Plan Reordering and Parallel Execution – A Parameterized Complexity View

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Introduction

We study the optimization problems for *partial-order* and *parallel* plans previously analyzed by Bäckström (JAIR-1998), but applying *parameterized complexity*.

- Analysis of *reordering* and *deordering* of a partial-order plan.
- Consider *parallel-length* of a partial-order plan, and use it as a criterion for plan optimization.

Planning Framework

PARTIAL-ORDER PLAN

- A *planning problem instance (*PPI*)* Π, contains *operator* specifications.
- A *partial-order plan* for Π is $P = \langle A, \prec \rangle$
  - A is a set of operator occurrences (or actions)
  - $\prec$ is an order on A, $a \prec b$ means action a has to be executed before action b.
- A partial-order plan P is Π-*valid* if every topological sorting of it is a solution for Π.

PARALLEL PLAN

- A tuple $P = \langle A, \prec, \# \rangle$ where
  - $\langle A, \prec \rangle$ is a partial-order plan.
  - $\#$ is an irreflexive and symmetric relation on A, called the *non-concurrency* relation. $a \# b$ means actions a and b cannot be executed simultaneously.

Parameterized Complexity Theory

Standard Complexity measures complexity as a function of the input size (n).

Tractable: solvable in time $O(n^c)$ for some constant c.

Parameterized Complexity measures complexity as a function of both input size (n) and a parameter (k) which is independent of n.

Fixed-parameter tractable: solvable in time $O(f(k) \cdot n^c)$, for some function f.

- ***Multi-parameter complexity analysis:*** parameter k is replaced by a list of parameters $k_1, k_2, \ldots, k_l$ and $f(k)$ is replaced by $f(k_1, k_2, \ldots, k_l)$.

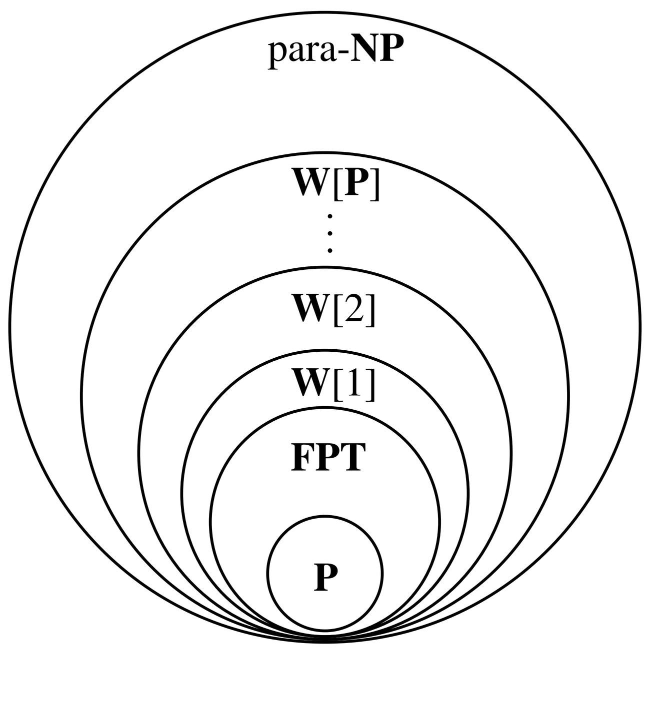
Parameterized Complexity Classes:

FPT: Fixed-parameter tractable problems

W[i]: Defined by WEIGHTED SATISFIABILITY PROBLEM (weight $\leq k$, literal alterations $\leq i$)

W[P]: Same as **W**[i] but with with unbounded alternations.

para-NP: Problems solvable in non-deterministic time $f(k) \cdot n^c$.



Least-constrained Plans

REORDERING

- For any two partial-order plans $P = \langle A, \prec \rangle$ and $Q = \langle A, \prec' \rangle$, and a PPI Π, Q is a *reordering* of P wrt. Π if and only if both P and Q are Π-valid.
- A special case of reordering where $\prec' \subseteq \prec$, is called *deordering*.

INSTANCE:

- A PPI Π.
- A Π-valid partial order plan $P = \langle A, \prec \rangle$.
- A positive integer k.

ADDITIONAL PARAMETERS:

- $n_\prec$: $|\prec|$, i.e. size of the partial order (number of relation tuples)
- $h_\prec$: height of $\prec$, i.e. size of its longest chain
- $w_\prec$: width of $\prec$, i.e. size of its largest antichain

$n_{\prec'}$, $h_{\prec'}$ and $w_{\prec'}$ are defined analogously for the desired order.

QUESTIONS:

MIN-CONSTRAINED REORDERING (MCR): Does P have a Π-valid reordering $Q = \langle A, \prec' \rangle$ such that $|\prec'| \leq k$?

MIN-CONSTRAINED DEORDERING (MCD): Similar question for deordering.

Least-constrained Plans – Results

Summary of the results for different parameter combinations:

<table>
<tr><th></th><th>MCD</th><th>MCR</th></tr>
<tr><td rowspan="2">para-NP
-hard</td><td>$\{h_\prec, h_{\prec'}\}$</td><td>$\{h_\prec\}$</td></tr>
<tr><td colspan="2">$\{w_\prec\}$</td></tr>
<tr><td rowspan="2">in W[P]

W[2]-hard</td><td></td><td>$\{n_\prec\}$</td></tr>
<tr><td colspan="2">$\{n_{\prec'}\}$
$\{n_{\prec'}, h_\prec, h_{\prec'}\}$
$\{n_{\prec'}, w_\prec, h_{\prec'}\}$</td></tr>
<tr><td>FPT</td><td>$\{n_\prec\}$</td><td></td></tr>
</table>

Reordering Parallel Plans

INSTANCE:

- A PPI Π.
- A Π-valid parallel plan $P = \langle A, \prec, \# \rangle$.
- A positive integer l_p.

PARAMETERS:

Every parameter applicable to MCR and PPL.

QUESTIONS:

MIN-PARALLEL REORDERING (MPR): Does P have a Π-valid reordering with a parallel execution of length at most l_p?

MIN-PARALLEL DEORDERING (MPD): Similar question for deordering.

Reordering Parallel Plans – Results

Summary of the results for different parameter combinations appear in the following table:

<table>
<tr><th></th><th>MPD</th><th>MPR</th></tr>
<tr><td>para-NP-hard</td><td colspan="2">$\{l_p, \Delta_\#, n_\prec, n_{\prec'}, h_\prec, h_{\prec'}\}$</td></tr>
<tr><td>in W[P]

W[2]-hard</td><td colspan="2">$\{n_\#, n_{\prec'}\}$
$\delta \cup \{h_\prec, h_{\prec'}\}$
$\delta \cup \{w_\prec, h_{\prec'}\}$</td></tr>
<tr><td>FPT</td><td>$\{n_\#, n_\prec\}$</td><td></td></tr>
</table>

where $\delta = \{l_p, n_\#, n_{\prec'}\}$.

Parallel Plan Length

INSTANCE:

- A PPI Π.
- A Π-valid parallel plan $P = \langle A, \prec, \# \rangle$.
- A positive integer l_p.

PARAMETERS:

- l_p: length of the parallel plan
- $n_\#$: size of the non-concurrency relation
- $\Delta_\#$: maximum degree of $G_\#$ (the graph of non-concurrency relation)
- k: number of processors/parallel actions

QUESTIONS:

PARALLEL ∞-PROC. PLAN LENGTH (PPL): Does P have a parallel execution of length at most l_p? (*unlimited processors/parallel actions*)

PARALLEL k-PROC. PLAN LENGTH (PPL$_k$): Does P have a k-processor parallel execution of length at most l_p?

Parallel Plan Length – Results

- The length of the parallel execution is closely related to the graph coloring problem: ($\chi_\#$ is the chromatic number of $G_\#$)

  $$\max\{h_\prec, \chi_\#\} \leq l_p \leq h_\prec + n_\#$$

  The above inequality is tight.
- Summary of the results for different parameter combinations appear in the following table:

	PPL	PPL$_k$
para-**NP**-hard	$\{l_p, \Delta_\#, n_\prec, h_\prec\}$	$\{l_p, n_\#, \Delta_\#\}$
W[2]-hard		$\{k\}$
FPT	$\{n_\#\}$	$\{l_p, n_\#, k\}$

References

- Christer Bäckström Computational aspects of reordering plans. *Journal of Artificial Intelligence Research*, 9:99–137, 1998.
- Bernhard Nebel and Christer Bäckström. On the Computational Complexity of Temporal Projection, Planning, and Plan Validation. *Artificial Intelligence*, 66(1):125-160, 1994.