = L(v) \quad \forall v \in V_h$$

V_h = finite dimensional subspace with basis $\{\phi_j\}_{j=1}^{n}$

$\Rightarrow$ $A\xi = b$ linear system of equations

$$U(x) = \sum_{j=1}^{n} \xi_j \phi_j(x)$$

$$\|u - U\|_E \leq \|u - v\|_E \quad \forall v \in V_h$$

Galerkin optimal in energynorm $\|w\|_E = \sqrt{a(w,w)}$

How to construct V_h?

(1) $\Omega \rightarrow \mathcal{T}_h$: mesh generation

(2) Choose basis on $\mathcal{T}_h$

Ex: $\{\phi_j\}$ hat/tent functions (cG(1) FEM)

How to construct A & b?

$$A = (a_{ij}) \; ; \; a_{ij} = a(\phi_j, \phi_i) \; ; \; b_i = L(\phi_i)$$

$$a_{ij} = a(\phi_j, \phi_i) = \sum_K a^K(\phi_j, \phi_i)$$ (Sum over elements in mesh $\mathcal{T}_h$)

Assemble algorithm:

Construct A by looping over all elements in the mesh to add each element contribution to global matrix A