A priori error estimation:

$$\|u - U\|_E \leq \|u - \pi_h u\|_E \leq C_i \|h D^2 u\|$$

A posteriori error estimation:

$$\|u - U\|_E \leq C_i \|h R(U)\| \quad , \; R(U) = f - A(u)$$

Error estimation using duality:

$$\|u - U\| \leq C_i S \|h^2 R(U)\|$$

$S = \|D^2 \varphi^*\|$ stability factor

φ solution to dual problem $A^* \varphi = e$

where $(Av, w) = (v, A^* w)$

Adaptive mesh refinement:

$$\|u - U\| \leq \sum_K \varepsilon_K = \sum_K C_i S \|h^2 R(U)\|_K$$

ε_K error contribution for element K

(1) Start from coarse mesh $\tau_h^{(0)}$, $i = 1$

(2) Solve $Au = f$

(3) Solve $A^* \varphi = e$

(4) If $\sum_K \varepsilon_K < TOL$ STOP, else

(5) Refine elements with largest ε_K

$\tau_h^{(i-1)} \rightarrow \tau_h^{(i)}$, then $i = i+1$ & goto (2).