Cost-optimal and Net-benefit Planning - A Parameterised Complexity View

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Overview

- Under standard complexity analysis, there is no difference in complexity for different choices of numeric domain for action costs.
- We use parameterised complexity analysis to do a more fine-grained analysis of Cost-optimal and Net-benefit planning:
  - The usage of *zero-cost* or *rational-cost* actions **does** change the parameterised complexity of planning
  - Analysis of a large number of subclasses, using both PUBS restrictions and restricting the number of preconditions and effects

Problem Definition

INSTANCE: $\mathbb{P} = \langle V, A, I, G, c \rangle$

- $\langle V, A, I, G \rangle \in C \subseteq \mathrm{SAS}^{+}$
  V, A, I and G are the set of variables, the set of actions, initial state and goal state, respectively.
- $c : A \to \mathbb{D}$ (numeric) is a cost function

PARAMETER: A non-negative integer k.
QUESTION:

COST-OPTIMAL PLANNING $\big(\mathrm{COP}(C, \mathbb{D})\big)$: Does $\mathbb{P}$ have a plan ω of cost $c(\omega) \leq k$?

Restrictions on C

P (post-unique): No two actions change the same variable to the same value.

U (unary): Each action has only one effect.

B (binary): Each variable takes only two values.

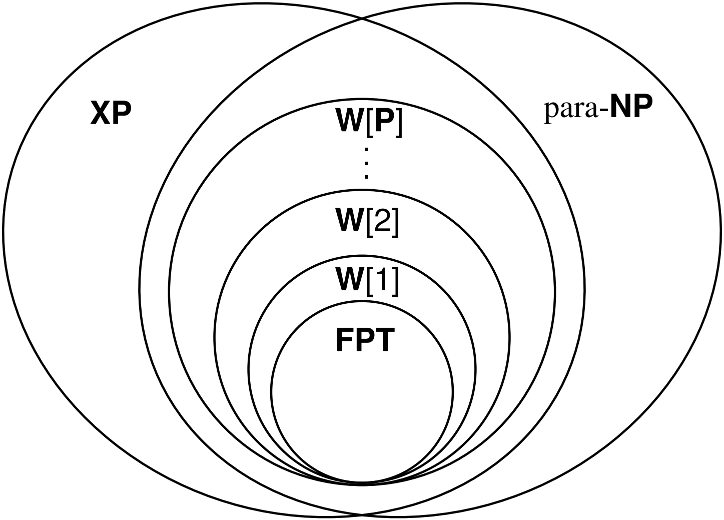
S (single-valued): When two actions have v as their precondition but not as effect, then they require the same value from v.

Parameterised Complexity Theory

Standard Complexity measures complexity as a function of the input size (n).
Tractable: solvable in time $O(n^c)$ for some constant c.
Parameterised Complexity measures complexity as a function of both input size (n) and a parameter (k) which is independent of n.
Fixed-parameter tractable: solvable in time $O(f(k) \cdot n^c)$.
Parameterised Complexity Classes:

- **FPT**: Fixed-parameter tractable problems
- **W**[i]: Defined by WEIGHTED SATISFIABILITY PROBLEM (weight $\leq k$, literal alterations $\leq i$)
- **W**[**P**]: Same as **W**[i] but with with unbounded alternations.
- para-**NP**: Problems solvable in non-deterministic time $f(k) \cdot n^c$.
- **XP**: Problems solvable in time $n^{f(k)}$.

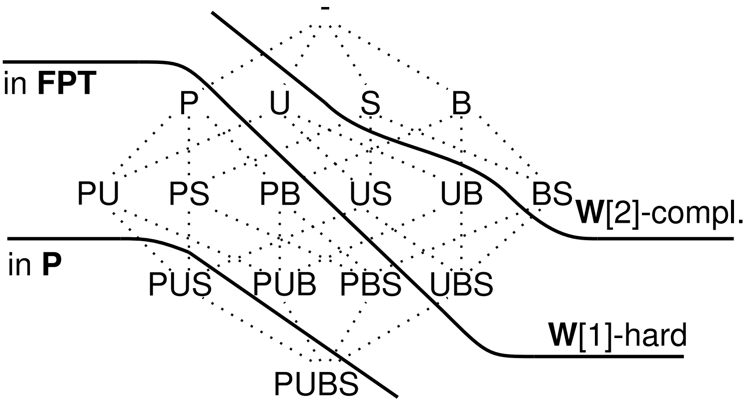
Parameterised Complexity Classes



Cost-optimal Planning (COP) Results

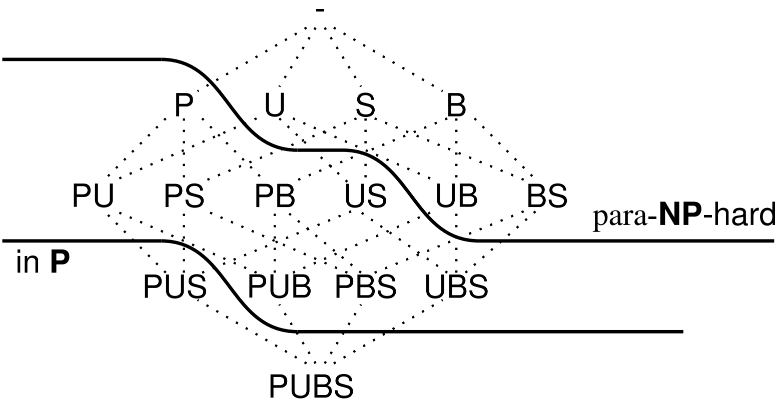
- COP($C, \mathbb{Z}_+$), COP($C, \mathbb{Z}_0$) and COP($C, \mathbb{Q}_+$) are all **PSPACE**-complete and hence cannot be distinguished using standard complexity.

Positive Integers: COP($C, \mathbb{Z}_+$)

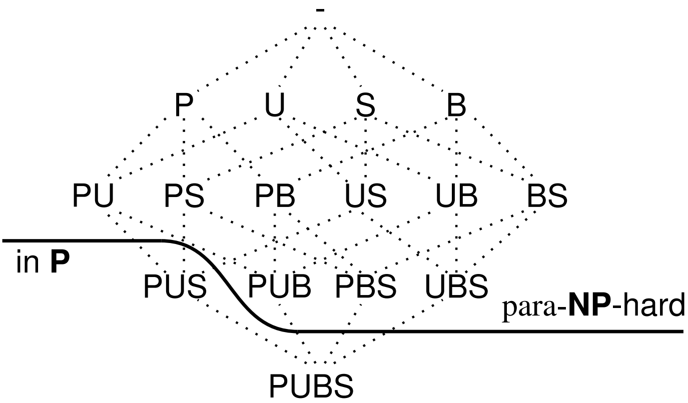


coincide

Non-negative Integers: COP(C, $\mathbb{Z}_0$)



Positive Rationals: COP(C, $\mathbb{Q}_+$)



COP Results - Pre/Eff Restrictions

$\mathrm{COP}(C, \mathbb{Z}_+)$

pre \ eff	1	≥ 3	$*$
0	P	**W**[1]-hard	**W**[2]-hard
≥ 1	**W**[1]-hard	**W**[1]-hard	**W**[2]-hard
$*$	**W**[1]-hard	**W**[1]-hard	**W**[2]-hard

$\mathrm{COP}(C, \mathbb{Z}_0)$

pre \ eff	1	≥ 3	$*$
0	P	**W**[1]-hard	**W**[2]-hard
≥ 1	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard
$*$	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard

COP Results - Pre/Eff Restrictions

$\mathrm{COP}(C, \mathbb{Q}_+)$

eff

pre

	1	≥ 2	$*$
0	P	para-**NP**-hard	para-**NP**-hard
≥ 1	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard
$*$	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard

Net-benefit Planning - Definition

INSTANCE: $\mathbb{P} = \langle V, A, I, G, c, U \rangle$

- $U : \mathrm{vars}(G) \to \mathbb{D}$ is a utility function

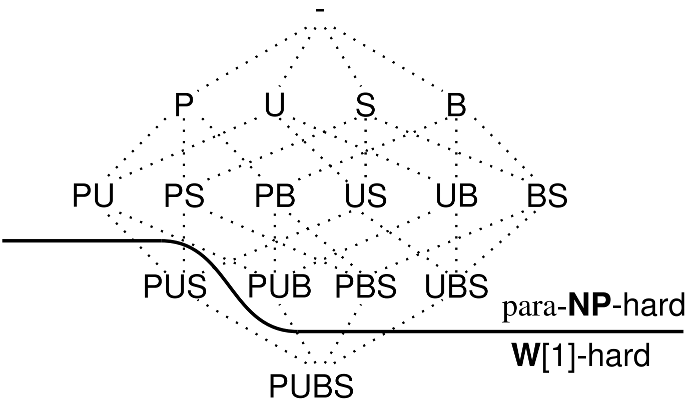
PARAMETER: A non-negative integer k.

QUESTION:

NET-BENEFIT PLANNING $(\mathrm{NBP}(C, \mathbb{D}))$: Is there a state $s \in S(V)$ and a plan ω from I to s such that $U(s) - c(\omega) \geq k$?

NBP Results - PUBS Restrictions

$$\text{NBP}(C, \mathbb{Z}_+),\ \text{NBP}(C, \mathbb{Z}_0),\ \text{NBP}(C, \mathbb{Q}_+)$$



NBP Results - Pre/Eff Restrictions

NBP($C, \mathbb{Z}_+$), NBP($C, \mathbb{Z}_0$), NBP($C, \mathbb{Q}_+$)

pre \ eff	1	≥ 2	$*$
0	P	?	?
≥ 2	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard
$*$	para-**NP**-hard	para-**NP**-hard	para-**NP**-hard

fpt Reduction

An *fpt reduction* from a parameterised language $L \subseteq \Sigma^* \times \mathbb{Z}_0$ to another parameterised language $L' \subseteq \Pi^* \times \mathbb{Z}_0$ is a mapping $R : \Sigma^* \times \mathbb{Z}_0 \rightarrow \Pi^* \times \mathbb{Z}_0$ such that:

(1) $\langle \mathbb{I}, k \rangle \in L \Leftrightarrow \langle \mathbb{I}', k' \rangle = R(\mathbb{I}, k) \in L'$

(2) There is a computable function f and a constant c such that R can be computed in time $f(k) \cdot |\mathbb{I}|^c$

(3) There is a computable function g such that $k' \leq g(k)$

Observation

A COP($\mathrm{SAS}^+, \mathbb{Q}_+$) instance can be polynomially reduced to a COP($\mathrm{SAS}^+, \mathbb{Z}_+$) instance by multiplying all costs and the parameter with a suitable value α. This is, however, **not an fpt reduction** since α will typically not depend on the parameter (only), which contradicts condition (3) for fpt reductions. Hence, the membership results for COP($C, \mathbb{Z}_+$) do not transfer to COP($C, \mathbb{Q}_+$).

Summary

- Cost-optimal planning is **W**[2]-complete for positive integer costs, the same as Length-optimal planning
- Cost-optimal planning is para-**NP**-hard for non-negative integers and positive rationals
- Helps to theoretically explain the differences observed in practice.

Future Work

- To consider additional parameters and do multi-parameter analyses for even more fine-grained results.

The End