ACCOUNTING FOR UNCERTAINTY IN MEDICAL DATA: A CUDA IMPLEMENTATION OF NORMALIZED CONVOLUTION

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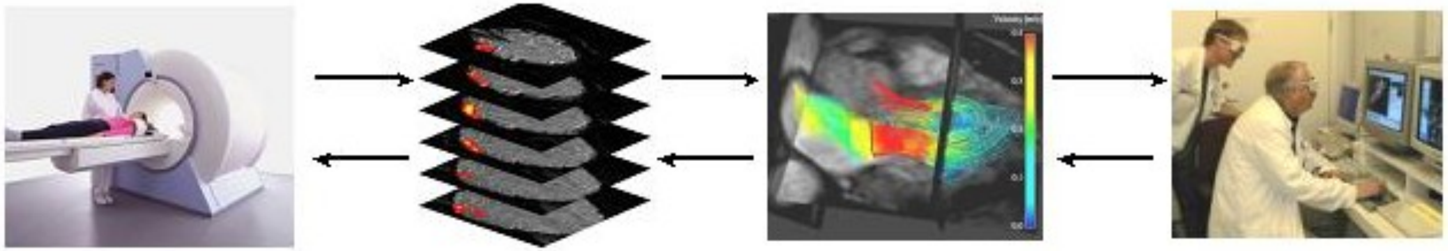
Linköping University

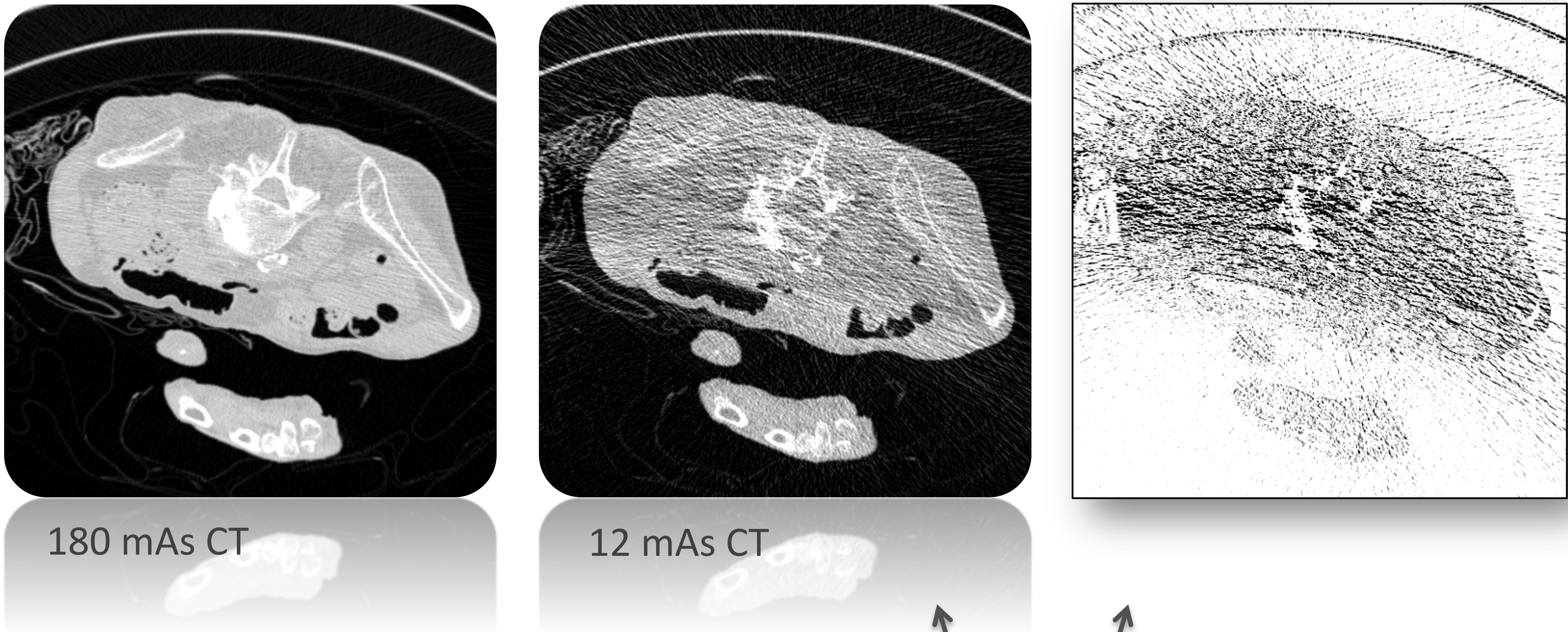
Edge Detection
Bilateral Filtering
Transfer Functions
Gradient Estimation

FILTERING MEDICAL IMAGES



UNCERTAINTY

KNOWN UNCERTAINTY

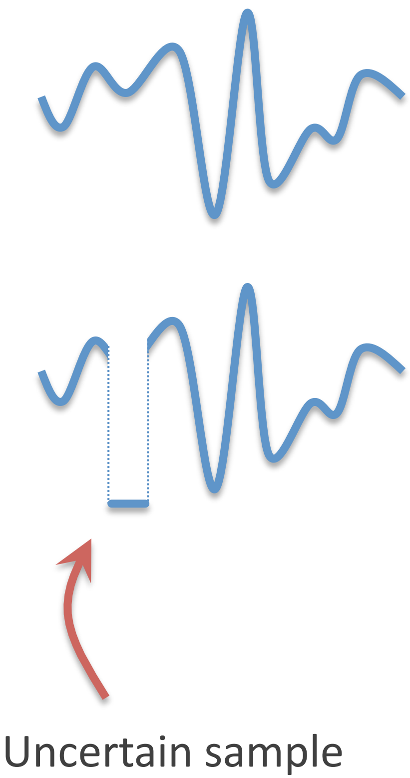


180 mAs CT

12 mAs CT

Notation: $s(k)$ $r(k)$

Known Uncertainty



Filtering a known signal is straightforward

..but..

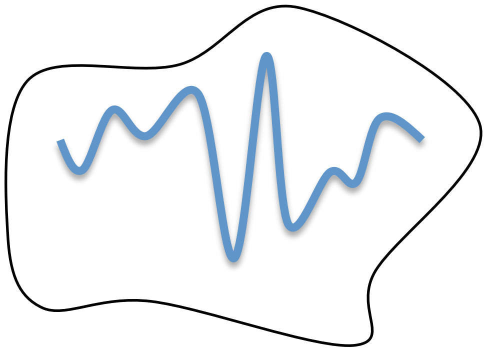
How do we treat uncertainty?

Local filter kernels, adaptive to the local signal uncertainty!

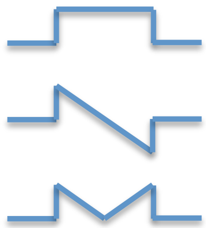
THEORY

Convolution

Polynomial expansion



$h = s * f$



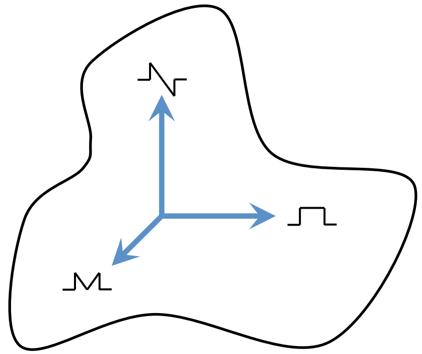
$f_1(l)$

$\vdots$

$f_M(l)$

$$h(k) = \sum_l s(k+l)\overline{f(-l)}$$

G



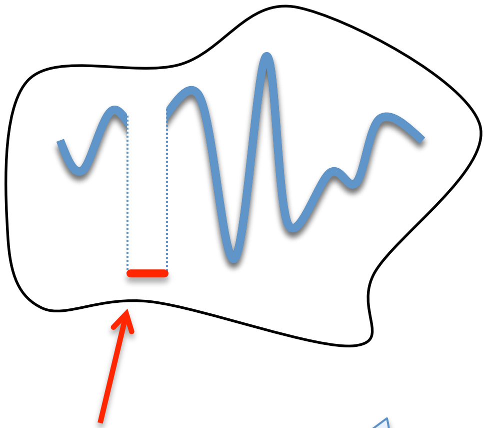
$$h_m(k) = \langle s(k+l) | \overline{f_m}(-l) \rangle$$

$$h_m(k) = \sum_l s(k+l) G_l \overline{f_m(-l)}$$

Interpret convolution as a projection

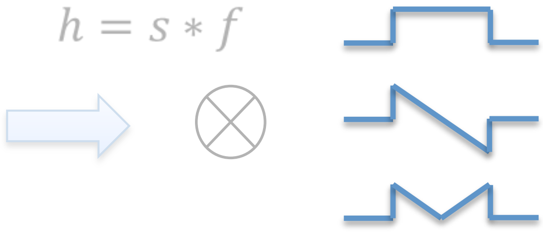
Normalized Convolution

Polynomial expansion



Uncertainty!

$h = s * f$



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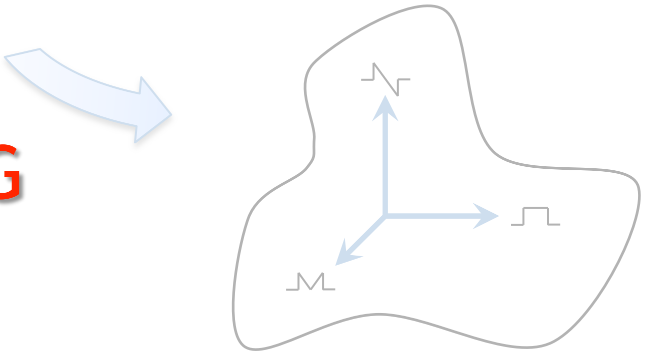
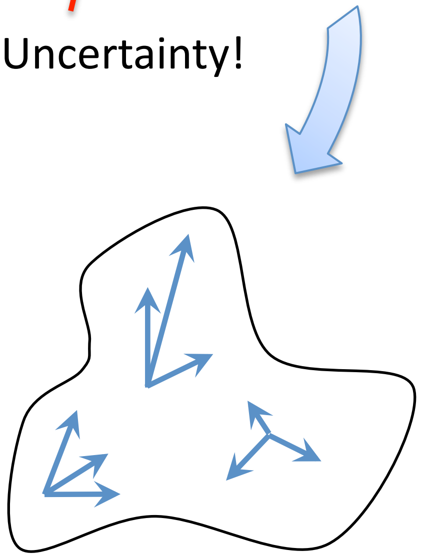


G



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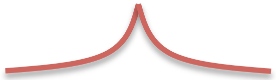


Adaptive metric!

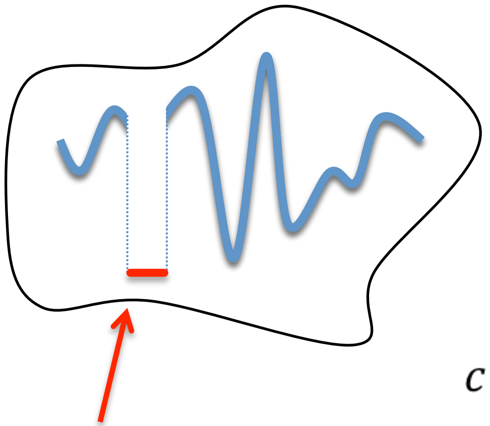
Spatially varying!

Applicability: $a(l)$

Signal certainty: $r(k)$

$$h_m(k) = \sum_l s(k+l) r(k+l) a(l) f_m(l)$$

Normalized Convolution

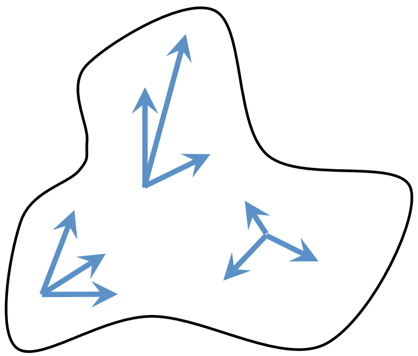


Uncertainty!

Polynomial Expansion for Orientation
and Motion Estimation. [Far02] Farnebäck

$$c = \begin{pmatrix} \langle a * b_1 * \overline{b_1} | r \rangle & \cdots & \langle a * b_1 * \overline{b_M} | r \rangle \\ \vdots & \ddots & \vdots \\ \langle a * b_M * \overline{b_1} | r \rangle & \cdots & \langle a * b_M * \overline{b_M} | r \rangle \end{pmatrix}^{-1} \begin{pmatrix} \langle a * b_1 | r * s \rangle \\ \vdots \\ \langle a * b_M | r * s \rangle \end{pmatrix}$$

One matrix inversion per pixel in the image!



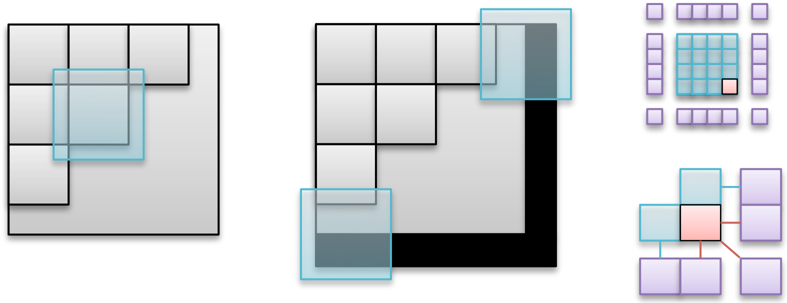
Adaptive metric!
Spatially varying!

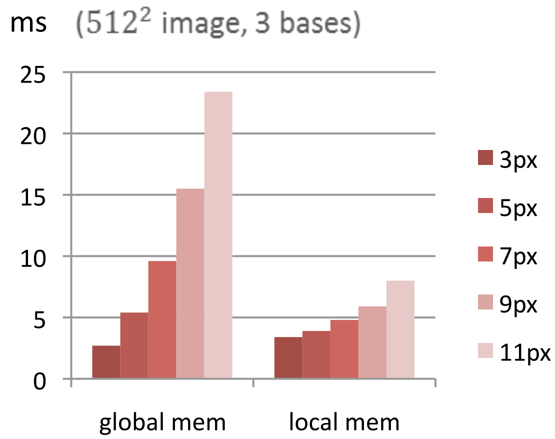
Pre-computation!
G is only *M x M*!

CUDA

CUDA

- Standard optimizations
  - 512 cores
  - Coalesced memory reads
  - Latency hiding
  - Loop unrolling
- Optimizations for Normalized Convolution
  - Templates for recursive matrix inversion
  - Pre-processor defines



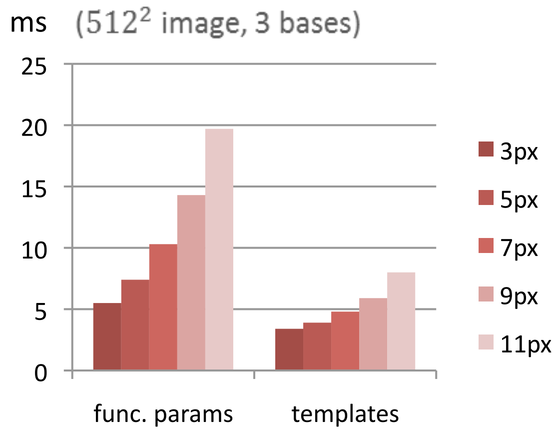


CUDA

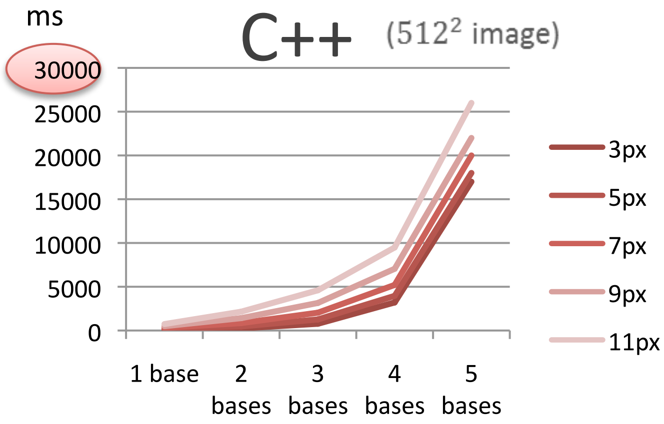
$$A^{-1} = \frac{1}{\det(A)} * Adj(A)$$

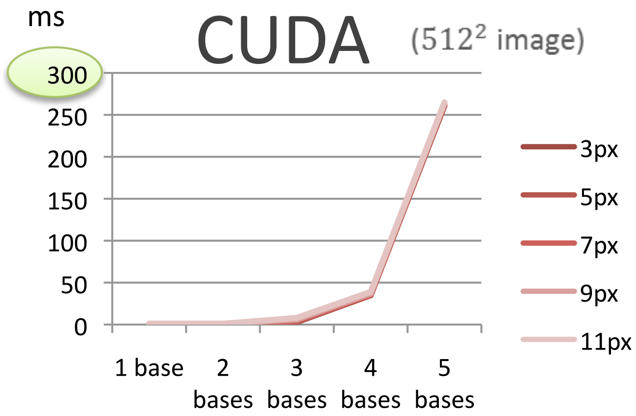
- Standard optimizations
  - 512 cores
  - Coalesced memory reads
  - Latency hiding
  - Loop unrolling
- Optimizations for Normalized Convolution
  - Templates for recursive matrix inversion
  - Pre-processor defines

```
template<int M> __device__ float Determinant()
                {... Determinant<M-1>() ...}
template<> __device__ float Determinant<3>();
template<> __device__ float Determinant<2>();
template<> __device__ float Determinant<1>();
```



CUDA





Conclusions

G is only $M \times M$

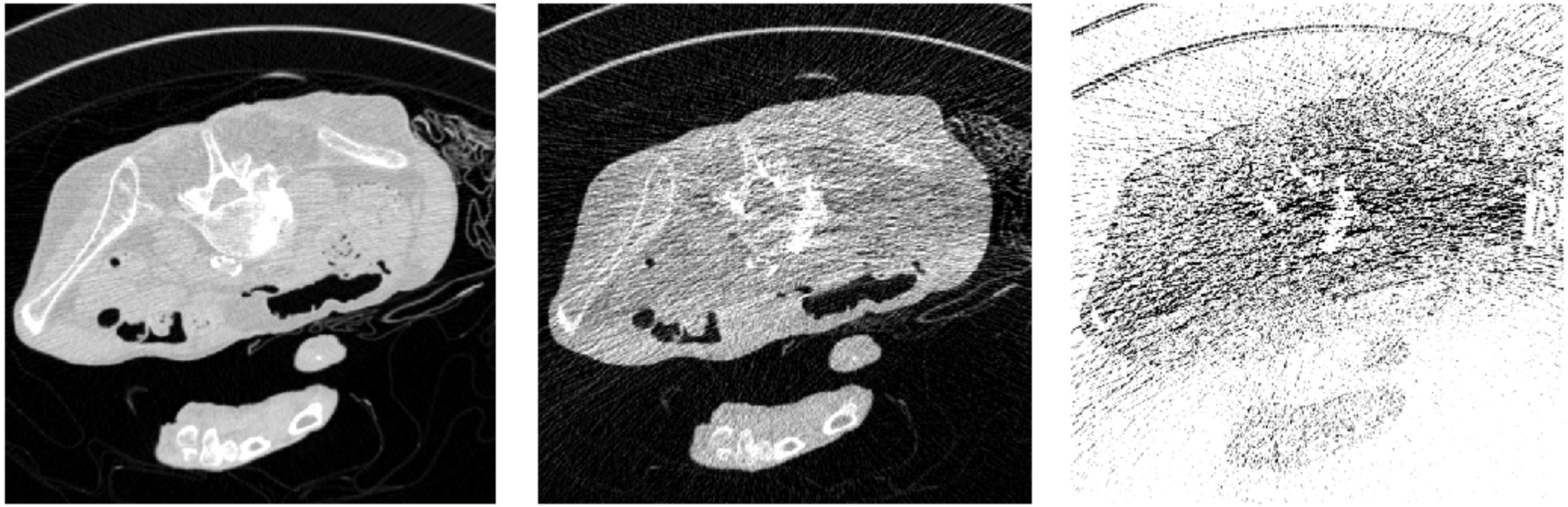
…but even $M \times M$ is costly

…and we need precision

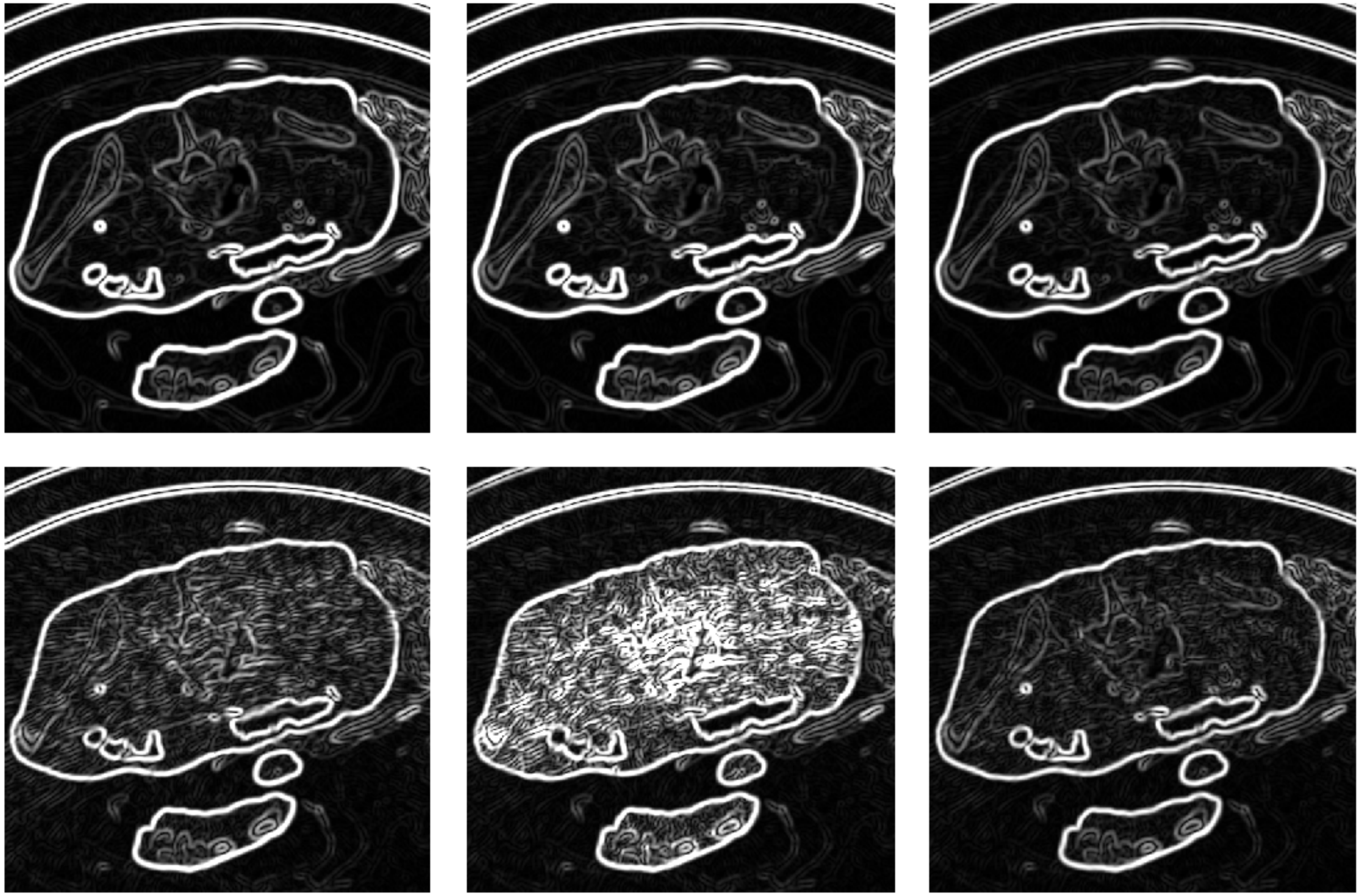
We can achieve real-time performance!

RESULTS

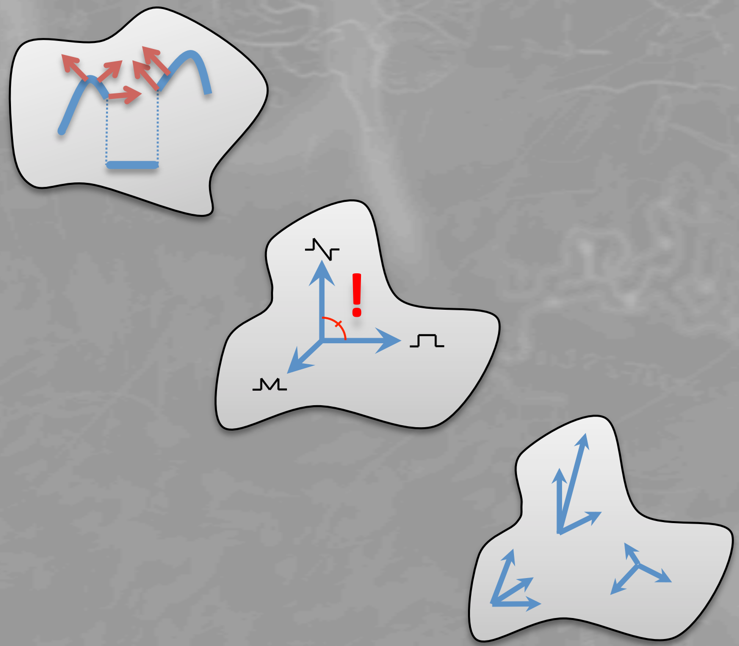
RESULTS



RESULTS



Thank You!



Linköping University
expanding reality

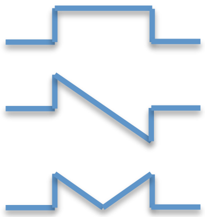
NORRKÖPING VISUALIZATION CENTER

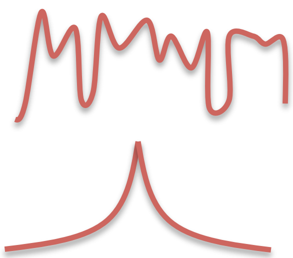
CRESEARCH

$$h_m(k) = \sum_l s(k+l)\overline{f_m(-l)}$$

$$h_m(k) = \sum_l s(k+l)G_l\overline{f_m(-l)}$$

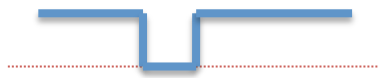
$$h_m(k) = \sum_l s(k+l)r(k+l)a(l)f_m(l)$$

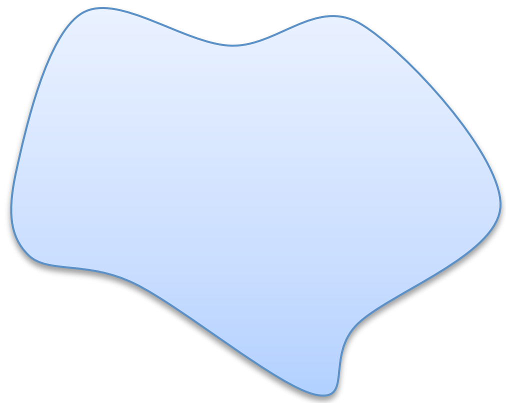




$$h(k) = \langle s(k+l) | f^*(-l) \rangle$$

$$c = G^{-1}$$
$$c = (B^* G_0 B)^{-1}$$



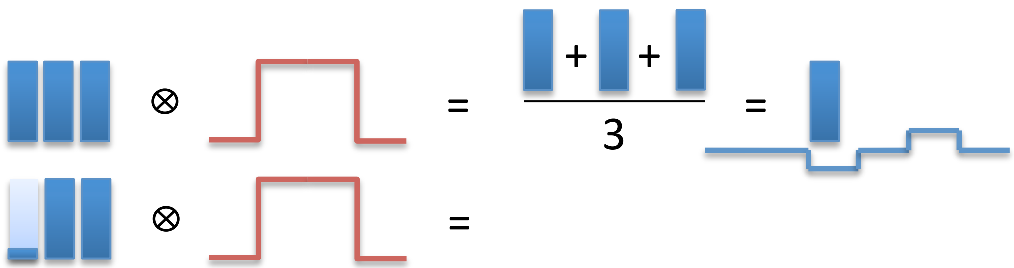


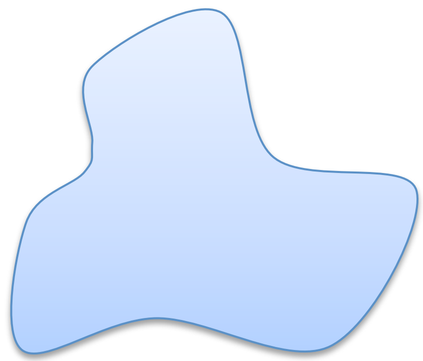
`[111] * [111] = (1+1+1)/3 = 1`

`[?11]`$_{[011]}$`* [111] =`

`(0+1+1)/3 = 0.67`

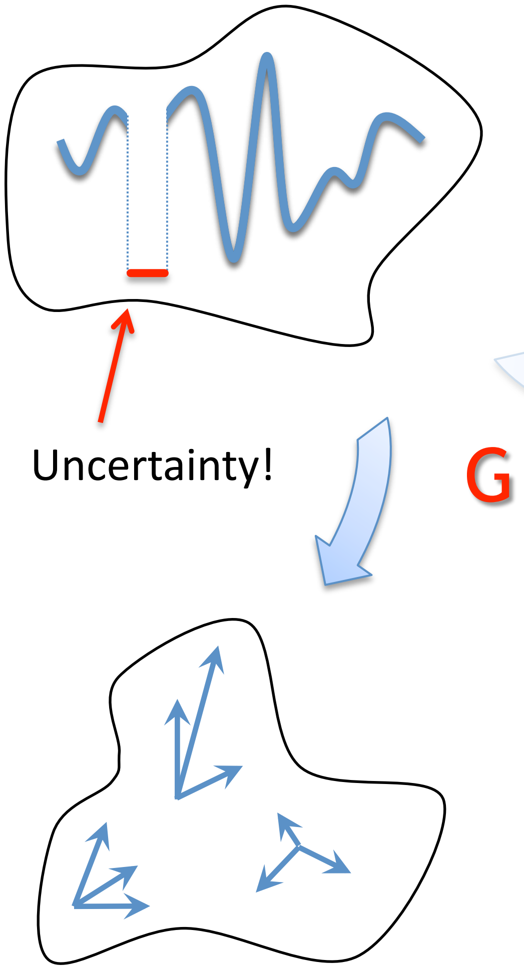
`(1+1)/2 = 1`





CONVOLUTION

Polynomial expansion



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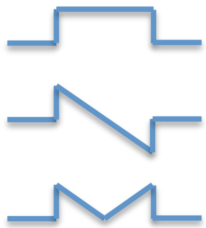
$\otimes$



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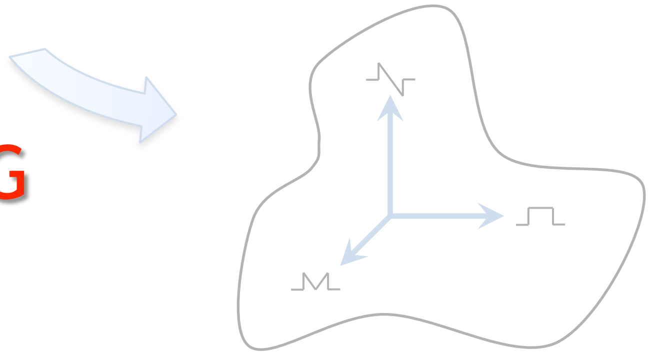

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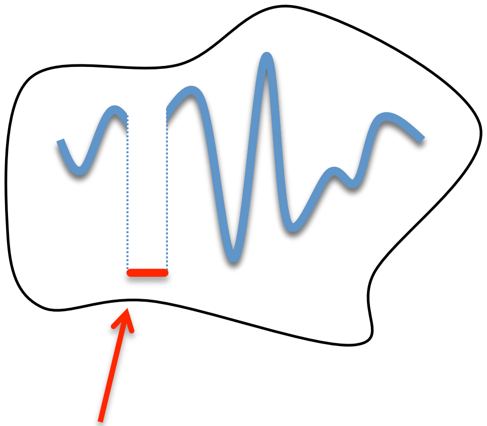
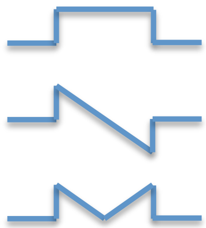
Normalized Convolution



Uncertainty!

Polynomial expansion



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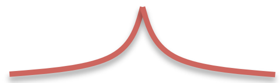

Applicability: a(l)

Signal certainty: $r(k)$

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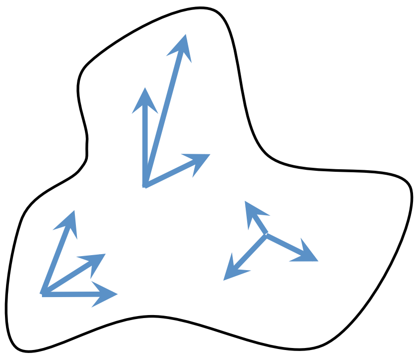
$$c = G^{-1}h_m$$

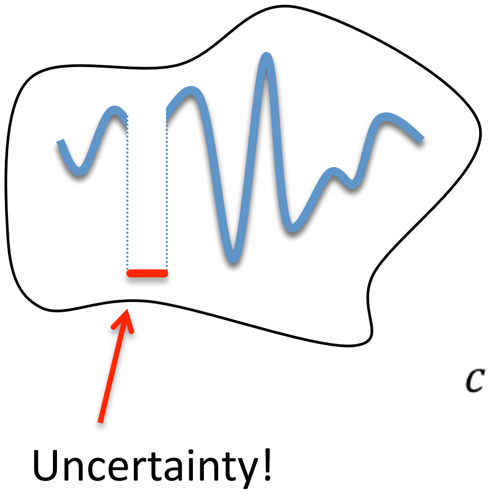
Weighted least squares



Different metric!

Spatially varying!

Normalized Convolution



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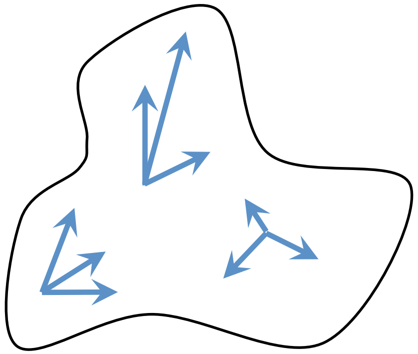
$$c = G^{-1} h_m$$



Different metric! ☹
Spatially varying! ☹

G is only M x M! ☺
Pre-computation! ☺