TYPICAL WRITTEN EXAMINATION QUESTIONS FOR DN2260: THE FINITE ELEMENT METHOD

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1. Computation questions

- Derive a basic interpolation estimate in 1D.
- Formulate the L2 projection for a function f into a piecewise linear function space V_h and show optimality.
- Formulate Galerkin methods for linear differential equations (DE) with different boundary conditions (Dirichlet, Neumann).
- Compute matrix and vector entries for linear basis functions on the interval (1D) and triangles (2D) for linear DE.
- Formulate an algorithm for assembling the linear system for Galerkin's method applied to a linear time-independent DE.
- Annotate/document/fill in blanks in a computer program implementing the general assembly algorithm
- Compute the affine mapping from a reference to a physical element.
- Show a priori and a posteriori error estimates for a linear elliptic DE in the energy norm in 1D.
- Show that Galerkin's method is optimal in the energy norm.
- Show an a posteriori error estimate for a linear DE for a quantity of interest of the error (e, ψ) using duality.
- Formulate a simple adaptive algorithm based on an error estimate
- Formulate the Rivara recursive bisection algorithm. Compute a few refinement steps by hand for a given mesh.
- Formulate the dG(0) and cG(1) methods for initial value ODE
- Formulate Galerkin's method for an initial boundary value PDE
- Show basic stability estimates for linear DE (heat equation, wave equation), given hints how to start (what to multiply with)
- Compare stability estimates for standard Galerkin's method and the streamline diffusion method for the convection-diffusion equation.
- State the Lax-Milgram theorem
- Show that the conditions of Lax-Milgram's theorem are satisfied for basic linear elliptic DE

2. Discussion questions

- Describe the physical meaning of boundary conditions for a given DE
- Discuss using piecewise polynomial (linear for example) basis functions versus using global basis functions with regard to the resulting linear system and adaptivity
- Discuss why it can be useful to represent the computed solution U as a function.
- Describe the different parts of a software system for computation of differential equations using the finite element method and how they fit together
- Discuss the advantages of adaptivity, and the cost of solving differential equations using the finite element method

- Discuss how it can be possible to a posteriori estimate/bound the error $e = u - U$ without knowing the exact solution u
- What is a stability estimate? What information does it give?
- Describe the streamline-diffusion method
- Discuss the role of the dual equation and solution when estimating the error. Show that the dual equation goes backward in time for the heat equation.