Lab2: TM wave scattering on PEC and dielectric cylinders; MoM analysis of patch antenna

1 DIY MoM Code for scalar wave scattering in 2D

The notes describe the Method of Moments method for the scattering problem for the Helmholtz equation. Your job is to write a matlab code which does the computations, using the pulse basis functions and collocation ("point matching") at the line element midpoints, as described for the Laplace equation in lecture notes and CEMbook.

Matlab functions are available on the home page for the exact solution (a series expansion in Bessel and Hankel functions). The notes give a matlab function

```
field(z,zprime,gamma,sigma,k)
```

which computes the field from the charge distribution on the curve on a grid of points; the helper functions `green` and `greender` compute the Green's function and its derivative. `field` assumes that the 2D geometry is represented as complex numbers. If you want to use it your code should also use complex numbers $z = x + iy$ for (x,y).
The notes propose exercises. Never mind the last exercise about memory savings, but *do* work the other two, and think about the jumps at the boundary of the field computed from the integral.
Program the visualization tools first so you can use them to debug the code. It helps to animate the fields; irregularities, waves moving in the wrong direction, discontinuities, etc., are more easily spotted by our vision system when there is motion. Animation is simple because the fields are time harmonic. About 20 snapshots are needed in a period (10 may be too few).

```
% set up grid and field
x = linspace(0,1,20);
[xx,yy]=meshgrid(x,x);
k = 5;
fi = pi/6;
E = exp(i*k*(cos(fi)*xx+sin(fi)*yy));
% animate
n = 20;
c = exp(i*2*pi*linspace(0,1-1/(n-1),n));
for k = 1:5*n
      surf(real(c(mod(k,n-1)+1)*E));
      pause(0.1)
end
```

When the code works, compare the l2 norm of the scattered field on some grid of points, for instance on a circle outside the scatterer, to that computed by the exact solution on some grid.

1.1 PEC case

Q1: Plot the field on a dense grid close to the source points, to see the singularity as the field point approaches the source points.

Q2: Plot l2 error vs. number of elements per wavelength for a case with $ka = 5$, where k is the wavenumber, and a is the radius of the circle. What order of convergence do you observe?

Q3: The internal resonance problem

Run a sequence of *k*-values with fixed *a*, and plot the l2-norm of the charge distribution (σ) vs. *k*. What is the smallest *k* for which the linear system is singular? At this *k* you should see very large σ, or maybe not: The coefficient matrix becomes singular with a rank-1 null space, but the equations still admit (among many others) the "correct" solution. However, picking the right one requires knowledge of the null space, i.e. the eigensolution, which is the one we try to find!

Q4 (voluntary: focusing)
Replace the circle by a concave parabolic mirror, send in a wave and observe the focusing of the reflected wave. Compute by geometrical optics where the focus should be and check.

Q5 (voluntary: far-field transform)
Compute the bi-static far-field scattering cross section of your mirror. Evaluate the computed field on a circle outside the scatterer and compute its Fourier coefficients. The large argument asymptotic expansion of Hankel functions can be found in e.g Abramowitz-Stegun, Handbook of Mathematical Functions, or other collection of formulas.

1.2 Dielectric scatterer

Experience shows that the coding takes great care with signs, especially in the diagonal elements. So, the hard part is getting it completely right. The relative permittivity of the scatterer is ε. Check your code by running with $\varepsilon = 1$ and also an $\varepsilon < 1$ (longer waves inside than outside). Convince yourself by plotting details of the fields at the scatterer surface that the waves inside and outside satisfy the correct interface conditions.

Q6: Repeat the convergence study from the PEC case and $\varepsilon = 4$. Now the waves inside the scatterer are shorter and set the requirement for the element size. How large ε can you run before matlab runs out of memory (or you run out of patience)?

Q7: With larger *ka* you will see more clearly the focusing of the transmitted wave. What should ε be to put the focus on the circle? Use rays and Snell's law of refraction to find where the focus is; when it is inside you only need consider *one* refraction. Run the case and check.

2 Analysis of a patch antenna by MoM software

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Good Luck!