

Example 1

$$-\frac{d}{dx} \left(k \frac{du}{dx} \right) = f \quad \text{in } 1 \leq x \leq 2 \quad \text{with } u(1) = 3 \text{ and } u'(2) = 4$$

Solve the problem with $k(x) = x$ and $f(x) = -4x$ using 3 linear finite elements with nodes in $x_1 = 1$, $x_2 = 1.3$, $x_3 = 1.5$ and $x_4 = 2$.

******* No answer written yet *******

(On the exercise we solved it using two different approaches:

- 1) the theoretical way, calculation all involved integrals over the entire domain.
- 2) the practical FEM way, calculating elementwise using matrix-notation.)

Example 2

$$-\nabla \cdot (k \nabla u) = f \quad \text{in } \Omega = \left\{ \begin{array}{l} 2 < x < 4 \\ 3 < y < 5 \end{array} \right\} \quad \text{with } u = g \text{ on } \Gamma$$

Solve the problem with 2D FEM and bilinear standard quad functions. Start with dividing each side in two, giving 4 quad elements. The refine by dividing each side by 2 again.. Use a 1-point (mid-point) quadrature formula.

- a) Calculate the stiffness matrix and load vector assuming $k = 1$, $f = 1$ and $g = -0.3x^2 - 0.2y^2$.
- b) Calculate the stiffness matrix and load vector assuming $k = x$, $f = -(y^2 + 2x^2)$ and $g = xy^2$.

******* No answer written yet *******

(On the exercise we solved part **b** using the practical FEM way, calculating elementwise using matrix-notation.)