MATEMATIK
Chalmers tekniska högskola

Hjälpmedel: Inga hjälpmedel är tillåtna
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2003–04–22 kl. 8.45-13.45

TMA372/MAN660 Partiella differentialekvationer TM, IMP, E3, GU

OBS! Skriv namn och personnummer på samtliga inlämnade papper.

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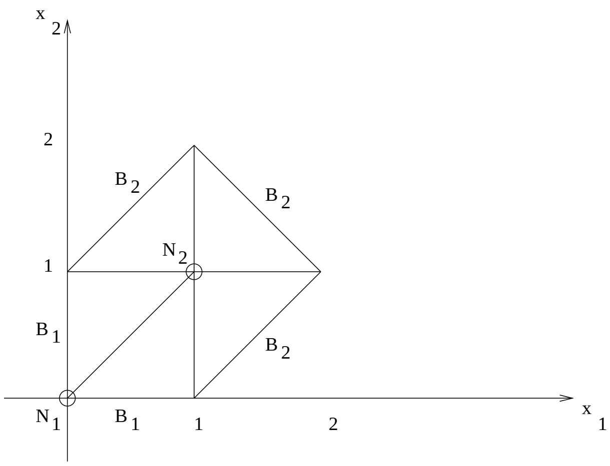
1. Let Ω be the triangulated domain below. Compute the cG(1) solution of $-\Delta u = 0$ in Ω with the Neumann data: $\partial_n u = 3$ on B_1 and Dirichlet condition: $u = 0$ on B_2.



2. Consider the one-dimensional heat equation:

$$\begin{cases} \dot{u} - u'' = f, & 0 < x < 1, \quad t > 0, \\ u(x,0) = u_0(x), & 0 < x < 1, \\ u(0,t) = u_x(1,t) = 0, & \quad t > 0. \end{cases}$$

a) Using appropriate variational forms show the stability estimates: $\|u(\cdot,t)\| \leq \|u_0\| + \int_0^t \|f(\cdot,s)\|\, ds$, and $\|u_x(\cdot,t)\|^2 \leq \|u_0'\|^2 + \int_0^t \|f(\cdot,s)\|^2\, ds$.
b. Give physical meaning to the equation when f=9-u.

3. Let a be a positive constant. Consider the boundary value problem (BVP)

$$-u''(x) + au(x) = f(x), \quad 0 < x < 1, \quad u(0) = u(1) = 0.$$

Formulate the corresponding variational formulation (VF), and the minimization problem (MP) and prove that $(BVP) \Longleftrightarrow (VF) \Longleftrightarrow (MP)$.

4. Prove an a priori and an a posteriori error estimate (in the H^1-norm: $\|u\|_{H^1}^2 = \|u'\|^2 + \|u\|^2$) for a finite element method for the problem

$$-u'' + 2xu' + 2u = f, \quad 0 < x < 1, \qquad u(0) = u(1) = 0.$$

5. Consider the boundary value problem

$$-div(\varepsilon \nabla u + \beta u) = f, \text{ in } \Omega, \quad u = 0, \text{ on } \partial\Omega,$$

where Ω is a bounded polygonal donmain in $\mathbb{R}^2$, $\varepsilon > 0$ is a constant, $\beta = (\beta_1(x), \beta_2(x))$, and $f = f(x)$. Give the conditions (based on Lax-Milgrams theorem) for existence of a unique solution for this problem. Derive stability estimates for u i terms of $||f||_{L_2(\Omega)}$, ε and $diam(\Omega)$.

MA