

# Advanced Computation in Fluid Mechanics

## Course PM 2011

Johan Hoffman

February 10, 2011

### Description

Advanced trans-disciplinary course approaching fundamental problems in fluid mechanics of practical importance by tools from mathematical analysis and numerical analysis, using modern computational technology.

### Prerequisites

The course DN2260 Finite element methods, or equivalent.

### Aim

The goal is that the students should be able to analyze and use General Galerkin (G2) adaptive finite element computational technology to model fluid flow at high Reynolds numbers. More precisely, the students should be able to:

- define the concepts weak solution and well-posedness,
- derive energy estimates for underlying equations and G2 approximations,
- derive a posteriori output error estimates for G2 using duality,
- analyze the global effect of friction boundary conditions in G2,
- formulate adaptive algorithms based on a posteriori error estimates,
- use G2 software for adaptive computation of turbulent flow.

Based on a review of research literature and the students own G2 computations, the student should be able to analyze the following fundamental (open) problems of state of the art fluid mechanics using G2 computation/analysis:

- well-posedness and computability of turbulence,
- boundary conditions, turbulent boundary layers and flow separation,
- computational prediction of aerodynamic forces,

with applications in a number of areas such as car-, ship- and aircraft industry. The purpose is to develop a critical approach with the possibility to question established truths, and to form new hypotheses.

## Examination

Below is listed the course examination together with respective deadlines:

Exam (4.0 hp)

- Problem sheet 1-10: Monday February 22 (50% of course grade)
- Oral exam (Problems 1-10): Tuesday February 22 (pass/fail)

Project (3.5 hp)

- Project report (max 10 pages/group: pdf-file): Monday March 8 (50% of course grade)
- Oral presentation of final report: Tuesday 13.15 March 8 (pass/fail)

Each part of the examination is mandatory to pass the course, and attendance to the seminars is also mandatory. All submissions should be sent by email to the course leader and the teaching assistant in time for the deadlines, and the only accepted format is pdf-files. Except for the problem sheet: here also paper submissions to the teaching assistant are accepted. The problem sheet is submitted individually, whereas the project reports are submitted one for each group.

The percentages refer to the respective weights the activities have for the total grade in the course. The quality of each activity is then also graded, so that the total grade is given as the sum of the percentages times the quality (0-100%) of each activity. The total grade for the Exam and Project is given according to the following scale:

- A 90%
- B 80%
- C 70%
- D 60%
- E 50%
- F <50%

Failure to comply with deadlines for submissions automatically reduces the quality of the activity.

## Project

The purpose of the course projects is for the students to use analysis, computation and available literature to study an (open) research problem in computational fluid dynamics. The course project consists of the following parts:

- 3 seminars: At a first introductory seminar the teacher gives a presentation followed by a project introduction. At the second seminar the project groups present their preliminary work together with a discussion. At the last seminar each group gives a presentation of their project where the other groups act as opponents.
- 1 project report by each group (max 10 pages: as pdf-file)
- 1 final presentation

The projects are done in groups (2-4 students), in one of the following themes:

1. Turbulence, computability and adaptive finite element methods
2. Turbulent flow separation and boundary layer modeling
3. Flight aerodynamics

### Project themes

Each group should use the computational software to address a set of basic problems connected to the group theme, and for the highest grades the group is also expected to study additional problems connected to the theme using computation. The basic problems for each theme are:

#### **Theme 1: Turbulence, computability and adaptive FEM**

1. Consider a circular cylinder with diameter  $D = 1$  and length  $4D$ : compute mean drag  $\bar{c}_D \equiv \int_I c_D(t) dt$  for the cylinder at  $Re = 3900$  using adaptive mesh refinement: is the value for  $c_D$  converging as you refine the mesh? If so, after how many mesh refinements?
2. Is the value for the velocity field in a point in the turbulent wake converging as you refine the mesh? Why?
3. Is the time series  $c_D(t)$  converging as you refine the mesh? How does the mean value  $\bar{c}_D$  change with the length of the time interval  $I$ ?

#### **Theme 2: Turbulent flow separation and boundary layer modeling**

1. Consider a circular cylinder with diameter  $D = 1$  and length  $4D$ : compute  $\bar{c}_D$  for  $Re = 10^4$  and  $10^6$  using adaptive mesh refinement with no slip boundary conditions.

2. Compute  $\bar{c}_D$  with skin friction boundary conditions on the cylinder and  $\nu = 0$ , how does  $\bar{c}_D$  change for different friction parameters  $\beta = 0.1, 10^{-2}, 10^{-3}, 0$ ? Why?
3. How does other flow features change with different friction parameters  $\beta = 0.1, 10^{-2}, 10^{-3}, 0$ ? Why?

### Theme 3: Flight aerodynamics

1. Consider a section of a NACA 0012 airfoil with  $\nu = 0$  and slip boundary conditions ( $\beta = 0$ ): compute lift and drag coefficients for different angles of attack  $\alpha = 2^\circ, 8^\circ, 14^\circ, 18^\circ$ . How does the flow change? Why?
2. At what angle of attack  $\alpha$  does the wing stall? Why? .
3. Change to no slip boundary conditions: how does this change the flow? Why?

### Final report

The project report should be maximum 10 pages, including a cover page and reference list, and should give a presentation of the group work on their theme; discussing the background of the problem, the computations (basic problems, together with other problems defined by the group), the literature and discussions from the seminars, and a summary and analysis by the group.

The format of the report should be the following:

1. Introduction (problem description)
2. Background (literature review, put problem in context)
3. Method (short description of computational method)
4. Results (computational results)
5. Summary and analysis (analysis of computational results with respect to problem and literature)

### Course plan

#### Seminar 1

- Thermodynamics, incompressible flow, stability, vorticity
- Problem sheet: 1-4
- Reading: [1] (chapters 1-12)

#### Seminar 2

- General Galerkin (G2) finite element method, weak solution, energy estimates, boundary conditions

- Problem sheet: 5-7
- Reading: [1] (chapters 13-20 (hard), 25-29)

#### Seminar 3

- Adaptive method, linearized dual problem, a posteriori error estimation
- Problem sheet: 8-10
- Reading: [1] (chapters 30,33)

#### Seminar 4

- Project seminar 1: introduction

#### Seminar 5

- Oral examination: Problem sheet 1-10

#### Seminar 6

- Project seminar 2: discussion on project drafts

#### Seminar 7

- Project seminar 3: presentations of final report

## References

- [1] J. HOFFMAN AND C. JOHNSON, *Computational Turbulent Incompressible Flow: Applied Mathematics Body and Soul Vol 4*, Springer-Verlag Publishing, 2006.

## Contact

Course leader is Johan Hoffman (*jhoffman@kth.se*), and teaching assistant is Niyazi Cem Degirmenci (*ncde@kth.se*).