

An Introduction to Spatial Sound (6):

Ambisonic Encoding/Decoding Calculations:

Example 1(a):

Encode the following mono source M_1 :

Azimuth = 45°

Elevation = 0°

Using:

$$W = \text{input signal} \times 0.707$$

$$X = \text{input signal} \times \cos(\varphi) \cdot \cos(\theta)$$

$$Y = \text{input signal} \times \cos(\varphi) \cdot \sin(\theta)$$

$$Z = \text{input signal} \times \sin(\varphi)$$

$$W = M_1 \times 0.707$$

$$X = M_1 \times \cos(0) \cdot \cos(45^\circ)$$

$$Y = M_1 \times \cos(0) \cdot \sin(45^\circ)$$

$$Z = M_1 \times \sin(0)$$

Therefore:

$$W = M_1 \times 0.707$$

$$X = M_1 \times 0.707$$

$$Y = M_1 \times 0.707$$

$$Z = 0$$

1(b) Decode M_1 with a cardioid speaker response for a square 4-speaker horizontal rig:

A cardioid response implies $d = 1$. Note that for a square 4-speaker horizontal rig the speakers are located as follows:

$$S_1 = 45^\circ$$

$$S_2 = 135^\circ$$

$$S_3 = 225^\circ$$

$$S_4 = 315^\circ$$

Using:

$$g_w = \sqrt{2} = 1.414$$

$$g_x = \cos(\varphi) \cdot \cos(\theta)$$

$$g_y = \cos(\varphi) \cdot \sin(\theta)$$

$$g_z = \sin(\varphi)$$

$$S_i = 0.5 \times [(2-d) \cdot g_w \cdot W + d \cdot (g_x \cdot X + g_y \cdot Y + g_z \cdot Z)]$$

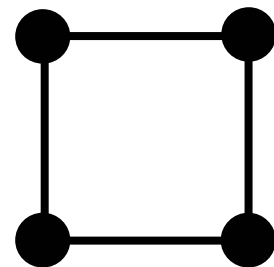
Therefore:

$$S_1 = 0.5 \times [1.414 \cdot W + \cos(45^\circ) \cdot X + \sin(45^\circ) \cdot Y]$$

$$S_2 = 0.5 \times [1.414 \cdot W + \cos(135^\circ) \cdot X + \sin(135^\circ) \cdot Y]$$

$$S_3 = 0.5 \times [1.414 \cdot W + \cos(225^\circ) \cdot X + \sin(225^\circ) \cdot Y]$$

$$S_4 = 0.5 \times [1.414 \cdot W + \cos(315^\circ) \cdot X + \sin(315^\circ) \cdot Y]$$



And so:

$$S_1 = 0.5 \times [1.414 \cdot W + 0.707 \cdot X + 0.707 \cdot Y]$$

$$S_2 = 0.5 \times [1.414 \cdot W - 0.707 \cdot X + 0.707 \cdot Y]$$

$$S_3 = 0.5 \times [1.414 \cdot W - 0.707 \cdot X - 0.707 \cdot Y]$$

$$S_4 = 0.5 \times [1.414 \cdot W + 0.707 \cdot X - 0.707 \cdot Y]$$

Notice the opposite polarity of the B-format gains.

Combining Encode and Decode Equations:

$$S_1 = 0.5 \times [1.414 \cdot (M_1 \times 0.707) + 0.707 \cdot (M_1 \times 0.707) + 0.707 \cdot (M_1 \times 0.707)]$$

$$S_2 = 0.5 \times [1.414 \cdot (M_1 \times 0.707) - 0.707 \cdot (M_1 \times 0.707) + 0.707 \cdot (M_1 \times 0.707)]$$

$$S_3 = 0.5 \times [1.414 \cdot (M_1 \times 0.707) - 0.707 \cdot (M_1 \times 0.707) - 0.707 \cdot (M_1 \times 0.707)]$$

$$S_4 = 0.5 \times [1.414 \cdot (M_1 \times 0.707) + 0.707 \cdot (M_1 \times 0.707) - 0.707 \cdot (M_1 \times 0.707)]$$

$$S_1 = 0.5 \times [M_1 + 0.5 \cdot M_1 + 0.5 \cdot M_1]$$

$$S_2 = 0.5 \times [M_1 - 0.5 \cdot M_1 + 0.5 \cdot M_1]$$

$$S_3 = 0.5 \times [M_1 - 0.5 \cdot M_1 - 0.5 \cdot M_1]$$

$$S_4 = 0.5 \times [M_1 + 0.5 \cdot M_1 - 0.5 \cdot M_1]$$

$$S_1 = 0.5 \cdot M_1 + 0.25 \cdot M_1 + 0.25 \cdot M_1$$

$$S_2 = 0.5 \cdot M_1 - 0.25 \cdot M_1 + 0.25 \cdot M_1$$

$$S_3 = 0.5 \cdot M_1 - 0.25 \cdot M_1 - 0.25 \cdot M_1$$

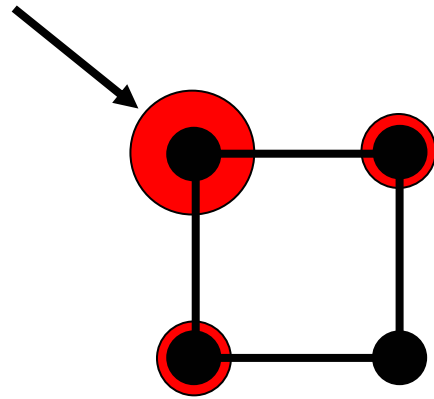
$$S_4 = 0.5 \cdot M_1 + 0.25 \cdot M_1 - 0.25 \cdot M_1$$

$$S_1 = M_1$$

$$S_2 = 0.5 \cdot M_1$$

$$S_3 = 0$$

$$S_4 = 0.5 \cdot M_1$$



2 - Encode and Decode source M_2 with a cardioid speaker response at position:

Azimuth = 90°

Elevation = 0°

Encode Using:

$$W = \text{input signal} \times 0.707$$

$$X = \text{input signal} \times \cos(\phi) \cdot \cos(\theta)$$

$$Y = \text{input signal} \times \cos(\phi) \cdot \sin(\theta)$$

$$Z = \text{input signal} \times \sin(\phi)$$

$$W = M_2 \times 0.707$$

$$X = M_2 \times \cos(0) \cdot \cos(90^\circ)$$

$$Y = M_2 \times \cos(0) \cdot \sin(90^\circ)$$

$$Z = M_2 \times \sin(0)$$

Therefore:

$$W = M_2 \times 0.707$$

$$X = 0$$

$$Y = M_2$$

$$Z = 0$$

Combining Encode and Decode Equations:

$$S_1 = 0.5 \times [1.414 \cdot (M_2 \times 0.707) + 0.707 \cdot (0) + 0.707 \cdot (M_2)]$$

$$S_2 = 0.5 \times [1.414 \cdot (M_2 \times 0.707) - 0.707 \cdot (0) + 0.707 \cdot (M_2)]$$

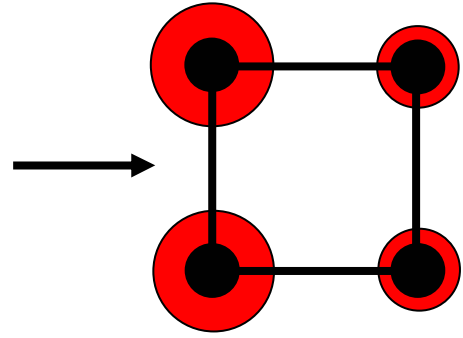
$$S_3 = 0.5 \times [1.414 \cdot (M_2 \times 0.707) - 0.707 \cdot (0) - 0.707 \cdot (M_2)]$$

$$S_4 = 0.5 \times [1.414 \cdot (M_2 \times 0.707) + 0.707 \cdot (0) - 0.707 \cdot (M_2)]$$

Therefore:

$$\begin{aligned} S_1 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) + 0.707 \cdot (M_2)] \\ S_2 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) + 0.707 \cdot (M_2)] \\ S_3 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) - 0.707 \cdot (M_2)] \\ S_4 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) - 0.707 \cdot (M_2)] \end{aligned}$$

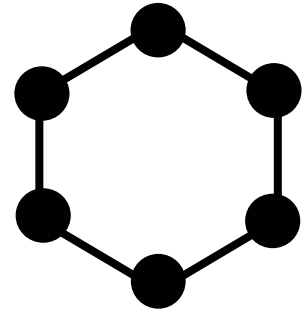
$$\begin{aligned} S_1 &= 0.8534 \cdot M_2 \\ S_2 &= 0.8534 \cdot M_2 \\ S_3 &= 0.1464 \cdot M_2 \\ S_4 &= 0.1464 \cdot M_2 \end{aligned}$$



3 - Decode M_1 and M_2 for a hexagonal 6-speaker horizontal rig:

A cardioid response implies $d = 1$. Note that for a square 6-speaker hexagonal rig the speakers are located as follows:

$$\begin{aligned} S_1 &= 0^\circ \\ S_2 &= 60^\circ \\ S_3 &= 120^\circ \\ S_4 &= 180^\circ \\ S_5 &= 240^\circ \\ S_6 &= 300^\circ \end{aligned}$$



Using:

$$\begin{aligned} g_w &= \sqrt{2} = 1.414 \\ g_x &= \cos(\varphi) \cdot \cos(\theta) \\ g_y &= \cos(\varphi) \cdot \sin(\theta) \\ g_z &= \sin(\varphi) \\ S_i &= 0.5 \times [(2-d) \cdot g_w \cdot W + d \cdot (g_x \cdot X + g_y \cdot Y + g_z \cdot Z)] \end{aligned}$$

Therefore:

$$\begin{aligned} S_1 &= 0.5 \times [1.414 \cdot W + \cos(0^\circ) \cdot X + \sin(0^\circ) \cdot Y] \\ S_2 &= 0.5 \times [1.414 \cdot W + \cos(60^\circ) \cdot X + \sin(60^\circ) \cdot Y] \\ S_3 &= 0.5 \times [1.414 \cdot W + \cos(120^\circ) \cdot X + \sin(120^\circ) \cdot Y] \\ S_4 &= 0.5 \times [1.414 \cdot W + \cos(180^\circ) \cdot X + \sin(180^\circ) \cdot Y] \\ S_5 &= 0.5 \times [1.414 \cdot W + \cos(240^\circ) \cdot X + \sin(240^\circ) \cdot Y] \\ S_6 &= 0.5 \times [1.414 \cdot W + \cos(300^\circ) \cdot X + \sin(300^\circ) \cdot Y] \end{aligned}$$

And so:

$$\begin{aligned} S_1 &= 0.5 \times [1.414 \cdot W + X] \\ S_2 &= 0.5 \times [1.414 \cdot W + 0.5 \cdot X + 0.866 \cdot Y] \\ S_3 &= 0.5 \times [1.414 \cdot W - 0.5 \cdot X + 0.866 \cdot Y] \\ S_4 &= 0.5 \times [1.414 \cdot W - X] \\ S_5 &= 0.5 \times [1.414 \cdot W - 0.5 \cdot X - 0.866 \cdot Y] \\ S_6 &= 0.5 \times [1.414 \cdot W + 0.5 \cdot X - 0.866 \cdot Y] \end{aligned}$$

For M_1 :

$$\begin{aligned} W &= M_1 \times 0.707 \\ X &= M_1 \times 0.707 \\ Y &= M_1 \times 0.707 \\ Z &= 0 \end{aligned}$$

Therefore:

$$\begin{aligned} S_1 &= 0.5 \times [1.414 \cdot (M_1 \times 0.707) + (M_1 \times 0.707)] \\ S_2 &= 0.5 \times [1.414 \cdot (M_1 \times 0.707) + 0.5 \cdot (M_1 \times 0.707) + 0.866 \cdot (M_1 \times 0.707)] \\ S_3 &= 0.5 \times [1.414 \cdot (M_1 \times 0.707) - 0.5 \cdot (M_1 \times 0.707) + 0.866 \cdot (M_1 \times 0.707)] \\ S_4 &= 0.5 \times [1.414 \cdot (M_1 \times 0.707) - (M_1 \times 0.707)] \\ S_5 &= 0.5 \times [1.414 \cdot (M_1 \times 0.707) - 0.5 \cdot (M_1 \times 0.707) - 0.866 \cdot (M_1 \times 0.707)] \\ S_6 &= 0.5 \times [1.414 \cdot (M_1 \times 0.707) + 0.5 \cdot (M_1 \times 0.707) - 0.866 \cdot (M_1 \times 0.707)] \end{aligned}$$

$$\begin{aligned} S_1 &= 0.8534 \cdot M_1 \\ S_2 &= 0.9827 \cdot M_1 \\ S_3 &= 0.6292 \cdot M_1 \\ S_4 &= 0.1464 \cdot M_1 \\ S_5 &= 0.017 \cdot M_1 \\ S_6 &= 0.3705 \cdot M_1 \end{aligned}$$

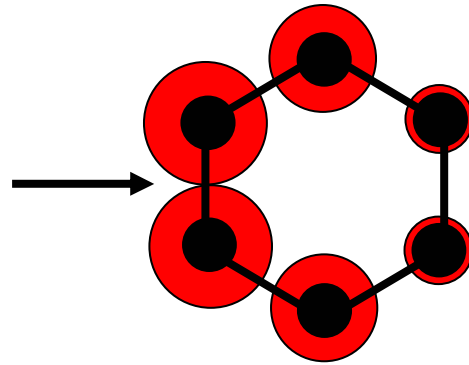
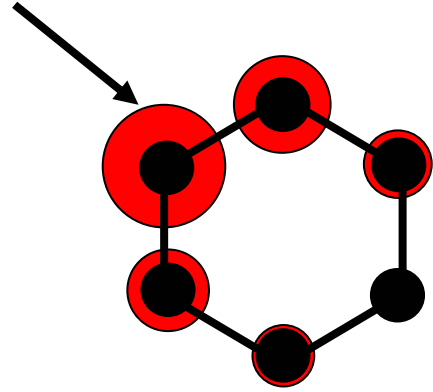
For M_2 :

$$\begin{aligned} W &= M_2 \times 0.707 \\ X &= 0 \\ Y &= M_2 \\ Z &= 0 \end{aligned}$$

Therefore:

$$\begin{aligned} S_1 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707)] \\ S_2 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) + 0.866 \cdot (M_2)] \\ S_3 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) + 0.866 \cdot (M_2)] \\ S_4 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707)] \\ S_5 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) - 0.866 \cdot (M_2)] \\ S_6 &= 0.5 \times [1.414 \cdot (M_2 \times 0.707) - 0.866 \cdot (M_2)] \end{aligned}$$

$$\begin{aligned} S_1 &= 0.5 \cdot M_2 \\ S_2 &= 0.933 \cdot M_2 \\ S_3 &= 0.933 \cdot M_2 \\ S_4 &= 0.5 \cdot M_2 \\ S_5 &= 0.067 \cdot M_2 \\ S_6 &= 0.067 \cdot M_2 \end{aligned}$$



4(a) Encode and Decode source M_3 with a cardioid speaker response at position:

**Azimuth = 0°
Elevation = 0°**

(a) Using:

$$\begin{aligned} W &= \text{input signal} \times 0.707 \\ X &= \text{input signal} \times \cos(\varphi) \cdot \cos(\theta) \\ Y &= \text{input signal} \times \cos(\varphi) \cdot \sin(\theta) \\ Z &= \text{input signal} \times \sin(\varphi) \end{aligned}$$

$$\begin{aligned} W &= M_3 \times 0.707 \\ X &= M_3 \times \cos(0) \cdot \cos(0^\circ) \\ Y &= M_3 \times \cos(0) \cdot \sin(0^\circ) \\ Z &= M_3 \times \sin(0) \end{aligned}$$

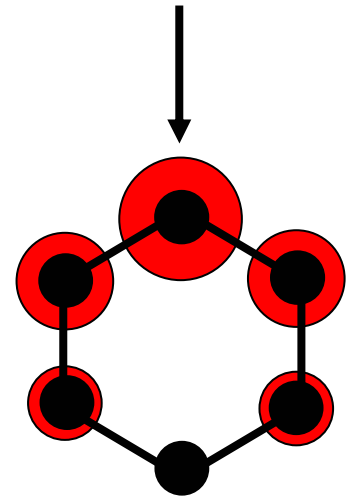
Therefore:

$$\begin{aligned} W &= M_3 \times 0.707 \\ X &= M_3 \\ Y &= 0 \\ Z &= 0 \end{aligned}$$

And so:

$$\begin{aligned} S_1 &= 0.5 \times [1.414 \cdot (M_3 \times 0.707) + M_3] \\ S_2 &= 0.5 \times [1.414 \cdot (M_3 \times 0.707) + 0.5 \cdot M_3] \\ S_3 &= 0.5 \times [1.414 \cdot (M_3 \times 0.707) - 0.5 \cdot M_3] \\ S_4 &= 0.5 \times [1.414 \cdot (M_3 \times 0.707) - M_3] \\ S_5 &= 0.5 \times [1.414 \cdot (M_3 \times 0.707) - 0.5 \cdot M_3] \\ S_6 &= 0.5 \times [1.414 \cdot (M_3 \times 0.707) + 0.5 \cdot M_3] \end{aligned}$$

$$\begin{aligned} S_1 &= M_3 \\ S_2 &= 0.75 \cdot M_3 \\ S_3 &= 0.25 \cdot M_3 \\ S_4 &= 0 \\ S_5 &= 0.25 \cdot M_3 \\ S_6 &= 0.75 \cdot M_3 \end{aligned}$$



4(b) What would this sound like? Interpret your results.