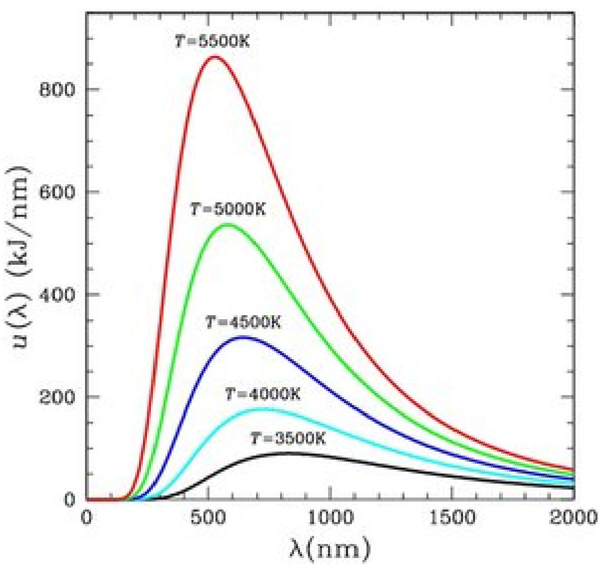
Mathematical Physics of BlackBody Radiation



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Preface

The mystery of blackbody radiation triggered the birth of modern physics in 1900, when Planck in an "act of despair" invented the idea of a smallest quantum of energy, which Nature assembles according to laws of statistics with high frequency high energy waves being rare, because they require many quanta. But Planck viewed quanta to be merely a mathematical trick to resolve a scientific deadlock of classical wave mechanics, a trick without real physical meaning.

Nevertheless, Einstein used a similar idea of "quanta of light" later called photons, to come up with a (simple) formula for the photoelectric effect, which gave him the Nobel Prize in 1921; for the formula but not its derivation based on quanta, because Swedish scientists did not believe in any reality of light quanta or light particles. In late years Einstein confessed that neither he believed in light quanta, but the reservations of the inventors were overwhelmed by the snowball of quantum mechanics starting to roll in the 1920s.

Hundred years later blackbody radiation is back at the center of discussion, now as the cornerstone of climate alarmism based on the idea of atmospheric "backradiation" from so-called "greenhouse gases" causing "global warming". The weakness of this cornerstone is exposed in the book *Slaying the Sky Dragon: Death of the Greenhouse Gas Theory* [19] using arguments from this book.

The basic idea is to use a classical deterministic continuum wave mechanics combined with a new feature of *finite precision computation*, which Nature is supposed to use in analog form and which can be modeled by a computer in digital form. This leads to a form of computational blackbody radiation with close connections to the computational thermodynamics and the 2nd Law of thermodynamics developed in the book Computational Thermodynamics [22].

Statistical models based on microscopic randomness were introduced in thermodynamics by Boltzmann in order to prove and explain the 2nd Law, which seemed impossible usinng classical deterministic continuum models. Planck used the same ”trick” to avoid the seemingly unavoidable “ultraviolet catastrophe” in classical deterministic continuum wave mechanics of blackbody radiation. However, it is in principle impossible to directly test the validity of a model of microscopic randomness, since that would require in microscopics of microscopics. On the other hand, the effect of finite precision computation (which can be viewed as a testable rudimentary form of statistics) can in be determined which makes model verification possible in principle.

The present book can be read as a more detailed account of my arguments in [19] related to radiation, but can also be seen as an attempt to resuscitate classical deterministic continuum mechanics (as opposed to statistical particle mechanics) from the ‘ultraviolet catastrophe” by fusing it with a new concept of finite precision computation.

Stockholm in November 2011

Claes Johnson

Part I

Old Picture

Chapter 1

Blackbody Radiation

> All these fifty years of conscious brooding have brought me no nearer to the answer to the question, "What are light quanta?". Nowadays every Tom, Dick and Harry thinks he knows it, but he is mistaken. (Einstein 1954)

> You are the only person with whom I am actually willing to come to terms. Almost all other fellows do not look from the facts to the theory but from the theory to the facts; they cannot extricate themselves from a once accepted conceptual net, but only flop about in it in a grotesque way. (Einstein to Schrödinger about the statistical "Copenhagen interpretation" of quantum mechanics)

> Would it not be possible to replace the hypothesis of light quanta by another assumption that would also fit the known phenomena? If it is necessary to modify the elements of the theory, would it not be possible to retain at least the equations for the propagation of radiation and conceive only the elementary processes of emission and absorption differently than they have been until now? (Einstein)

1.1 Birth of Modern Physics

Modern physics in the form of quantum mechanics and relativity theory was born in the beginning of the 20th century from an apparent collapse of classical deterministic physics expressing in mathematical terms the rationality of the Enlightenment and scientific revolution as Euler-Lagrange differential equations of Calculus, crowned by *Maxwell's equations for electromagnetics* formulated in 1865 by the British physicist Clerk Maxwell.

The collapse resulted from a couple of scientific paradoxes, which appeared unsolvable using classical physics, both connected to light as electromagnetic waves described by Maxwell's equations:

- *ultra-violet catastrophy* of *blackbody radiation*: infinite energy,
- *non-existence of an aether* as a medium carrying electromagnetic waves.

Theoretical science cannot tolerate paradoxes or contradictions, because in a contradictory mathematical theory everything is both true and false at the same time, and thus a paradox presented by some critics of a theory must be handled one way or the other by the proponents of the theory. A paradox can be deconstructed by showing that it is only an apparent paradox, not a real paradox, which is the only scientifically acceptable solution.

The ultra-violet catastrophegave birth to quantum mechanics and the non-existence of an aether to relativity theory. Today, hundred years and two World Wars later, modern physics has again reached an impasse described in David Lindleys *The End of Physics: The Myth of a Unified Theory* with string theory as an ultimate expression of a depart from rationality in modern physics.

1.2 Planck, Einstein and Schrödinger

The task of resolving the paradox of the ultraviolet catastrophe of blackbody radiation was taken on by the young ambitious physicist Max Planck in his role as scientific leader of the emerging German Empire. After much agony and battle with his scientific soul, in 1900 Planck came up with a resolution which involved a depart from the concept of light as a deterministic wave phenomenon described by Maxwell's equations, to a description by statistics of particles or *quanta* of energy named *photons*.

Planck thus returned to Newton's corpuscular theory of light, which had been replaced by Maxwell's wave theory in the late 19th century, now in a combination with the new particle statistics of thermodynamics developed by Ludwig Boltzmann. In an "act of despair" Planck gave up deterministic continuum physics for statistics of of particles and thus opened the door to modern physics with *wave-particle duality* viewed as a resolution of the inescapable contradiction between wave and particle.

Einstein picked up Planck's quanta as a patent clerk in one of his five articles during his "annus mirabilis" in 1905, and suggested an explanation of a law of photoelectricity which had been discovered experimentally. This gave Planck's quanta a boost and in 1923 Einstein the Nobel Prize in Physics, not for his explanation based on light as particles, which the Nobel Committee did not buy, but for the "discovery" of a law which had already been discovered experimentally.

Both Planck and Einstein introduced discrete quanta of energy as a "mathematical trick" without physical reality in order to avoid the ultraviolet catastrophelong before the quantum mechanics of atoms was formulated in the 1920s in the form of Schrödinger's wave equation, even before the existence of atoms had been experimentally confirmed.

Planck, Einstein and Schrödinger refused to embrace the new quantum mechanics with the wave function as the solution of the Schrödinger's wave equation being interpreted as a probability distribution of discrete particles. They were therefore left behind as modern physics took off on a mantra of wave-particle duality into a new era of atomic physics, with the atomic bomb as evidence that the direction was correct.

The inventors of quantum mechanics were thus expelled from the new world they had created, but the question remains today: Is light waves or particles? What is really wave-particle duality?

There is massive evidence that light is waves, well described by Maxwell's equations. There are some aspects of light connected to the interaction of light and matter in emission and absorption of light which are viewed to be difficult to describe as wave mechanics, with *blackbody radiation* as the basic problem.

If blackbody radiation captured in *Planck's Law of Radiation* can be derived by wave mechanics, then a main motivation of particle statistics disappears and a return to rational determinism may be possible. And after all Schrödinger's equation is a wave equation and Schrödinger firmly believed that there are no particles, only waves as solutions of his wave equation.

1.3 Finite Precision Computation

In this book I present an analysis of blackbody radiation with a new proof of Planck's Law based on a deterministic wave model subject to a certain limitation of *finite precision computation* which replaces the full particle statistics used by Planck in his original proof. Finite precision computation models physics as an analog computational process with input data being transformed to output data, which can be simulated by digital computation. The idea of finite precision computation is also used in an alternative theory of thermodynamics in the form of Computational Thermodynamics [22], without any statistics.

I follow up in Many-Minds Quantum Mechanics [3] following the original idea of Schrödinger to view the Schrödinger wave equation as a model of interacting electrons and atomic in the form of a coupled set wave functions without any need of statistical interpretation. The idea of many-minds is also also used in a new approach to relativity [4] without the paradoxes of Einsteins special theory of relativity.

Chapter 2

Blackbody as Blackpiano

> Experiments on interference made with particle rays have given brilliant proof that the wave character of the phenomena of motion as assumed by the theory does, really, correspond to the facts. The de Broglie-Schrodinger method, which has in a certain sense the character of a field theory, does indeed deduce the existence of only discrete states, in surprising agreement with empirical facts. It does so on the basis of differential equations applying a kind of resonance argument. (Einstein, 1927)

A blackbody is a theoretical idealized object described as something "absorbing all incident radiation" commonly pictured as a cavity or empty bottle/box in which waves/photons are bouncing back and forth between walls at a certain temperature defining the temperature of the cavity. The bottle has a little peephole through which radiation is escaping to be observed, as indicated in the above common illustration of a blackbody.

A blackbody is supposed to capture an essential aspect of the radiation from a real body like the visible glow from a lump of iron at 1000 C, the Sun at 6000 C or the invisible infrared faint glow of a human body at 37 C.

But why is a lump of iron, the Sun or a human body thought of as an empty bottle with a peephole?

Yes, you are right: It is because Planck used this image in his proof of Planck's Law of blackbody radiation based on statistics of energy quanta/photons in a box. Planck's mathematical proof required a certain set up and that set up came to define the idealized concept of a blackbody as an empty bottle with peephole. But to actually construct anything near such a blackbody is impossible.

It is natural to ask if with another proof of Planck's Law, the concept of blackbody would be different, possibly closer to reality?

This book gives a positive answer in a different proof of Planck's Law with a different concept of blackbody as a lattice of vibrating atoms absorbing and emitting radiation as electromagnetic waves, which models a real body like a lump of iron, and not a fictional empty bottle with a peephole. We shall see that here is a close acoustical analog of a such a blackbody in terms of the strings and soundboard of a grand piano: a blackbody as a blackpiano!

This shows the role of mathematics in the formation of concepts of the World:

- With a strange mathematical proof the World may appear strange and incomprehensible.
- With a natural mathematical proof the World my become comprehensible.

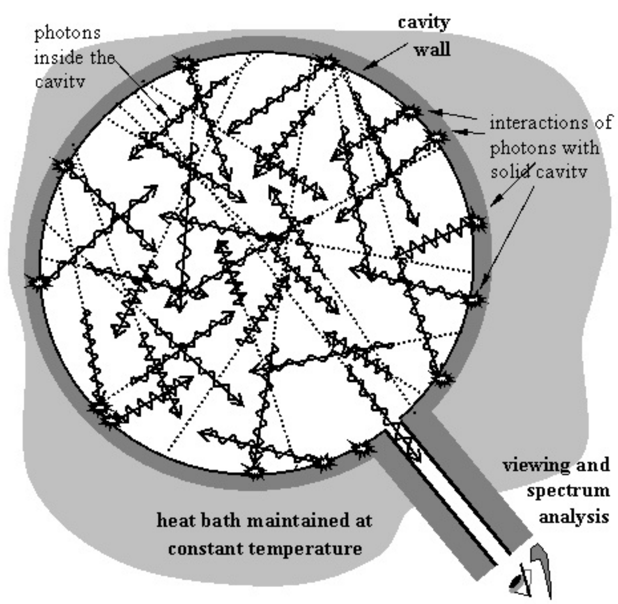

Figure 2.1: Blackbody as a cavity filled with photons bouncing back and forth.

Figure 2.2: Blackbody as strings and soundboard of a grand piano.

Chapter 3

Interaction Light-Matter

> Light and matter are both single entities, and the apparent duality arises in the limitations of our language. (Heisenberg)

> What we observe as material bodies and forces are nothing but shapes and variations in the structure of space. Particles are just schaumkommen (appearances).The world is given to me only once, not one existing and one perceived. Subject and object are only one. The barrier between them cannot be said to have broken down as a result of recent experience in the physical sciences, for this barrier does not exist. (Schrödinger)

One of the big mysteries of physics is the interaction between immaterial light and material matter, or in a wider context the interaction between immaterial soul/mind and material body.

Descartes believed that the interaction soul-body took place in the little pineal gland in the center of the brain. Modern neurobiology does not give much support to Descartes' idea but has not really any better theory and so the mystery of how soul and body interact remains to be resolved.

What does then modern physics say about the interaction of light and matter? There are two competing theories depending on the nature of light as Deterministic electromagnetic waves described by Maxwell's equations. Statistics of massless particles. 2. connects to Newton's old corpuscular theory of light, which was revived by Einstein in 1905 after it had been declared dead and had been replaced by Maxwell's wave theory in the late 19th century.

2. became popular because it offered a resolution to the light-matter interaction problem by simply side-stepping the whole question by claiming

that everything is (statistics of) particles: If light is not an immaterial electromagnetic wave phenomenon, but simply some sort of material particles (albeit without mass, but never mind) then there is no wave-matter problem to resolve!

Clever, but maybe too clever since after all light is an electromagnetic wave phenomenon. This brings us back to 1. and the real question of how an immaterial wave can interact with a material body?

In Mathematical Physics of Blackbody Radiation I suggest a resolution with immaterial waves interacting with matter by wave resonance and statistics replaced by finite precision computation. This is a resolution in terms of waves with electromagnetic wave motion interacting with wave motion in matter ultimately also consisting of electromagnetic waves.

The wave-matter interaction problem is thus in this case resolved by understanding that everything is (finite precision) wave and wave resonance, both light and matter. In the wider context: everything is soul and soul resonance.

We have thus two possible solutions of the light-matter interaction problem:

- Everything is (finite precision) deterministic wave and wave resonance.
- Everything is statistics of particles and collision of particles.

Maxwell and Schrdinger said 2. This book says 2.

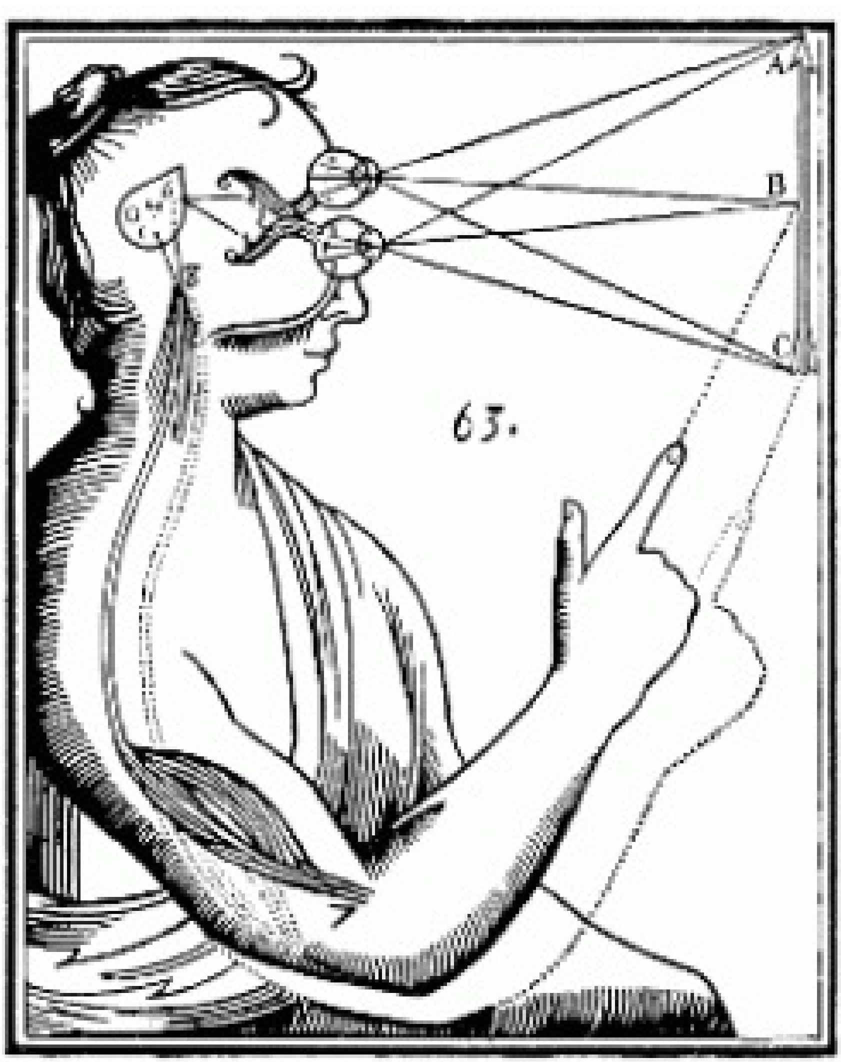

Figure 3.1: Interaction of soul and body through the pineal gland according to Descartes.

Chapter 4

Planck-Stefan-Boltzmann Laws

> The spectral density of black body radiation ... represents something absolute, and since the search for the absolutes has always appeared to me to be the highest form of research, I applied myself vigorously to its solution. (Planck)

4.1 Planck's Law

The particle nature of light of frequency ν as a stream of *photons* of energy $h\nu$ with h Planck's constant, is supposed to be motivated by Einstein's model of the photoelectric effect [6] viewed to be impossible [31] to explain assuming light is an electromagnetic wave phenomenon satisfying Maxwell's equations. The idea of light in the form of energy quanta of size $h\nu$ was introduced by Planck [5] in "an act of despair" to explain the *radiation energy* $R_\nu(T)$ emitted by a blackbody as a function of frequency ν and temperature T, per unit frequency, surface area, viewing solid angle and time:

$$R_\nu(T) = \gamma T \nu^2 \theta(\nu, T), \quad \gamma = \frac{2k}{c^2}, \tag{4.1}$$

with the *high-frequency cut-off* factor

$$\theta(\nu, T) = \frac{\frac{h\nu}{kT}}{e^{\frac{h\nu}{kT}} - 1}, \tag{4.2}$$

where c is the speed of light in vacuum, k is Boltzmann's constant, with $\theta(\nu, T) \approx 0$ for $\frac{h\nu}{kT} > 10$ say and $\theta(\nu, T) \approx 1$ for $\frac{h\nu}{kT} < 1$. Since $h/k \approx 10^{-10}$, this effectively means that only frequencies $\nu \leq T10^{11}$ will be emitted, which fits with the common experience that a black surface heated by the high-frequency light from the Sun, will not itself shine like the Sun, but radiate only lower frequencies. We refer to $\frac{kT}{h}$ as the *cut-off* frequency, in the sense that frequencies $\nu > \frac{kT}{h}$ will be radiated subject to strong damping. We see that the cut-off frequency scales with T, which is *Wien's Displacement Law*. In other words, the cut-off distance in terms of wave-length scales with $\frac{1}{T}$ as shown in Fig. 4.1.

Below we shall for simplicity leave out the constant of proportionality in (4.1) and write $R_\nu(T) \sim T\nu^2\theta(\nu, T)$ expressing the dependence on T and ν, with $\sim$ denoting proportionality. But it is important to note that the constant $\gamma = \frac{2k}{c^2}$ is very small: With $k \approx 10^{-23}\, J/K$ and $c \approx 3 \times 10^8\, m/s$, we have $\gamma \approx 10^{-40}$. In particular, $\gamma\nu^2 << 1$ if $\nu \leq 10^{18}$ including the ultraviolet spectrum, a condition we will meet below.

4.2 Stefan-Boltzmann's Law

By integrating/summing over frequencies in Plancks radiation law (4.1), one obtains *Stefan-Boltmann's Law* stating the the total radiated energy $R(T)$ per unit surface area emitted by a black-body is proportional to T^4:

$$R(T) = \sigma T^4 \tag{4.3}$$

where $\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-8}\, W^{-1}m^{-2}K^{-4}$ is *Stefan-Boltzmann's constant*.

On the other hand, the classical *Rayleigh-Jeans Radiation Law* $R_\nu(T) \sim T\nu^2$ without the cut-off factor, results in an "ultra-violet catastrophy" with infinite total radiated energy, since $\sum_{\nu=1}^{n} \nu^2 \sim n^3 \rightarrow \infty$ as $n \rightarrow \infty$.

Stefan-Boltzmann's Law fits (reasonably well) to observation, while the Rayleigh-Jeans Law leads to an absurdity and so must somehow be incorrect. The Rayleigh-Jeans Law was derived viewing light as electromagnetic waves governed by Maxwell's equations, which forced Planck in his "act of despair" to give up the wave model and replace it by statistics of "quanta" viewing light as a stream of particles or photons. But the scientific cost of abandoning the wave model is very high, and we now present an alternative way of avoiding the catastropheby modifying the wave model by *finite precision computation*, instead of resorting to particle statistics.

We shall see that the finite precision computation introduces a high-frequency cut-off in the spirit of the finite precision computational model

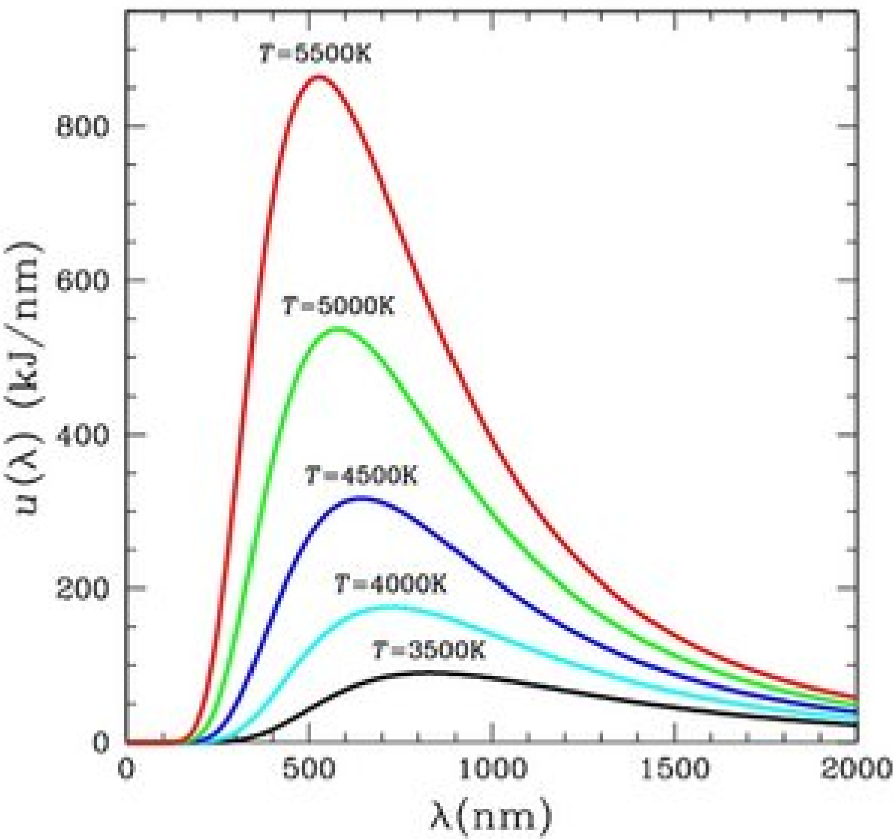

Figure 4.1: Radiation Energy vs wave length at different temperatures of a radiating body, per unit frequency. Observe that the cut-off shifts to higher frequency with higher temperature according to Wien's Displacement Law

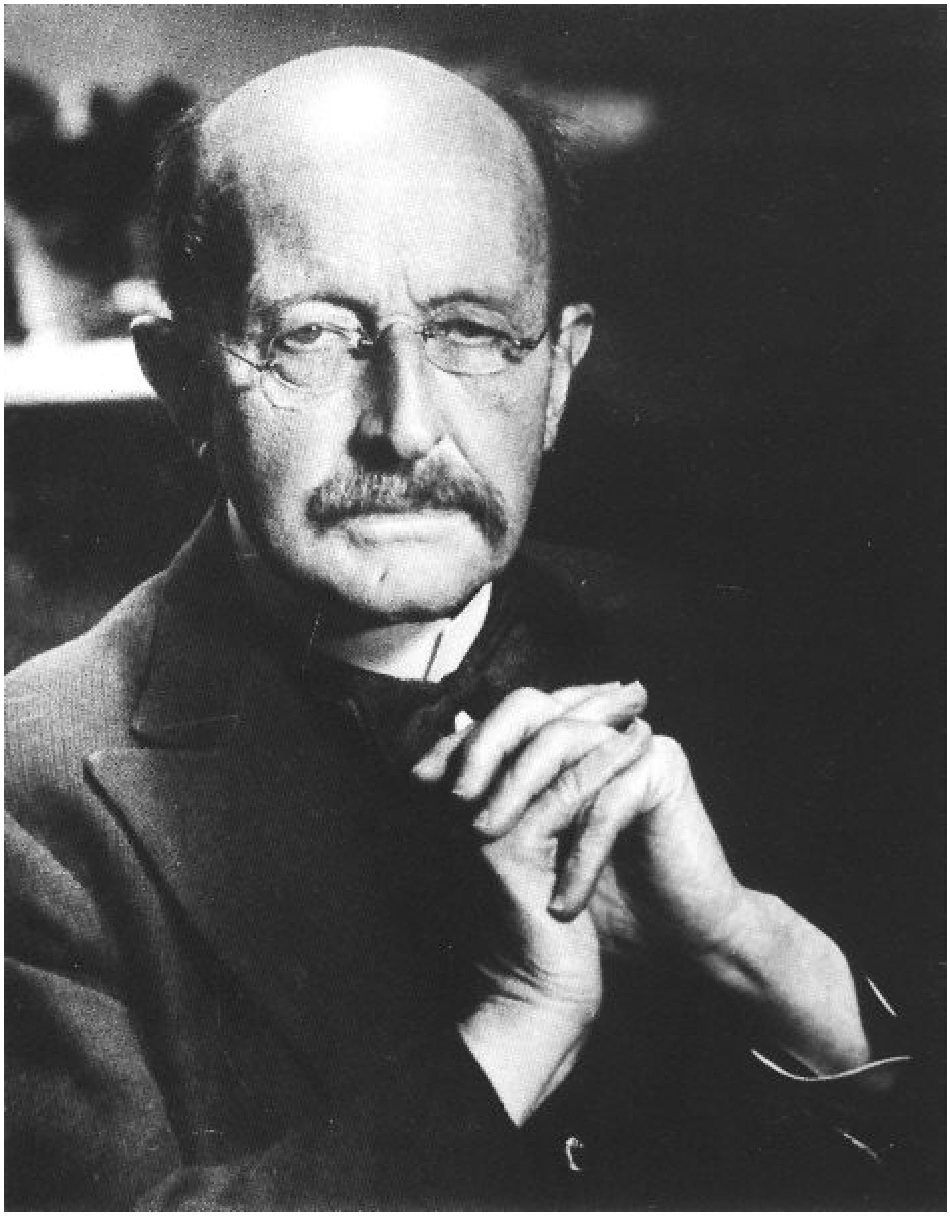

Figure 4.2: Planck 1900: *...the whole procedure was an* **act of despair** *because a theoretical interpretation had to be found at any price, no matter how high that might be...*

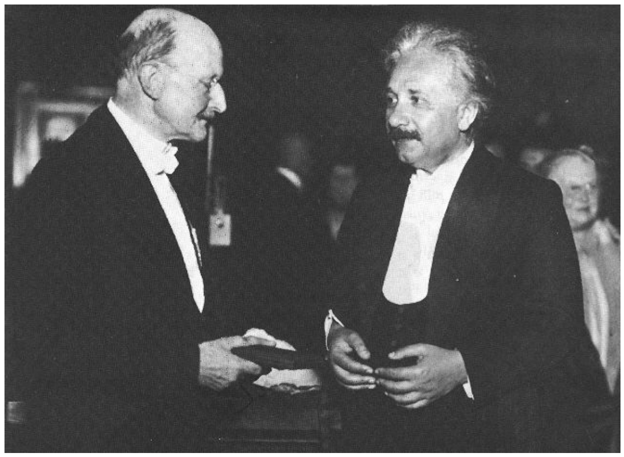

Figure 4.3: Planck to Einstein: *I hereby award you the Planck Medal because you expanded my desperate idea of quantum of energy to the even more desperate idea of quantum of light.*

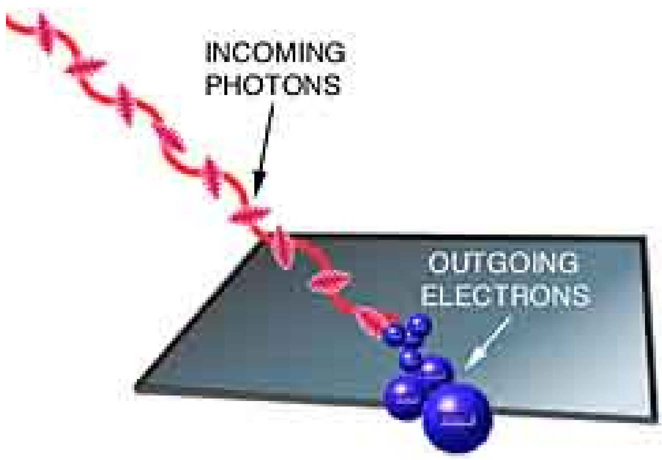

Figure 4.4: The photoelectric effect according to Einstein.

for thermodynamics presented in [22].

The photon is considered to be a "particle" with no mass and no charge, but to observe individual photons appears to be extremely difficult. In fact, the existence of photons seems to be highly hypothetical with the main purpose of explaining black-body radiation and the photoelectric effect. If explanations of these phenomena may be given using classical wave mechanics, maybe the existence of photons as particles without both mass and charge may be seriously questioned, including statistical particle mechanics, as Einstein himself did during the later half of his life [16, 10, 11, 12, 14, 18].

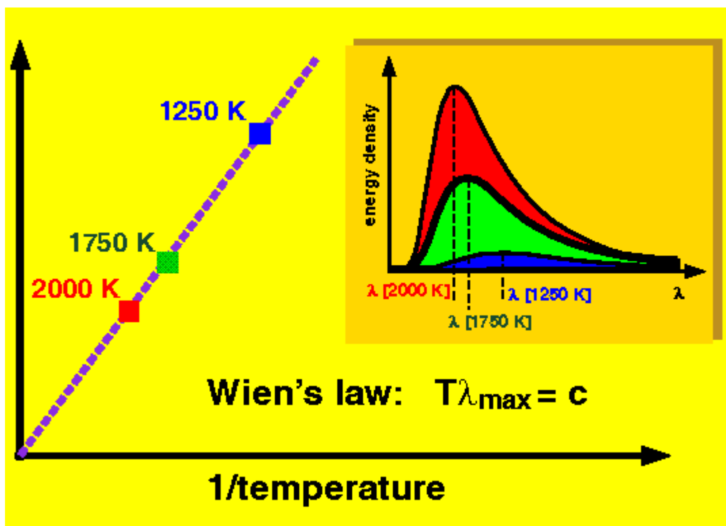

Figure 4.5: Wien's Displacement Law.

The scientific price of resorting to statistical mechanics is high, as was clearly recognized by Planck and Einstein, because the basic assumption of statistical mechanics of microscopic games of roulette seem both scientifically illogical and impossible to verify experimentally. Thus statistical mechanics runs the risk of representing *pseudo-science* in the sense of Popper [**?**] because of obvious difficulties of testability of basic assumptions.

The purpose of this note is to present an alternative to statistics for black-body radiation based on finite precision compuation in the form of General Galerkin G2. We also extend to include aspects of photo-electricity and the Compton effect.

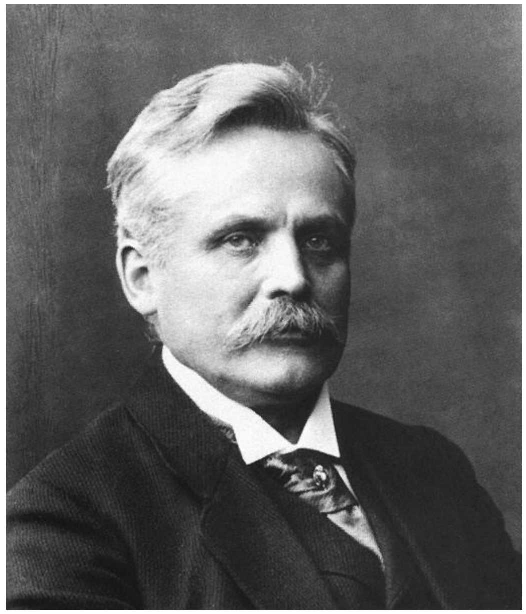

Figure 4.6: Wilhelm Wien (1864-1928).

4.3 The Enigma of the Photoelectric Effect

The most convincing evidence of the particle nature of light is supposed to be that the photoelectric effect has a dependence on the frequency of the incident light, which is not present in a basic linear wave mechanical model without viscosity effects: A electric current of electrons ejected by incident light requires the frequency to be larger than a certain threshold, and morover the kinetic energy of the ejected electrons scales with the frequency above the threshold. Photoelectricity thus has a frequency dependence, which is not present in a linear wave model with solutions scaling with the intensity of the forcing.

But a wave model with a small viscosity acting on derivatives of the state, also exhibits frequency dependence, and can be designed to model basic aspects of photoelectricity. In particular we shall find that G2 finite precision computation introduces a frequency dependent viscosity acting above a certain threshold and thus shares features with photoelectricity. The argument that the only particle models are capable of describing photoelectricity thus is weak. Since this is the main argument, it appears that the evidence against the particle nature of light suggested by Newton, presented by Young, Fresnel and Maxwell in the 19th century, is still strong.

4.4 The Enigma of Blackbody Radiation

The basic enigma of blackbody radiation can be given different formulations:

- Why is a blackbody black/invisible, by emitting infrared radiation when "illuminated" by light in the visible spectrum?
- Why is radiative heat transfer between two bodies always directed from the warmer body to the colder?
- Why can high frequency radiation transform to heat energy?
- Why can heat energy transform to radiation only if the temperature is high enough?
- Why is low-frequency radiative heating inefficient?

We shall find that the answer is *resonance in a system of resonators* (oscillating molecules):

- incoming radiation is absorbed by resonance,
- absorbed incoming radiation is emitted as outgoing radiation, or is stored as internal/heat energy,
- outgoing radiation has a frequency spectrum $\sim T\nu^2$ for $\nu \lesssim T$, assuming all frequencies ν have the same temperature T, with a cut-off to zero for $\nu \gtrsim T$,
- incoming frequencies below cut-off are emitted,
- incoming frequencies above cut-off are stored as internal heat energy.

4.5 Confusion in Media

The mystery of blackbody radiation opened to the mystery of quantum mechanics permeating modern physics:

- Einstein vs Niels Bohr

- Solvay Conference 1927
- Max Planck and Blackbody Radiation 1
- Max Planck and Blackbody Radiation 2
- Theory of Heat Radiation by Max Planck

4.6 Confessions by Confused Scientists

To motivate that a renewed analysis of blackbody radiation is needed, 110 years after Planck, we recall some statements of famous scientists indicating what they really think about the quantum mechanics, light quanta and photons forming the basis of Planck's description of blackbody radiation (including the introductory quotes by Einstein and Planck's "act of despair").

> If we are going to have to put up with these damned quantum jumps, I am sorry that I ever had anything to do with quantum mechanics. (Schrödinger to Bohr 1926)

> To derive the Planck radiation law, it is essential that the energy of the atom have discrete values and changes discontinuously. (Bohr to Schrödinger 1926).

> The discussion between Bohr and Schrödinger began at the railway station in Copenhagen and was carried out every day from early morning to late night. Bohr appeared to me like a relentless fanatic, who was not prepared to concede a single point to his interlocutor or to allow him the slightest lack of precision. It will scarely be possible to reproduce how passionate the discussion was carried out from both sides. (Heisenberg in Der Teil und das Ganze)

> Niels Bohr brainwashed a whole generation of theorists into thinking that the job of interpreting quantum theory was done 50 years ago. (Murray Gell-Mann)

> It is nonsense to talk about the trajectory of an electron inside an atom (Schrödinger to Born 1927 at the 5th Solvay Conference in Brussels).

Contemporary physicists come in two varieties. Type 1 physicists are bothered by EPR and Bell's Theorem. Type 2 (the majority) are not, but one has to distinguish two subvarieties. Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein) or to contain physical assertions that can be shown to be false. Type 2b are not bothered and refuse to explain why. (David Mermin)

I think it is safe to say that no one understands quantum mechanics. Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will go 'down the drain' into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that. (Richard Feynman)

Einstein presented an extended critique of the Copenhagen interpretation of quantum mechanics (at the 5th Solvay Conference 1927 in Brussels), and his debate with Bohr continued inside and outside the conference rooms, It provided the greatest excitement of the meeting and was a historic occasion, a battle of titans over the epistemological foundations of physics and over the way in which scientists should understand the world. When the meeting ended, however, most of the physicists departed with the belief that the positivist Copenhagen view had prevailed, a belief nourished by the anti-realist philosphical tradition of central Europe. But Einstein, de Broglie and Schrödinger were not convinced, and they left what Einstein once called "the witches sabbath at Brussels" with a resolve to fight another day. (Moore in A Life of Erwin Schödinger)

I reject the basic idea of contemporary statistical quantum theory, insofar as I do not believe that this fundamental concept will provide a useful basis for the whole of physics...one is driven to the conviction that a complete description of a single system should, after all, be possible, but for such complete description there is no room in the conceptual world of statistical quantum theory... If it should be possible to move forward to a complete description, it is likely that the laws would represent relations among all the conceptual elements of this description which, per se, have nothing to do with statistics. (Einstein's Reply to Criticisms in Albert Einstein: Philosopher-Scientist, Library of Living Philosophers Series, Cambridge University Press, 1949)

4.7 Towards Enigma Resolution

We shall find that finite precision computation, in G2 appearing from residual stabilization, as a small-coefficient viscosity acting on higher derivatives of the state function, makes it possible to avoid the seemingly unsurmountable difficulties hampering classical continuum mechanics in the late 19th century, including "d'Alembert's paradox" of turbulent fluid mechanics, the "reversibility paradox" of the 2nd Law of thermodynamics and the "ultraviolet catastrophe" of blackbody radiation. The difficulties arise from unresolved microscopics in macroscopic continuum models and the only way out was believed to be by modeling the microscopics by statistics of pointlike particles, and this became the mantra of 20th century physics.

However, a medication with microscopic particle statistics comes along with several side effects, so severe that e.g. Einstein and Schrödinger refused to accept it, and thus a return to deterministic continuum models would seem desirable, if only the paradoxes and catatsrophies can be dealt with in a resonable way. We have shown in [21] that finite precision computation allows a resolution of d'Alembert's paradox and several of the mysteries of turbulent fluid mechanics, as well as a formulation of the 2nd Law without entropy statistics in [22] , and in this book we use the same general approach for the wave mechanics of radiation.

One can view deterministic finite precision computation as a primitive form of statistics, so primitive that the side effects do not show up, while the positive effect remains.

Chapter 5

Planck/Einstein Tragedy

5.1 James Jeans

Sir James Jeans states in *The Growth of Physical Science* shortly before his death in 1947:

- *The radiation from a red-hot body presented the same difficulty in a slightly different form. The theorem of equipartition showed that the radiation from such a body ought to consist almost entirely of waves of the shortest possible wave-length. Experiment showed the exact opposite to be the case.*

- *The first move to end the deadlock was made by Max Planck, Professor in Berlin University, and subsequently in the Kaiser Wilhelm Institute. In an epoch-making paper which he published in 1900, he imagined all matter to consist of vibrators, each having its own particular frequency of vibration, and emitting radiation of this frequency, just as a bell emits sound of its own frequency of vibration.*

- *This was completely in accordance with current ideas, but Planck now introduced the startling assumption that the vibrators did notemit energy in a continuous stream, but by a series of instantaneous gushes. Such an assumption was in flagrant oppositionto Maxwell's electromagnetic laws and to the Newtonian mechanics; it dismissed continuity from nature, and introduced a discontinuity for which there was so far no evidence. Each vibrator was supposed to have a certain unit of ra-*

diation associated with it, and could emit radiation only in complete units.

- *It could never emit a fraction of a unit, so that radiation was assumed to be atomic. Such an assumption naturally led to very different results from the Newtonian mechanics, but Planck was able to show that nature sided with him. His theory predicted the observed emission of radiation from a hot body exactly. Planck described his units of radiation as* **quanta**.

- *The amount of energy in any unit depended on the vibrator from which the unit came being equal to the frequency ofits vibrations multiplied by a constant h, which is generally known as Planck's constant; this has proved to be one of the fundamental constants of the universe like the charge on an electron or the mass of a proton. Through all the changes which the quantum theory has experienced and they are many h has stood firm as a rock, but we now associate it with radiation rather than with vibrators.*

5.2 Max Planck

We cite from [5]:

- *We shall now derive strange properties of heat radiation described by electromagnetic wave theory.*

- *We shall assume that the radiation in one direction is completely independent of the radiation in a different direction, even opposite.*

- *Zur radikalsten Affassung neigt J.J Thompson und A. Einstein, welche glauben, das die Fortpflanzung der elektromagnetischen Wellen nicht genau nach den Maxwellshen Feldgleichungen, sondern nach gewissen Energiequanten* $h\nu$ *erfolgt. Ich meine dagegen, dass man einstweilen noch nicht genotig ist, so revolutionär vorzugehen, sondern das mann damit auskommen dürfte, die Bedeutung des Energiequantums* $h\nu$ *lediglich in den Wechselwirkungen zu suchen, mit denen die Resonatoren einander beeinflussen. Eine definitive Entscheidigung über diese prinzipiellen Fragen können aber erst weiter Erfahrungen bringen.*

Figure 5.1: Max Planck 1901 being struck with the idea of energy quanta: *We shall now derive strange properties of heat radiation described by electromagnetic wave theory.*

Planck here describes that the seemingly absurd consequences of electromagnetic wave theory can be handled by introducing finite energy quanta, but he is not willing to pay the prize of viewing light as a stream of particles. Instead he klings to a faint hope that somehow wave theory can be saved by some form of interaction between the resonators. What we will now do is to give substance to this hope by replacing "finite energy quanta" by finite precision wave mechanics.

We have seen that Planck was ambigous: He could not believe in light as streams of discrete quanta but yet he made the basic assumption that radiation in different directions, even opposite, is fully independent, which can only be motivated from a particle nature of light. This double-play has become a principle of modern physics: light is both waves and particles and you are free to choose whatever description that serves you the best in every specific case. Planck's scientific conscience protested against the double-play, but was overuled by its effectiveness: It took physics out of its late 19th century trauma. Over time the double-play has become a virtue, and is today questioned by few physicists. But double-play is double-play and fair-play is more honorable, even in science.

Planck's idea of independent radiation in opposite directions, is today being used by some climate scientists to sell the idea of "backradiation" as a basis of "global warming" with the radiation from the Earth surface absorbed by the atmosphere being "backradiated" to and thus heating the Earth surface. We will below show that the warming effect of "backradiation" is fictitious and by Ockham's razor can be moved to the wardrobe of non-physical physics.

5.3 Planck and Einstein

Both Planck and Einstein struggled with the particle concept of energy and quanta, by realizing its use as a "mathematical trick" to resolve an apparent paradox of wave mechanics and at the same time being unable to give up deterministic wave mechanics by particle statistics:

- *We therefore regard - and this is the most essential point of the entire calculation - energy to be composed of a very definite number of equal packages* (Planck 1900).
- *The wave theory of light, which operates with continuous spatial func-*

tions, has worked well in the representation of purely optical phenomena and will probably never be replaced by another theory (Einstein).

- *I do not seek the meaning of "quantum of action" (light quantum) in the vacuum but at the site of absorption and emission* (Planck 1907).
- *Despite the apparently complete success of the Einstein equation (for the photoelectric effect), the physical theory on which it was designed to be the symbolic expression, is found so untenable that Einstein himself, I believe, no longer holds to it* (Millikan).
- *My futile attempts to fit the elementary quantum of action into classical theory continued for a number of years and cost me a great deal of effort. Many of my collegues saw in this something bordering on a tragedy* (Planck shortly before his death).
- *Einstein is increasingly aloof and sceptical (about the quantum discoveries he pioneered). Many of us regards this as a tragedy* (Born).

Note further that the equation of quantum mechanics, Schrödinger's equation, is a wave equation over a continuum. Thus quantum mechanics, despite its confusing name, is not a mechanics of discrete "quanta" but a wace mechanics, which can only be decribed as a "a tragedy". We present below a happy end to the tragedy.

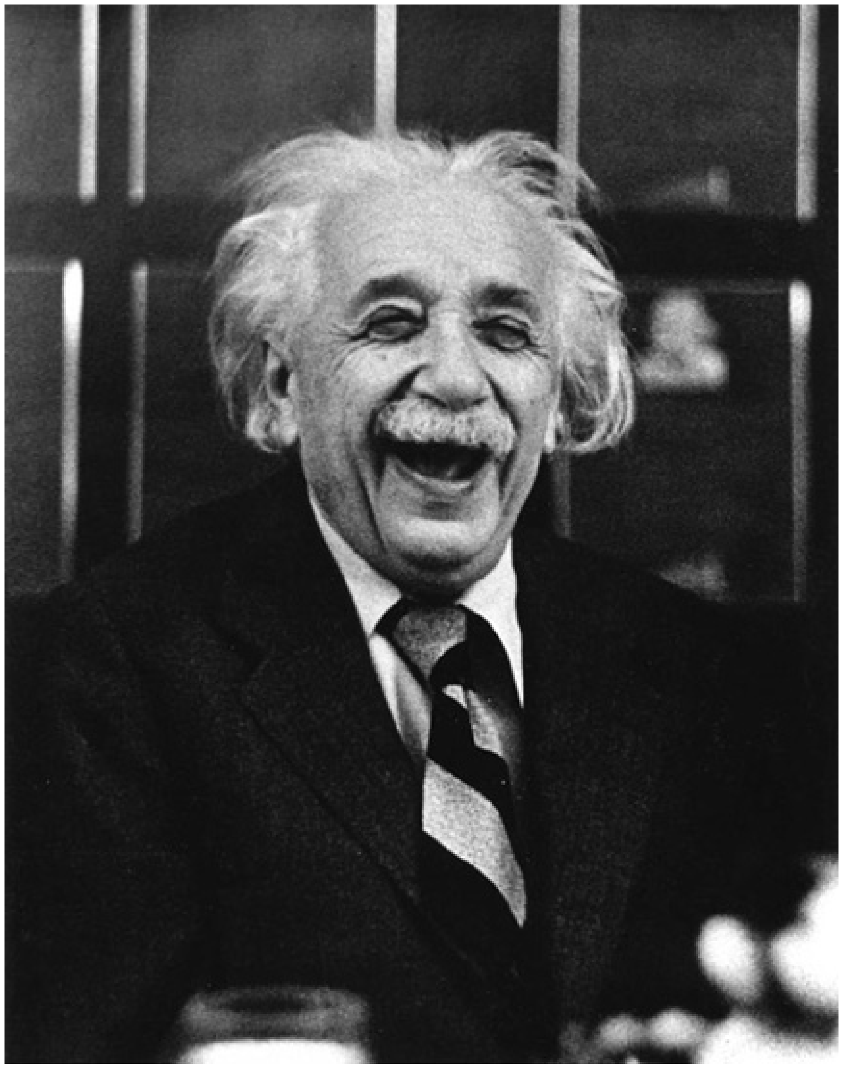

Figure 5.2: Einstein: *The more success the quantum mechanics has, the sillier it looks.*

Chapter 6

Classical Derivation of Rayleigh-Jeans Law

> Sciences usually advances by a succession of small steps, through a fog in which even the most keen-sighted explorer can seldom see more than a few paces ahead. Occasionally the fog lifts, an eminence is gained, and a wider stretch of territory can be surveyedsometimes with startling results. A whole science may then seem to undergo a kaleidoscopic rearrangement, fragments of knowledge sometimes being found to fit together in a hitherto unsuspected manner. Sometimes the shock of readjustment may spread to other sciences; sometimes it may divert the whole current of human thought. (James Jeans)

6.1 Counting Cavity Degrees of Freedom

The classical derivation of Rayleigh-Jeans Law is based on computing the number of standing waves in a resonating cubical cavity of side π, per unit frequency ω, of the form

$$u(x,t) = \sin(\nu_1 x_1)\sin(\nu_2 x_2)\sin(\nu_3 x_3)\sin(\omega t), \quad x = (x_1, x_2, x_3), \tag{6.1}$$

as solutions of the wave equation

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} = \frac{\partial^2 u}{\partial t^2} \tag{6.2}$$

with the n_i natural numbers satisfying

$$n_1^2 + n_2^2 + n_3^2 = \omega^2. \tag{6.3}$$

The number of modes $N(\omega)$ of frequency less than ω scales like the volume of a sphere of radius ω and thus $N(\omega) \sim \omega^3$, which gives

$$\frac{dN}{d\omega} \sim \omega^2. \tag{6.4}$$

Postulating *equipartition in energy* that is that all modes radiate the same energy kT, gives Rayleigh-Jeans Law on the form $R_\omega(T) \sim kT\omega^2$, which is the same form as that derived above with a different argument.

We thus arrive at the same Rayleigh-Jeans formula using two fundamentally different approaches, and one may ask which argument is the better in the sense that it best explains the physical mechanism behind the formula?

The classical argument connects radiance to the number of modes without specifying the coupling mechanism and a mechanism for equipartition and in this sense is *ad hoc*. This connects to arguments used in statistical mechanics based on computing numbers of permutations assigned equal probability.

The above argument based on the wave equation with a radiation term includes more physics and and will be complemented with model for equilibration in frequency below. This argument thus may be less ad hoc than the classical argument.

6.2 Dependence of Space Dimension

The classical derivation of Rayleigh-Jeans Law counts the number of modes in a three-dimensional cavity and would give a different law in one and two dimensions. In contrast our derivation based on a wave equation with radiation gives the same law in any dimension, since the law reflects the form of the radiation term which is the same in all dimensions.

Chapter 7

Statistics vs Computation

> This inhibits us from accepting in a naive way a "blurred model" as an image of reality...There is a difference between a shaky or not sharply focussed photograph and a photograph of clouds and fogbanks. (Schrödinger about the Copenhagen interpretation)

7.1 Cut-Off by Statistics

The Rayleigh-Jeans Law leads to an "ultraviolet catastrophy" because without some form of high-frequency limitation, the total raditation will be unbounded. Classical wave mechanics thus appears to lead to an absurdity, which has to be resolved in one way or the other. In an "act of despair" Planck escaped the catastropheby an Alexander Cut simply replacing classical wave mechanics with a new statistical mechanics where high frequencies were assumed to be rare; "a theoretical interpretation had to be found at any price, no matter how high that might be...". It is like kicking out a good old horse which has served fine for many purposes, just because it has a tendency to "go to infinity" at a certain stimulus, and replacing it with a completely new wild horse which you don't understand and cannot control.

7.2 Cut-Off by Finite Precision Computation

The price of throwing out classical wave mechanics is very high, and it is thus natural to ask if this is really necessary? Is there a form of classical mechanics without the ultraviolet catastrophy? Can a cut-off of high frequencies be performed without an Alexander cut-off?

We believe this is possible, and it is certainly highly desirable, because statistical mechanics is difficult to both understand and apply. We shall thus present a resolution where Planck's statistical mechanics is replaced by deterministic mechanics viewing physics as a form of *analog computation with finite precision* with a certain dissipative diffusive effect, which we model by digital computational mechanics coming along with a certain numerical dissipation.

It is natural to model finite precision computation as a viscous dissipative effect, since finite precision means that small details are lost as in smoothing by damping of high frequencies which is the effect of viscous dissipation.

We consider computational mechanics in the form of the *General Galerkin (G2)* method for the wave equation, where the dissipative mechanism arises from *weighted least squares residual stabilization* [21]. We shall first consider a simplified form of G2 with least squares stabilization of one of the residual terms appearing as a viscosity acting only on high frequencies. We then comment on full G2 residual stabilization.

Figure 7.1: A blackbody acts like a censor or high-pass filter which transforms coherent high-frequency high-interest information into incoherent noise, while it lets low-frequency low-interest information pass through.

Part II

New Analysis

Chapter 8

Wave Equation with Radiation

There are no quantum jumps, nor are there any particles. (H.D. Zeh [44])

8.1 A Basic Radiation Model

We consider the wave equation with radiation, for simplicity in one space dimension assuming periodicity: Find $u = u(x,t)$ such that

$$\ddot{u} - u'' - \gamma \dddot{u} = f, \quad -\infty < x, t < \infty \tag{8.1}$$

where (x,t) are space-time coordinates, $\dot{v} = \frac{\partial v}{\partial t}$, $v' = \frac{\partial v}{\partial x}$, $f(x,t)$ models forcing in the form of incoming waves, and the term $-\gamma \dddot{u}$ models outgoing radiation with $\gamma > 0$ a small constant.

This models, in the spirit of Planck [5] before collapsing to statistics of quanta, a system of resonators in the form of a vibrating string absorbing energy from the forcing f of intensity f^2 and dissipating energy of intensity $\gamma \ddot{u}^2$ as radiation, while storing or releasing vibrational (heat) energy in energy balance.

The wave equation (8.1) expresses a force balance in a vibrating system of charged particles with u representing the displacement from a reference configuration with $\dot{u}$ velocity and $\ddot{u}$ accelleration, and $-\gamma \dddot{u}$ represents the *Abraham-Lorentz recoil force* from an accellerating charged particle [43]. Energy balance follows from the force balance by multiplication by $\dot{u}$ followed by integration, which gives the dissipated radiated energy $\gamma \ddot{u}^2$ by integration by parts (from $-\gamma \dddot{u}$ multiplied by $\dot{u}$), referred to as *Lamours formula* [43].

In a mechanical analog the dissipative radiation term $-\gamma\dddot{u}$ is replaced by the dissipative viscous term $\mu\dot{u}$ with $\mu > 0$ a viscosity, with now dissipated energy $\mu\dot{u}^2$.

In both cases the model includes a dissipative mechanism describing energy loss (by radiation or viscosity) in the system, but the model does not describe where the lost energy ends up, since that would require a model for the receptor. The mechanical model has a direct physical representation as a forced vibrating string subject to a viscous damping force $\mu\dot{u}$. The radiative model is to be viewed as a conceptual model with radiative damping from an Abraham-Lorentz recoil force $-\gamma\dddot{u}$.

We shall see that the form of the damping term determines the energy spectrum, which thus is fundamentally different in the viscous and the radiative case.

We shall see that the equation modeling a vibrating string with radiative damping can be used as a concrete mathematical model of universal blackbody radiation with the coefficient γ chosen maximal as reference. We can view this model as concrete realization open to analysis of the standard conceptual model as an empty cavity with the property of absorbing (and re-emitting) all incident radiation. By studying the model we can explore aspects of radiation including universality of blackbody radiation.

8.1.1 Basic Energy Balance

Multiplying (8.1) by $\dot{u}$ and integrating by parts over a space period, we obtain

$$\int (\ddot{u}\dot{u} + \dot{u}'u')\, dx - \int \gamma\dddot{u}\dot{u}\, dx = \int f\dot{u}\, dx,$$

which we can write

$$\dot{E} = a - r \tag{8.2}$$

where

$$E(t) \equiv \frac{1}{2}\int (\dot{u}(x,t)^2 + u'(x,t)^2)\, dx \tag{8.3}$$

is the internal energy viewed as heat energy, and

$$a(t) = \int f(x,t)\dot{u}(x,t)\, dx, \quad r(t) = -\int \gamma\dddot{u}(x,t)\dot{u}(x,t)dx, \tag{8.4}$$

is the absorbed and radiated energy, respectively, with their difference $a - r$ driving changes of internal energy E.

Assuming time periodicity and integrating in time over a time period, we have integrating by parts in time,

$$R \equiv \int r(t)\,dt = \int\int \gamma\ddot{u}(x,t)^2 dxdt \geq 0 \tag{8.5}$$

showing the dissipative nature of the radiation term.

If the incoming wave is an emitted wave $f = -\gamma\dddot{U}$ of amplitude U, then

$$A - R \equiv \int\int (f\dot{u} - \gamma\ddot{u}^2) dxdt = \int\int \gamma(\ddot{U}\ddot{u} - \ddot{u}^2)\,dx \leq \frac{1}{2}(\bar{R}_{in} - \bar{R}), \tag{8.6}$$

with $R_{in} = \int\int \gamma\ddot{U}^2\,dxdt$ the incoming radiation energy, and R the outgoing. We conclude that if $E(t)$ is increasing, then $\bar{R} \leq \bar{R}_{in}$, that is, in order for energy to be stored as internal/heat energy, it is required that the incoming radiation energy is bigger than the outgoing.

Of course, this is what is expected from conservation of energy. It can also be viewed as a 2nd Law of Radiation stating that radiative heat transfer is possible only from warmer to cooler. We shall see this basic law expressed differently more precisely below.

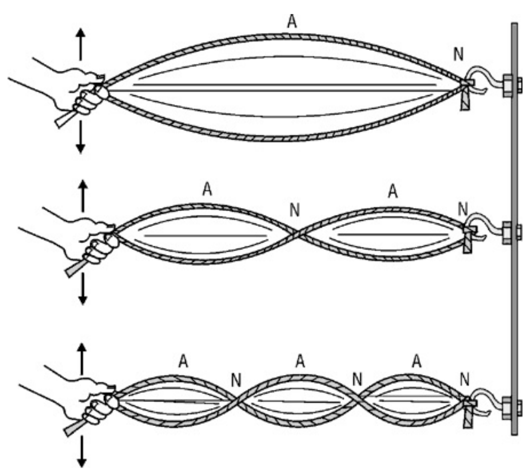

Figure 8.1: Standing waves in a vibrating rope.

Chapter 9

Spectral Analysis of Radiation

> But the conception of localized light-quanta out of which Einstein got his equation must still be regarded as far from established. Whether the mechanism of interaction between ether waves and electrons has its seat in the unknown conditions and laws existing within the atom, or is to be looked for primarily in the essentially corpuscular Thomson-Planck-Einstein conception of radiant energy, is the all-absorbing uncertainty upon the frontiers of modern Physics (Robert A Millikan in *The electron and the light-quanta from the experimental point of view*, Nobel Lecture, May 23, 1923).

9.1 Basic Energy Balance $R = F$

We shall now prove a basic balance in stationary equilibrium between the forcing F and the radiation, or more precisley the *radiance*, R in the wave equation (8.1), which we express as $R = F$ in the form:

$$R \equiv \int\int \gamma \ddot{u}^2 dxdt = \epsilon \int\int f^2 dxdt \equiv \epsilon \|f\|^2 \equiv F, \qquad (9.1)$$

where $\epsilon \lessapprox 1$ is a coefficient of emission independent of f, assuming f satisfies a certain condition of *near resonance* and we assume time-periodicity with $\int \dot{E}\, dt = 0$.

We will below identify an (ideal) blackbody by the relation $R = \|f\|^2$, thus by $\epsilon = 1$, and we will find that the wave model satisfying $R = \epsilon\|f\|^2$ with $\epsilon \lessapprox 1$ thus as announced above is a realization of an ideal blackbody.

We observe that $\epsilon \leq 1$ because the radiance R cannot exceed the forcing measured by $\|f\|^2$ as the maximal flux of incidient energy.

A blackbody with $\epsilon = 1$ will thus absorb/emit all incident energy while the case $\epsilon < 1$ with partial absorption/emission will represent a *greybody.*

The prove (9.1) we first make a spectral decomposition in x, assuming periodicity with period 2π:

$$\ddot{u}_\nu + \nu^2 u_\nu - \gamma \dddot{u}_\nu = f_\nu, \quad -\infty < t < \infty, \quad \nu = 0, \pm 1, \pm 2, ..., \tag{9.2}$$

into a set of forced damped linear oscillators with

$$u(x,t) = \sum_{\nu=-\nu_m}^{\nu_m} u_\nu(t) e^{i\nu x},$$

where ν_m is a fixed maximal frequency and $\gamma \nu_m^2 < 1$. We then use Fourier transformation in t,

$$u_\nu(t) = \int_{-\infty}^{\infty} u_{\nu,\omega} e^{i\omega t} d\omega, \quad u_{\nu,\omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} u_\nu(t) e^{-i\omega t}\, dt,$$

to get, assuming $u_\nu^{(3)}$ can be replaced by $-\nu^2 \dot{u}_\nu$:

$$(-\omega^2 + \nu^2) u_{\nu,\omega} + i\omega\gamma\nu^2 u_{\nu,\omega} = f_{\nu,\omega}.$$

We have by Parseval's formula,

$$\overline{u_\nu^2} \equiv \int_{-\infty}^{\infty} |u_\nu(t)|^2\, dt = 2\pi \int_{-\infty}^{\infty} |u_{\nu,\omega}|^2\, d\omega$$

$$= 2\pi \int_{-\infty}^{\infty} \frac{|f_{\nu,\omega}|^2\, d\omega}{(\nu - \omega)^2 (\nu + \omega)^2 + \gamma^2 \nu^4 \omega^2}$$

$$\approx \frac{2\pi}{\nu^2} \int_{-\infty}^{\infty} \frac{|f_{\nu,\omega}|^2\, d\omega}{4(\nu - \omega)^2 + \gamma^2 \nu^4} = \frac{2\pi}{\gamma \nu^4} \int_{-\infty}^{\infty} \frac{|f_{\nu,\nu+\gamma\nu^2\bar{\omega}}|^2\, d\bar{\omega}}{4\bar{\omega}^2 + 1},$$

where we used the change of integration variable $\omega = \nu + \gamma \nu^2 \bar{\omega}$.

We now assume as the definition of *near-resonance*, that

$$|f_{\nu,\omega}|^2 \approx \frac{1}{\pi^2} \overline{f_\nu^2} \quad \text{for} \quad |\nu - \omega| \leq \frac{\pi}{4}, \tag{9.3}$$

which requires that $|f_{\nu,\omega}|^2$ is small else, where we use $\approx$ to denote proportionality with constant close to 1. With this assumption we get noting that $\gamma\nu^2 < 1$,

$$\overline{u_\nu^2} \approx \frac{1}{\gamma\nu^4}\overline{f_\nu^2},$$

that is,

$$R_\nu \equiv \gamma\overline{\dot{u}_\nu^2} \approx \gamma\nu^4\overline{u_\nu^2} \approx \gamma\overline{\dot{u}_\nu}\nu^2 = \gamma T_\nu \nu^2 \approx \overline{f_\nu^2}, \tag{9.4}$$

where R_ν is the intensity of the radiated wave of frequency ν, and we view $T_\nu = \frac{1}{2}(\overline{\dot{u}_\nu^2} + \nu^2\overline{u_\nu^2}) \approx \overline{\dot{u}_\nu^2}$ as the temperature of the corresponding frequency.

The constant of proportionality in $R_\nu \approx \overline{f_\nu^2}$ is the *emissivity* denoted by ϵ to give $R_\nu = \epsilon\overline{f_\nu^2}$. We note that ϵ is only weakly dependent on γ and ν (through the near resonance condition), which means that we can attribute a certain emissivity close to one to the string and thus view the string with radiation as model of an ideal blackbody with $\epsilon = 1$.

Summing over frequencies recalling that $R = 2\pi\sum_\nu R_\nu$ and $\|f\|^2 = 2\pi\sum_\nu \overline{f_\nu^2}$, we get the desired energy balance (9.1):

$$R = \int_0^{2\pi} \gamma\overline{\dot{u}^2}\,dx = \epsilon\int_0^{2\pi} \overline{f^2}\,dx = \epsilon\|f\|^2, \tag{9.5}$$

stating that the intensity of the total outgoing radiation R is proportional to the intensity of the incoming radiation as measured by $\|f\|^2$.

The spectral analysis is performed with Fourier transformation in time over the real line with t ranging from $-\infty$ to ∞. A similar analysis can be done in the time-periodic case with integration in time over a period. We collect results for both cases in:

Theorem 3.1: The radiation R_ν of the solution u_ν of (9.8) satisfies $R_\nu = \gamma\overline{\dot{u}_\nu^2} = \epsilon\overline{f_\nu^2}$ with $\epsilon \lessapprox 1$, if f_ν satisfies (9.3) and $\gamma\nu^2 < 1$. Accordingly, the radiance R of the solution u of the wave equation (8.1) satisfies $R = \int\int \gamma\dot{u}^2 dxdt = \epsilon\|f\|^2 \equiv F$ assuming $\gamma\nu_m^2 < 1$ with ν_m a maximal frequency.

In the next chapter we make a connection to near-resonance in acoustics appearing in the tuning of a piano with the three strings for each tone (except the single string bass tones) tuned with an offset of about 0.5 Hz, with the effect of a longer sustain and singing quality of the piano. In this perspective the radiation of blackbody is like the thick chord obtained by pressing all the keys of a piano.

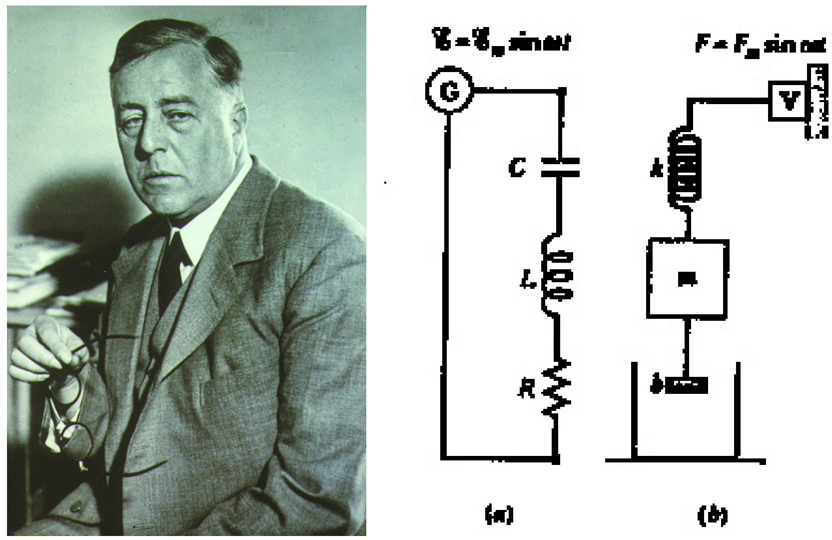

Figure 9.1: Sir James H. Jeans (1877-1946) and forced damped oscillators as electromagnetic circuit with capacitor (condenser), inductor and resistor, and as mechanical system with spring, mass and viscous shock absorber.

9.2 Rayleigh-Jeans Law

We read from (9.4) that

$$R_\nu \equiv \gamma \overline{\ddot{u}_\nu^2} \approx \gamma T_\nu \nu^2, \tag{9.6}$$

which can be viewed to express Rayleigh-Jeans Law, here as a direct conseuquence of the form of the radiation term $-\gamma \dddot{u}$. We see that the radiation energy increases linearly with temperature T_ν, at a given frequency: A hotter string will emit more radiation energy.

Theorem 3.1 gives the further information that if $\overline{f_\nu^2} \approx \gamma T \nu^2$, then also $R_\nu \approx \gamma T \nu^2$ if $T_\nu \sim T$. The emitted radiation will thus mimic an incoming Rayleigh-Jeans spectrum, in *thermal equilibrium* with $T_\nu = T$ for all frequencies ν. We shall below use this fact in an analysis of the interaction between two blackbodies through a common force f. We shall below motivate temperature equilibration as an effect of near-resonance.

Below we will extend Rayleigh-Jeans Law in the form $R = F$ to a Planck Law in the form $R + H = F$ where H represents internal heating with a temperature dependent switch from outgoing radiation R to H.

9.3 Radiation from Near-Resonance

The spectral analysis shows how radiation arises from a phenomenon of near-resonance: Each frequency f_ν of the incoming wave f excites resonant vibrations of the string with radiation $R_\nu = \epsilon\overline{f_\nu^2}$ and temperature T_ν determined by $R_\nu = \gamma T_\nu \nu^2$.

The effect of the near-reonance with γ small is that the solution u_ν will be nearly *in-phase* with f_ν, that is $\dot{u}_\nu$ is nearly *out-of-phase* with f_ν with a phase shift of a quarter of a period. This is to be compared with the case γ not small ($\gamma \approx 1$ say) when instead $\dot{u}_\nu$ will be in-phase with f_ν.

Accordingly, the absorption $\overline{f_\nu \dot{u}_\nu}$ is much smaller, relatively speaking, in the case of small damping. More precisley, we have the energy balance obtained by multiplying (9.8) by $\dot{u}_\nu$ and integrating in time:

$$\overline{f_\nu \dot{u}_\nu} = R_\nu = \epsilon\overline{f_\nu^2}, \tag{9.7}$$

even if $\overline{\dot{u}_\nu^2} = \overline{f_\nu^2}/\gamma\nu^2 >> \overline{f_\nu^2}$ if $\gamma\nu^2 << 1$. The effect can be seen in the force balance of the damped harmonic oscillator

$$\ddot{u}_\nu + \nu^2 u_\nu - \gamma \dddot{u}_\nu = f_\nu, \tag{9.8}$$

where f_ν in the case γ is not small is balanced mainly by the damping force $-\gamma\dddot{u}_\nu$, and in the case γ is small mainly by the oscillator. The Rayleigh-Jeans Law results from a non-trivial interaction of radiation with a background of near-resonant vibration.

Since γ is small, $-\gamma\dddot{u}$ represents in one sense a small perturbation of the wave equation, or a small damping of the harmonic oscillator, but this is compensated by the third order derivate in the radiation term, with the effect that the radiated energy is not small, but equal to the incoming energy measured by $\epsilon\|f\|^2$ with $\epsilon \approx 1$.

9.4 Thermal Equilibrium from Near-Resonance

We have seen that the assumption $T_\nu = T$ of thermal equilibrium underlies the formulation of the Rayleigh-Jeans Law in the form $R_\nu = \gamma T \nu^2$. We shall now argue that thermal equilibrium can be seen as a consequence of near-resonance. The idea is that since $T_\nu \approx \overline{\dot{u}_\nu^2}$ represents the internal energy of the oscillator of frequency ν, near-resonance means that oscillators of nearly the same frequency interact and interaction can be expected to decrease differences in oscillator energy as a form of spectral diffusion and thus tend to distribute energy evenly over different frequencies towards thermal equilibrium.

We shall see below that two blackbodies in radiative interaction will tend towards thermal equilibrium of equal temperature by sharing a common force f. Similarly, two different oscillators of a blackbody of nearly the same frequency ν can be expected to tend towards a common temperature by sharing the forcce f_ν in near-resonance interaction .

9.5 The Poynting Vector vs $\|f\|^2$

The *Poynting vector* $E \times H$ of an electromagnetic field (E, M) with E the electric and H the magnetic field, measures the energy flux of (E, M), which connects to our meaure of $\overline{f_\nu^2}$ and $\|f\|^2$ as a measure of the energy (intensity) of the forcing f. This motivates the restriction $\epsilon \leq 1$ in the energy balance $R = \epsilon\|f\|^2$ since the absorption/emission cannot exceed the electromagnetics energy flux, and of course to definie a blackbody by $\epsilon = 1$ expressing that all incident energy is absorbed/emitted.

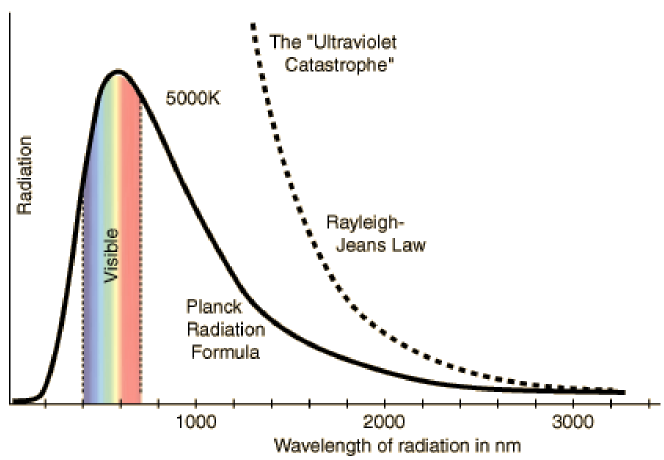

Figure 9.2: The ultraviolet catastropheof the Rayleigh-Jeans Law.

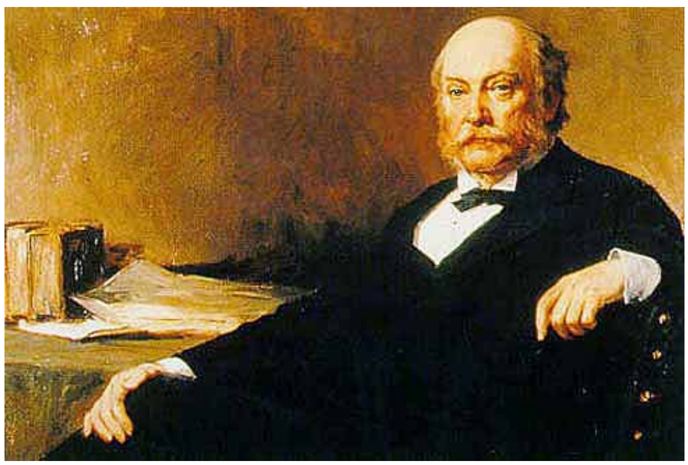

Figure 9.3: Lord Rayleigh (1842-1919).

Chapter 10

Acoustic Near-Resonance

> Examples ... show how difficult it often is for an experimenter to interpret his results without the aid of mathematics. (Lord Rayleigh)

10.1 Radiation vs Acoustic Resonance

We have seen that the Rayleigh-Jeans Radiation Law expresses near-resonance in a wave model with damping from a 3rd order time derivative and we now make a connection to acoustic resonance in string instruments modeled by a wave equation with viscous damping from a 1st order time derivative.

10.2 Resonance in String Instrument

Let us illustrate the basic phenomenon of near-resonance in the acoustic resonance of a string instrument modeled by the classical damped harmonic oscillator

$$\ddot{u}(t) + \nu^2 u(t) + \gamma \dot{u}(t) = f(t), \quad -\infty < t < \infty, \tag{10.1}$$

where $\dot{u} = \frac{du}{dt}$, $\ddot{u} = \frac{d2u}{dt^2}$, ν is a given moderate to large frequency, $\gamma > 0$ is a damping parameter and $f(t)$ is a periodic forcing. Here f represents the outgoing sound from a soundboard (guitar body) and the damping term the force from a vibrating string in contact with the soundboard.

We are interested in the outgoing sound (forcing f) resulting from the force interaction between the string (damping $\gamma\dot{u}$) and the soundboard (oscillator $\ddot{u} + \nu^2 u$). We seek periodic solutions and measure the relation between the forcing and the damping by the *efficiency* $E = \frac{F}{R}$ with

$$F = \int f^2(t)\,dt, \quad R = \int \gamma \dot{u}^2(t)\,dt\,, \tag{10.2}$$

with integration over a time period. If the forcing $f(t)$ is periodic with the resonance frequency ν, referred to as *perfect resonance*, then $\dot{u}(t) = \frac{1}{\gamma}f(t)$, which gives $E = \gamma$ with $\dot{u}$ in phase with $f(t)$.

We shall distinguish two basic different cases with the forcing $f(t)$ balanced by the harmonic oscillator term $\ddot{u}(t) + \nu^2 u(t)$ and the damping term $\gamma\dot{u}(t)$ in two different ways:

1. $\gamma \approx 1$ with $\gamma\dot{u} \approx f(t)$ and $|\ddot{u}(t) + \nu^2 u(t)| << |f(t)|$,

2. $\gamma\nu < 1$ with $|\gamma\dot{u}(t)| << |f(t)|$ and $\ddot{u}(t) + \nu^2 u(t) \approx f(t)$,

with the case 2. representing near-resonance with small damping, as the case of most interest. We shall define near-resonance at a given frequency ν by flat spectrum centered at ν of width 1. A spectrum of width $\gamma << 1$ would then correspond to sharp resonance (with γ not very small this is sometimes referred to as broad resonance).

In case 1. the damping is large and the force $f(t)$ is balanced by the damping $\gamma\dot{u}(t)$ with $\dot{u}$ in phase with $f(t)$. In this case trivially $E \approx 1$.

In case 2. with near resonance and small damping, $f(t)$ is balanced by the oscillator with $\dot{u}$ out-of-phase with $f(t)$, and we shall see that also in this case $E \approx 1$. The case of near-resonance is to be compared with the case of perfect resonance with $f(t)$ again balanced by $\gamma\dot{u}$, with now $\dot{u}(t)$ in phase and $E = \gamma$.

If γ is small there is thus a fundamental difference betwen the case of near-resonance with $E \approx 1$ and the case of perfect resonance with $E = \gamma << 1$.

In applications to blackbody radiation we may view F as input and R as output, but it is also possible to turn this around view R as the input and F as the output, with $\frac{1}{E} = \frac{R}{F}$ representing emissivity $\epsilon \approx 1$ in the case of near-resonance.

In the case of near-resonance the force $f(t)$ is balanced mainly by the excited harmonic oscillator with a small contribution from the damping term, which gives $E \approx 1$.

In the case of perfect resonance the oscillator does not contribute to the force balance, which requires a large damping term leading to small efficiency.

The above discussion concerns time-periodic (equilibrium) states attained after a transient start-up phase, with the forcing now F in-phase with the velocity $\dot{u}$, in contrast to out-of-phase in equilibrium.

The discussion in this note connects to apects of wave vs particle modeling of light and sound [20, 15, 16, 23, 24, 5].

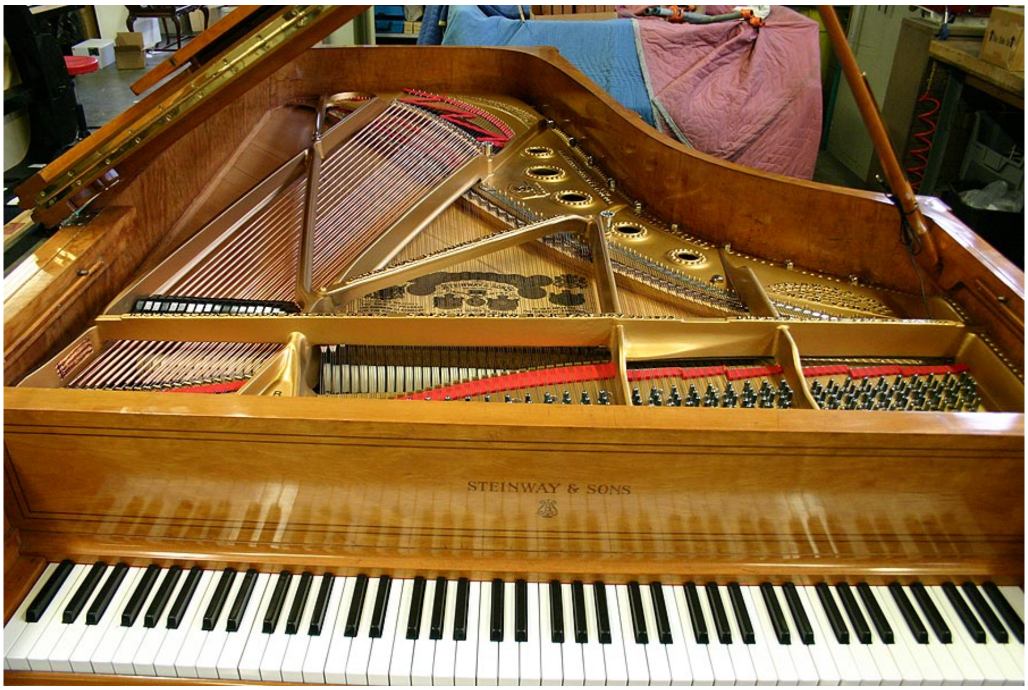

Figure 10.1: A Steinway Grand Piano as a set of strings over a resonating board.

10.3 Fourier Analysis of Near-Resonance

Although (10.1) is a maybe the most studied model of all of physics, it appears that the phenomenon of near-resonance has received little attention. As above we use Fourier transformation in t of (10.1), writing

$$u(t) = \int_{-\infty}^{\infty} \hat{u}(\omega) e^{i\omega t} d\omega, \quad \text{with} \quad \hat{u}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(t) e^{-i\omega t}\, dt,$$

to get

$$(-\omega^2 + \nu^2)\hat{u}(\omega) + i\gamma\omega\hat{u}(\omega) = \hat{f}(\omega).$$

We then use Parseval's formula, to seek a relation between the mean value of $u^2(t)$ and $f^2(t)$, assuming $\hat{f}(\omega)$ is supported around $\omega = \nu$:

$$\overline{u^2} \equiv \int_{-\infty}^{\infty} |u(t)|^2\, dt = 2\pi \int_{-\infty}^{\infty} |\hat{u}(\omega)|^2\, d\omega = 2\pi \int_{-\infty}^{\infty} \frac{|\hat{f}(\omega)|^2\, d\omega}{(\nu-\omega)^2(\nu+\omega)^2 + \gamma^2\omega^2}$$

$$\approx \frac{2\pi}{\nu^2} \int_{-\infty}^{\infty} \frac{|\hat{f}(\omega)|^2\, d\omega}{4(\nu-\omega)^2+\gamma^2} = \frac{2\pi}{\gamma\nu^2} \int_{-\infty}^{\infty} \frac{|\hat{f}(\nu+\gamma\bar{\omega})|^2)\, d\bar{\omega}}{4\bar{\omega}^2+1},$$

where we used the change of integration variable $\omega = \nu + \gamma\bar{\omega}$.

We now assume that $|\hat{f}(\omega)|^2 \sim \frac{1}{\pi^2}\overline{f^2}$ for $|\nu - \omega| \leq \frac{\pi}{4}$ as an expression of near-resonance, and that $|\hat{f}(\omega)|$ is small elsewhere. With this assumption we get

$$\gamma\overline{\dot{u}^2} \approx \gamma\nu^2\overline{u^2} \approx \overline{f^2},$$

that is $R \approx F$ and thus $E \approx 1$ as stated.

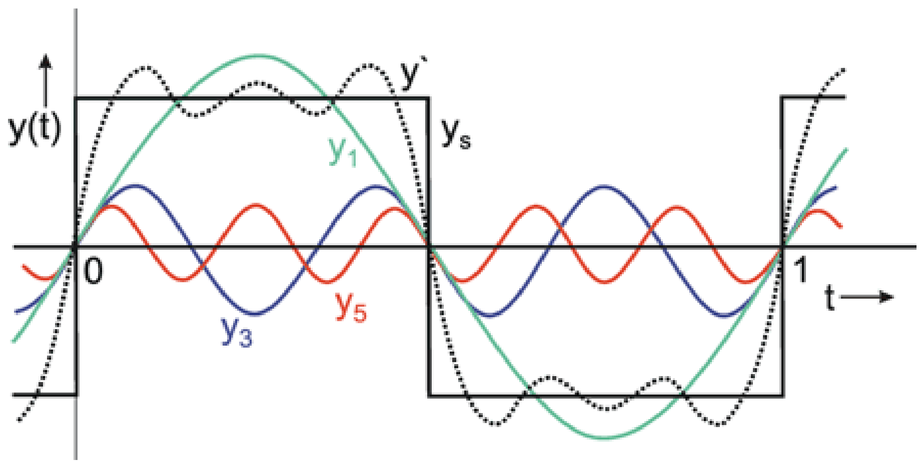

Figure 10.2: Fourier decomposition of square wave.

10.4 Application to Acoustical Resonance

A musical string instrument consists in principle of a vibrating string and a resonating body or soundboard, where we model the resonator with input from the string, as the force f during start-up and as a viscous force in equlibrium.

In the case of near resonance in equilibrium the input from the vibrating string is amplified by the resonator to an efficiency index $E \sim 1$, while perfect resonance would give $E << 1$, with small damping.

During start-up we consider the forcing f to be given by the vibrating string (without damping) and acting in-phase with the velocity $\dot{u}$ thus is pumping vibrational energy from the string into the body. Once equilibrium is reached, we shift view and consider f as the output from the body, which is sustained by a still vibrating string generating the viscous force. This mean that during both start-up and equilibrium the string vibrates in-phase with the body, by pumping energy into the body during start-up, and sustaining the output from the body in equilibrium.

The importance of near-reonance forcing is well-known to a piano-tuner, who tunes the three strings of a tone (except single stringed bass tones) at slightly different pitches (of about 0.5 Hz), which gives a longer sustain and a singing quality to the piano.

10.5 Computational Resonance

We show in Fig. 1-5 some computations with $\gamma = 0.001$, $\nu = 20$ and $\nu = 100$ with the following near-resonance forcing:

$$f(t) = \sum_{k=-5,k\neq 0}^{5} \sin((n + \frac{k}{10})t)\,0.1 \tag{10.3}$$

starting with the intial data $u(0) = 0$ and $\dot{u}(0) = 15$ and computing for $t > 0$. The efficiency index is computed as the mean value over the entire time interval. We see as expected from the Fourier analysis that the efficiency index $E \approx 1$ and that the forcing is out-of-phase with the damping, as the main characteristics of near-resonance with small damping.

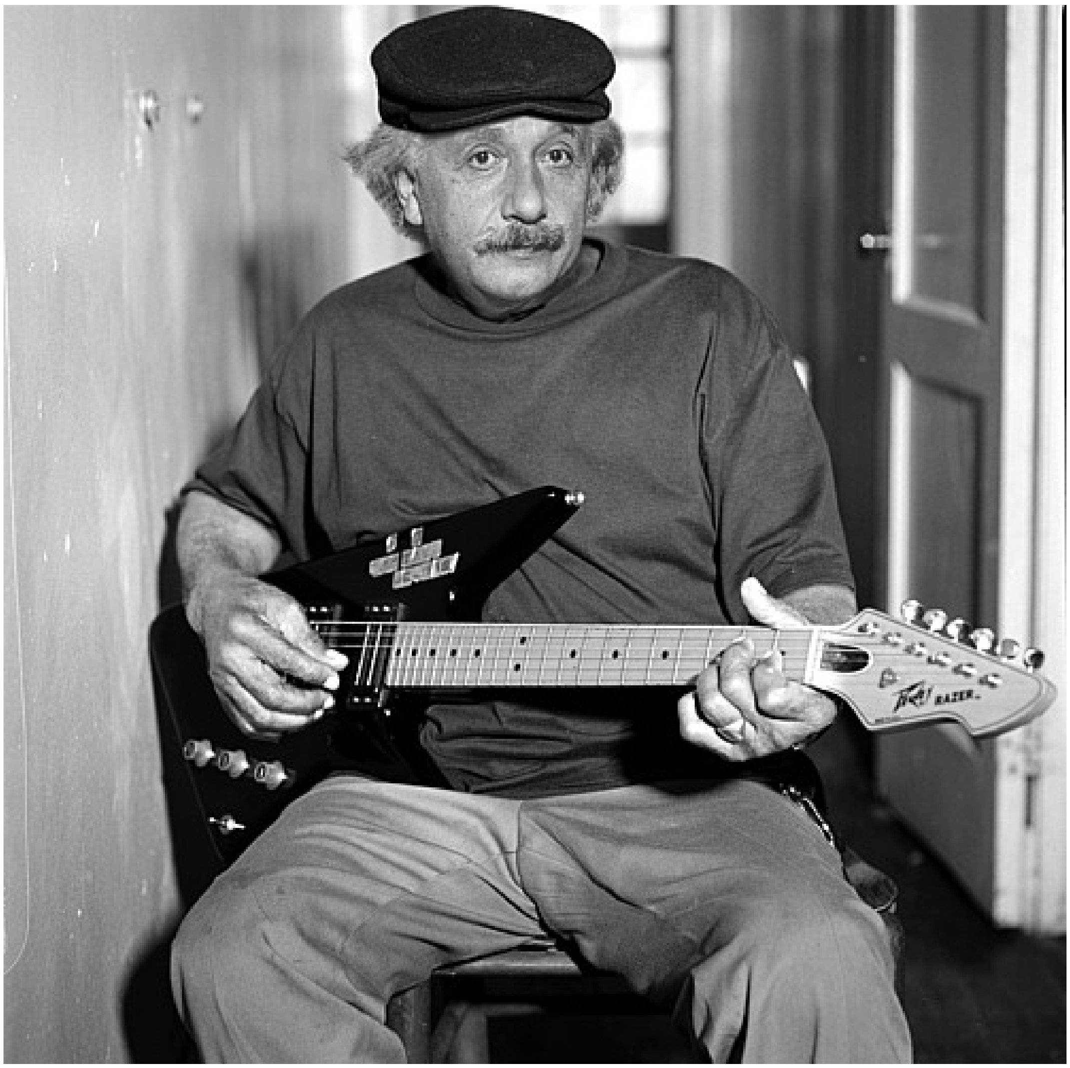

Figure 10.3: Einstein testing his theory of quanta to the resonance of his new Razor Atomic Guitar

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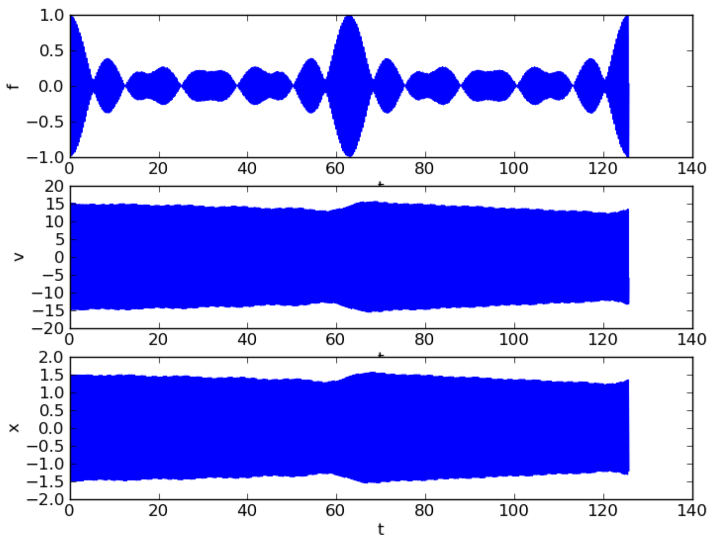

Figure 10.4: Position $x = u$, velocity $v = \dot{u}$ and forcing f with $n = 20$ over scaled time.

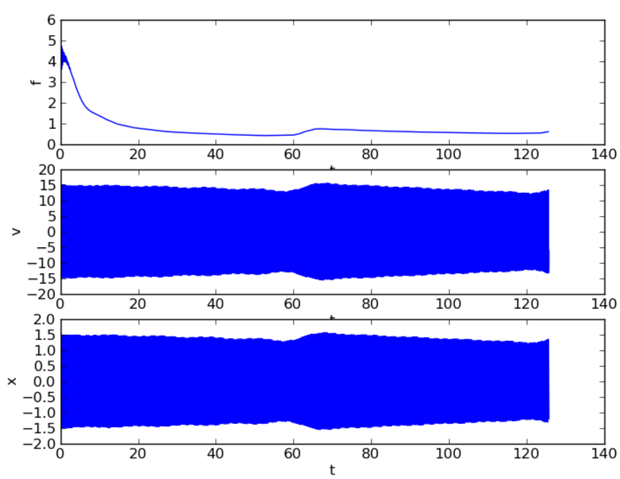

Figure 10.5: Position $x = u$, velocity $v = \dot{u}$ and efficiency $f = E$ with $n = 20$ over scaled time.

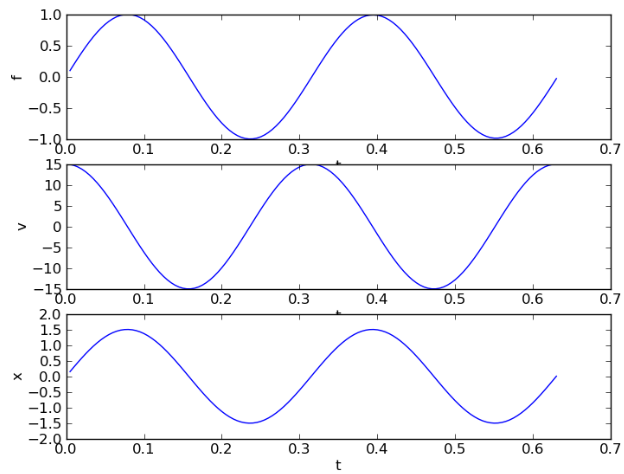

Figure 10.6: Position $x = u$, velocity $v = \dot{u}$ and forcing $f = E$ with $n = 20$ over ahort time. Notice a time lag of a quarter of a period between $\dot{u}$ and forcing f, representing out-of-phase forcing.

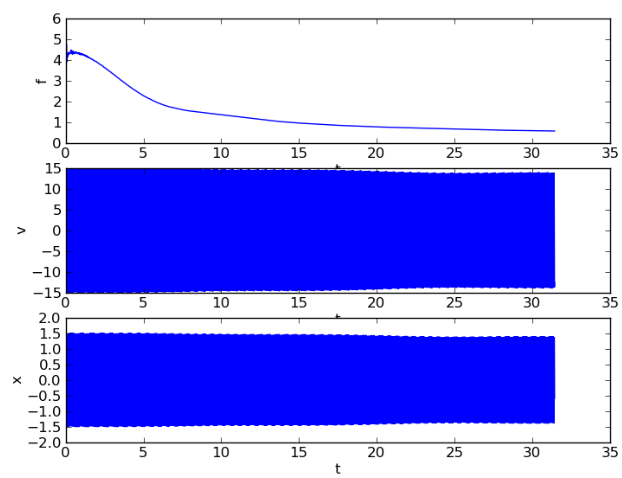

Figure 10.7: Position $x = u$, velocity $v = \dot{u}$ and efficiency $f = E$ with $n = 100$ over scaled time.

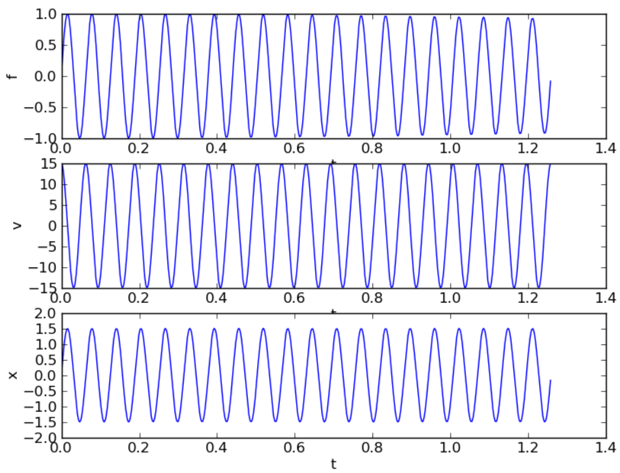

Figure 10.8: Position $x = u$, velocity $v = \dot{u}$ and forcing $f = E$ with $n = 100$ over short time.

Chapter 11

Model of Blackbody Radiation

> Despite the great success that the atomic theory has so far enyoyed, utimately it will have to be abandoned in favor of the assumption of continuous matter (wave mechanics) (Planck 1882).

11.1 Finite Precision Computation

We will model the effect of computing solutions of the wave equation with finite precision by G2 as a viscous force $-\delta^2\dot{u}''$ with viscosity coefficient δ^2 effectively limiting the resolution to a smallest *coordination length* δ with corresponding largest resolved frequency $\frac{1}{\delta}$.

We shall choose $\delta = \frac{h}{T}$ where h is a fixed *precision* parameter reflecting atomic dimensions in the physical model and T is temperature. The highest frequency which can be represented as a coherent wave motion is thus represented by $\frac{T}{h}$ scaling with T, in accordance with *Wien's displacement law*.

The choice $\delta = \frac{h}{T}$ reflects that finite precision computation requires sufficient variation of a wave u over the coordination length δ to allow coherent emission, with sufficient variation expressed as the condition $|\dot{u}|\delta > \sqrt{h}$ since $T \approx \sqrt{h}|\dot{u}|$ as shown in the next section.

As an illustration one may think of"the Mexican wave" around a stadium which cannot be sustained unless people raise hands properly; the smaller the "lift" is (with lift as temperature), the longer is the required coordination length or wave length.

The viscosity introduces dissipation of energy of intensity $\delta^2(\dot{u}')^2$ which will show up as a contribution to the internal heat energy E.

The effect of the G2 viscosity from residual stabilization is restricted to high frequencies leaving lower frequencies without dissipation. Accordingly, we shall through a spectral decomposition restrict δ to act only on high frequencies ν by setting $\delta = 0$ for $|\nu| \leq \frac{T}{h}$. On the other hand, for $|\nu| \leq \frac{T}{h}$ we will have $\gamma < \delta^2$ since $\gamma\nu_m^2 < 1$ and $\frac{1}{\delta} = \frac{T}{h} < \nu_m$, and we will then effectively set $\gamma = 0$ when $\delta = \frac{h}{T}$. The wave equation with radiation will thus be subject to the following switch at $\frac{T}{h}$ in a spectral representation:

$$\begin{aligned} &\gamma > 0, \quad \delta = 0 \quad \text{if} \quad |\nu| \leq \frac{T}{h}, \\ &\gamma = 0, \quad \delta = \frac{h}{T} \quad \text{if} \quad \frac{T}{h} < |\nu| < \nu_m, \end{aligned} \tag{11.1}$$

where $\gamma\nu_m^2 < 1$. This is a sharp switch of dissipation from exterior radiation to internal heating for frequencies above a certain threshold scaling with temperature. The switch in actual G2 computation is less sharp with a continuous transition from exterior radiation to internal heating over a certain frequency band.

11.2 Radiation and Heating

We consider the wave equation (8.1) with radiation augmented by a viscosity term acting in space with coefficient δ^2 and an equation for internal heat energy E:

$$\begin{aligned} &\ddot{u} - u'' - \gamma\dddot{u} - \delta^2\dot{u}'' = f, \quad -\infty < x, t < \infty, \\ &\dot{E} = \int f\dot{u}\,dx - \int \gamma\ddot{u}^2\,dx, \quad -\infty < t < \infty, \end{aligned} \tag{11.2}$$

where $\delta = \frac{h}{T}$ subject to the switch (11.1) and we define $T \equiv \sqrt{hE}$. The dependent variables in (11.2) are thus u and E with the switch defined by $\frac{T}{h} \approx \sqrt{\frac{E}{h}}$ where h is a fixed constant.

Here E is the total internal heat energy as the sum of the internal energy E_ν for each frequency defined by

$$E_\nu = \frac{1}{2}(\overline{\dot{u}_\nu^2} + \nu^2\overline{u_\nu^2}) \tag{11.3}$$

plus a contribution from the finite precision dissipation $\int \delta^2 \dot{u}' dxdt$. With $E_\nu \approx T$ and the cut-off $|\nu| < \frac{T}{h}$ we obtain $E \approx \frac{T^2}{h}$ and accordanc with the definition of $T = \sqrt{hE}$.

11.3 Planck as Rayleigh-Jeans with Cut-off

Theorem 3.1 gives directly a Planck Law as a Rayleigh-Jeans Law with cut-off:

$$R_\nu(T) \equiv \gamma \overline{\dot{u}_\nu^2} \approx \gamma T \nu^2 \theta_h(\nu, T) \tag{11.4}$$

where $R_\nu(T) = R_\nu = \gamma \overline{\dot{u}_\nu^2}$ with $\overline{\dot{u}_\nu^2} = T$.

$$\begin{aligned} \theta_h(\nu, T) &= 1 \quad \text{for} \quad |\nu| \leq \frac{T}{h} \\ \theta_h(\nu, T) &= 0 \quad \text{for} \quad |\nu| > \frac{T}{h}. \end{aligned} \tag{11.5}$$

We compare with Planck's own version of the law:

$$R_\nu(T) = \gamma T \nu^2 \theta(\nu, T), \quad \gamma = \frac{2}{c^2}, \tag{11.6}$$

with the exponential cut-off

$$\theta(\nu, T) = \frac{\frac{h\nu}{T}}{e^{\frac{h\nu}{T}} - 1}, \tag{11.7}$$

where c is the speed of light in vacuum, and we have normalized Boltzmann's constant k to 1, with $\theta(\nu, T) \approx 1$ for $|\nu| < \frac{T}{h}$ and $\theta(\nu, T) \approx 0$ for $|\nu| > \frac{10T}{h}$. We recall that the cutoff distance in terms of wave length $\frac{1}{\nu}$ is proportional to $\frac{1}{T}$.

11.4 Planck's Law: $R + H = F$

Using that the dissipative effects of $-\gamma \dddot{u}$ and $-\delta^2 \dot{u}'$ are similar, we have by the proof of Theorem of 3.1 for frequencies with $|\nu| > \frac{T}{h}$ and $\epsilon \lessapprox 1$:

$$H_\nu \equiv \delta^2 \overline{(\dot{u}')^2} = \epsilon \overline{f_\nu^2} \tag{11.8}$$

expressing that the internal heating above cut-off equals the the forcing. We thus obtain a balance of radiation and heating with forcing

$$R_\nu + H_\nu = \epsilon \overline{f_\nu^2} \quad \text{for all } \nu \tag{11.9}$$

or by summation

$$R + H = \epsilon \|f\|^2 \equiv F, \tag{11.10}$$

where $H = \int\int \delta^2(\dot{u}')^2 dxdt$ and as above $R = \int\int \gamma \ddot{u}^2 dxdt$.

We summarize in the following formulation of Planck's Law:

Theorem 8.1: The radiation R and heating H in the wave equation (11.2) with forcing f and switch (11.1) satisfies $R + H = \epsilon\|f\|^2 = F$, where $R_\nu = \gamma T \nu^2$ for $|\nu| \leq \frac{T}{h}$ and $H_\nu = \delta^2 T \nu^2$ for $|\nu| > \frac{T}{h}$ with $R_\nu H_\nu = 0$, $\epsilon \lessapprox 1$ and $\overline{\dot{u}_\nu^2} = T$.

Note that the absorption as the work done by force f on the velocity $\dot{u}$ equals $A = \int\int f\dot{u}dxdt$ which thus can be transformed into outgoing radiation R or internal heating H.

11.5 Connection to Uncertainty Principle

We now make a connection between the finite precision switch from radiation to internal heating at $\nu = \frac{T}{h}$ and Heisenberg's Uncertainty Principle. Since $T = \overline{\dot{u}_\nu^2}$ and $\overline{\dot{u}_\nu^2} \approx \nu^2 \overline{u_\nu^2}$ the switch can be written $\overline{\dot{u}_\nu^2} = h\nu$ or $\overline{u_\nu^2} \approx \frac{h}{\nu}$, from which follows by multiplication

$$\overline{\dot{u}_\nu^2}\,\overline{u_\nu^2} \approx h^2, \tag{11.11}$$

which can be interpreted as a variant of Heisenberg's Uncertainty Principle: The smaller the amplitude $|u_\nu|$ (the position) is, the larger must the velocity $|\dot{u}_\nu|$ (the momentum) be, in order for a coherent wave to be emitted/radiated.

11.6 Stefan-Boltzmann's Law

Summing over frequencies with $|\nu| \leq \frac{T}{h}$, we obtain the total radiation R as

$$R = 2\pi \sum_{|\nu| < \frac{T}{h}} \gamma T \nu^2 = \sigma T^4 \tag{11.12}$$

with $\sigma \approx \frac{2\pi\gamma}{3h^3}$ a constant. This is Stefan-Boltzmann's Radiation Law with σ Stefan-Boltzman's constant. The total radiation of the blackbody model thus scales like T^4 with T the common temperature of all frequencies below cut-off $\frac{T}{h}$.

11.7 Radiative Interaction

We now consider the interaction of two blackbodies indexed by 1 and 2 with solutions u_1 and u_2 of (11.2) starting at different different temperatures $T_1 < T_2$ at an initial time. We assume the bodies interact by sharing a common forcing f. For frequencies ν with $\frac{T_1}{h} < |\nu| < \frac{T_2}{h}$, we have by the above analysis

$$R_{2,\nu} \equiv \gamma_2 \overline{\ddot{u}_2} = \epsilon \overline{f_\nu^2} = \delta^2 \overline{(\dot{u}_1')^2} \equiv H_{1,\nu}. \tag{11.13}$$

which expresses a transfer of radiation energy $R_{2,\nu}$ into internal energy $H_{1,\nu}$ with a corresponding increase of T_1 until $T_1 = T_2$. For frequencies ν with $|\nu| < \frac{T_1}{h}$ we have $R_{1,\nu} = R_{2,\nu}$ while frequencies with $|\nu| > \frac{T_2}{h}$ cannot be present without additional external forcing. Planck's Law for two blackbodies in radiative interaction can thus be expressed as

$$R_1 + H_1 = R_2 + H_2. \tag{11.14}$$

11.8 Heat Capacity

We have seen that $E \sim T^2$, which gives a heat capacity $c(T) \equiv \frac{dE}{dT} \sim T$ which fits with experiments for metals like copper and silver for $T < 200$ K and for diamond for $K < 500$ K, as shown in Fig. 11.1.

We compare with an Einstein-Debye model with $c(T) = aT + bT^3$ assuming $b << a$.

There is another way of determining the heat capacity of a blackbody by equating the energy content of the blackbody with its emission $R = \sigma T^4$, which gives $c(T) = \frac{dR}{dT} \sim T^3$ as a part of the Einstein-Debye model with $a << b$.

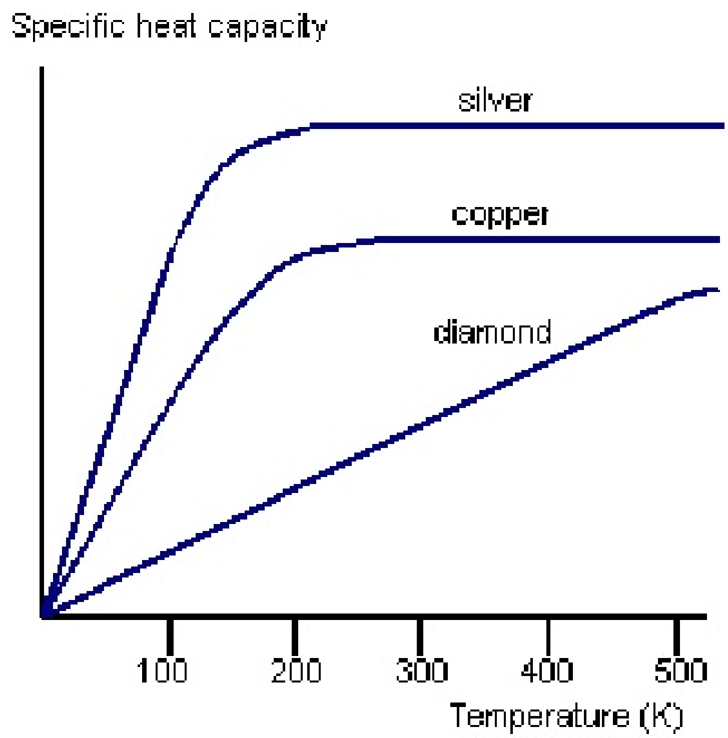

Figure 11.1: Heat capacity.

11.9 Radiative Cooling

If the forcing f is terminated then the blackbody will start cooling according to

$$\int \dot{E} dt = -\int \gamma \ddot{u}^2 dx dt = -R \tag{11.15}$$

with E defined by (8.3). With $E \sim T^2$ and $R \sim T^4$, this gives assuming $f(t) = 0$ for $t > 0$

$$E(t) = \frac{1}{1 + C\,t} E(0) \tag{11.16}$$

with C a positive constant, and thus the following cooling curve

$$T(t) = T(0)(1 + C\,t)^{-\frac{1}{2}}. \tag{11.17}$$

For an individual frequency ν we have since $E_\nu = T_\nu$

$$\dot{E}_\nu = -R_\nu \approx -\gamma\nu^2 T_\nu = -\gamma\nu^2 E_\nu \tag{11.18}$$

indicating a decay of $E_\nu = T_\nu$ according to $\exp(-\gamma\nu^2 t)$ with faster decay for higher frequencies. To maintain that $T_\nu = T$ for all ν, requires near-resonance with tendency towards thermal equilibration.

Note that with the version of Stefan-Boltzmann's Law of statistical mechanics, the internal energy E is proportional to T (and not T^2 as above), which gives a somewhat different cooling curve

$$T(t) = T(0)(1 + C\,t)^{-\frac{1}{3}}, \tag{11.19}$$

with less rapid decay with time. Experiments [40] appear to favor (11.17) before (11.19), see Fig. 11.2.

11.10 Interaction by Shared Force

Note that we consider the intercation of two blackbodies to be established through a (non-zero) shared force f. The above case of cooling with $f = 0$ thus represents an extreme case with one the of the bodies kept at 0 K.

The normal case is thus $f \neq 0$ described by Planck's Law as $R + H = F$ which covers interaction of two blackbodies indexed by 1 and 2 as $R_1 + H_1 = R_2 + H_2$.

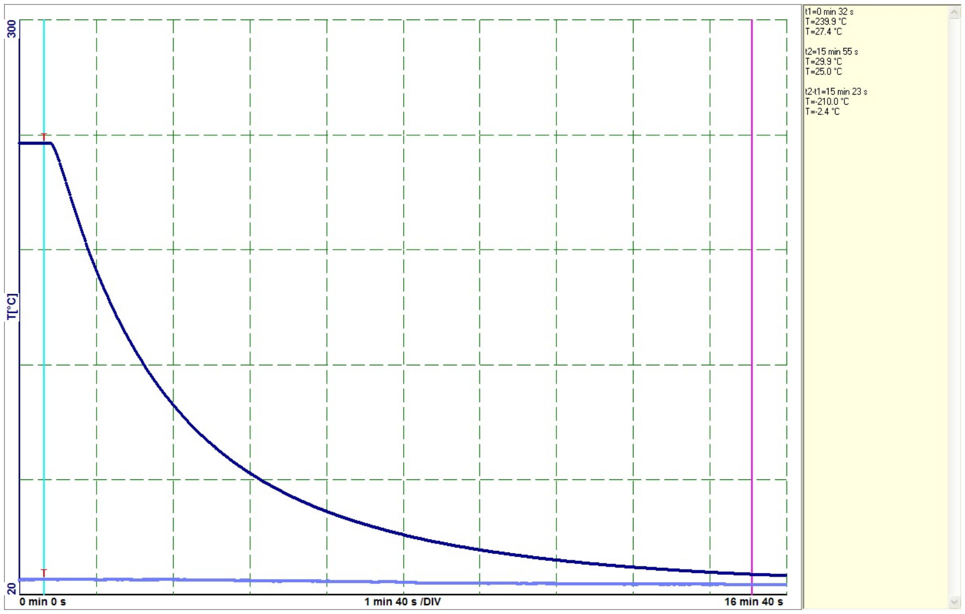

Figure 11.2: Experimental cooling curve.

11.11 Generic Nature of Blackbody

Consider two blackbodies labeled 1 and 2 described by the model (11.2) with different defining parameters (γ_1, h_1) and (γ_2, h_2) which are interacting by a shared force f. Suppose the interaction bodies has reached a state of radiative equilibrium at constant temperatures T_1 and T_2 without energy transfer between the bodies. The cut-off frequency then must be the same for both bodies, that is $\frac{T_1}{h_1} = \frac{T_2}{h_2}$ and for frequencies ν below the common cut-off Planck's law states that $\gamma_1 T_1 \nu^2 = \gamma_2 T_2 \nu^2$, that is $\gamma_1 T_1 = \gamma_2 T_2$.

We conclude that $\gamma_1 h_1 = \gamma_2 h_2$, which means that the two blackbodies have the same effective physical properties, with the cut-off condition $\frac{T_1}{h_1} = \frac{T_2}{h_2}$ coordinating the temperature scales of the bodies.

The wave model (11.2) thus describes a generic blackbody defined by the effective parameter γh, which connects radiative damping to finite precision. The model allows blackbodies with the same γh to reach radiative equilibrium with the same energy spectrum and cut-off frequency without energy transfer over frequencies.

Two blackbodies with different γh in radiative of equilibrium will have different cut-off frequencies and energy balance then requires transfer of energy over frequencies.

11.12 Cut-Off by Residual Stabilization

The discretization in G2 is accomplished by residual stabilization of a Galerkin variational method and may take the form: Find $u \in V_h$ such that for all $v \in V_h$

$$\int (A(u) - f)v\, dxdt + \delta^2 \int (A(u) - f)A(V)\, dxdt = 0, \tag{11.20}$$

where $A(u) = \ddot{u} - u'' - \gamma\dddot{u}$ and V is a primitive function to v (with $\dot{V} = v$), and V_h is a a space-time finite element space continuous in space and discontinuous in time over a sequence of discrete time levels.

Here $A(u) - f$ is the residual and the residual stabilization requires $\delta^2(A(u) - f)^2$ to be bounded, which should be compared with the dissipation $\delta\ddot{u}^2$ in the analysis with $\ddot{u}^2$ being one of the terms in the expression $(A(u) - f)^2$. Full residual stabilization has little effect below cut-off, acts like simplified stabilization above cut-off, and effectively introduces cut-off to zero for $|\nu| \geq \nu_m$ since then $\gamma|\dddot{u}| \sim \gamma\nu^2|\dot{u}| = \frac{\nu^2}{\nu_m^2}|\dot{u}| \geq |\dot{u}|$, which signifies massive dissipation.

11.13 Cordination Length

Frequencies below cut-off will be absorbed and radiated as *coherent* waves, while frequencies above cut-off will be absorbed and transformed into internal energy in the form of incoherent waves. which are not radiated. High frequencies thus may heat the body and thereby decrease the coordination length and thereby allow absorption and emission of higher frequencies.

Chapter 12

Universal Blackbody

> I had always looked upon the search for the absolute as the noblest and most worth while task of science.... My original decision to devote myself to science was a direct result of the discovery which has never ceased to fill me with enthusiasm since my early youth - the comprehension of the far from obvious fact that the laws of human reasoning coincide with the laws governing the sequences of the impressions we receive from the world about us; that, therefore, pure reasoning can enable man to gain an insight into the mechanism of the latter. In this connection, it is of paramount importance that the outside world is something independent from man, something absolute, and the quest for the laws which apply to this absolute appeared to me as the most sublime scientific pursuit in life. (Planck)

12.1 Kirchhoff and Universality

Kirchoff got hooked on an idea of universality of blackbody radiation with radiation only depending on temperature and frequency, independent of the composition of the emitting body, an idea which he transferred to his student Planck. For a physicist working with big ideas at the turn to modernity at the end of the 19th century, universality as the incarnation of the absolute was highly valued and thus attractive. But in the laboratory Kirchoff observed different materials displaying different emission spectra which were not simply related to temperature changes and so seemed to contradict universality.

However, Kirchhoff observed that graphite was special with a smooth spectrum with a distinct connection to temperature, and so the spectrum of graphite was chosen as model by Kirchoff followed by Stefan, Wien and Planck, but the problem of universality of course remained. Kirchhoff now asked if graphite could be eleveated to universilty?

Kirchhoff manufactured a box from graphite plates into which he placed various radiating objects and observed the resulting radiation through a small peep hole, through which radiation escaped. To his satisfaction Kirchhoff found that the the form of the radiation spectrum was dependent only on temperature and frequency and not of the body put into the box, and so found evidence of universality.

But Kirchhoff could not reproduce his results with a box with fully reflecting metallic walls, since in this case the emitted spectrum depended on the object put into the box. Kirchhoff then inserted a small piece of graphite into the perfectly reflecting enclosure and again obtained universality with the graphite apparently acting as "catalyst" towards universality.

We shall now analyze Kirchhoff's procedure for reaching universality. We shall then find that the graphite box is chosen as a reference blackbody as a physical body characterized by

- temperature equilibration (all frequencies have the same energy),
- absorption of all incident radiation,
- maximal high-frequency cut-off.

The graphite box will then act as a reference thermometer measuring the temperature of the body put into the box as the temperature of the reference blackbody in radiative equilibrium. The effect of the graphite would thus be to equilibrate the radiation in frequency and to determine a maximal cut-off.

We understand that this way universality is achieved by chosing a specific blackbody as reference and then referring other bodies to the reference body. Universlity is thus achieved by chosing a certain universal standard rather than observing that all bodies radiate in the same way.

We now look into the details of this procedure which represents a form of standarization rather than true universality.

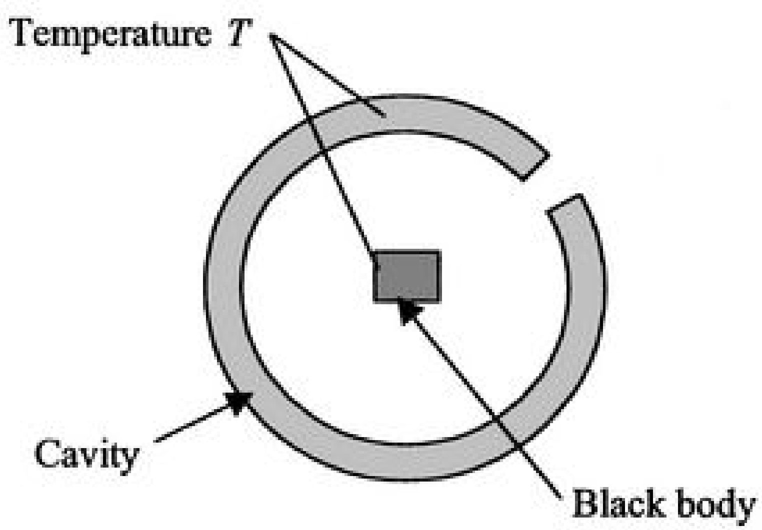

Figure 12.1: Model of Universal Blackbody.

12.2 Blackbody as Cavity with Graphite Walls

Fig. 12.1 is used to convey the idea of universality of blackbody radiation: The radiation spectrum from the cavity (with walls of graphite) observed through the peep-hole of the cavity, is observed to only depend on the temperature of the body placed in the cavity and not the nature of the body. Questions:

- Why is the model of a blackbody a cavity with peep-hole?
- What is the role of the graphite walls of the cavity?

Planck's Law expresses the radiated energy $E(T,\nu)$ of frequency ν from a blackbody of temperature T as

$$E(T,\nu) = \gamma T \nu^2, \tag{12.1}$$

where γ is supposed to be a universal constant, which is the same for all blackbodies independent of their composition. But how can the radiated energy be independent of the physics of the radiating body?

Chapter 13

Model of Universal Blackbody

Kirchhoff formed a conceptual model of a universal blackbody as a cavity with the property of absorbing all incident radiation. We shall now see how universality can be captured in our wave model with its apparent dependence on the pair of coefficients (γ, h), by choosing a specific pair $(\bar{\gamma}, \bar{h})$ as reference or universal standard. We recall the elements of our wave model:

- $U_{tt} - U_{xx} - \gamma U_{ttt} - \delta^2 U_{xxt} = f$: force balance,
- $U_{tt} - U_{xx}$: material force from vibrating string with U displacement,
- $-\gamma U_{ttt}$ is Abraham-Lorentz (radiation reaction) force with γ a small positive parameter,
- $-\delta^2 U_{xxt}$ is a friction force acting on frequencies larger than the cut-off frequency $\frac{T}{h}$ and then contributing to internal heating,
- $\delta = \frac{h}{T}$ is a smallest coordination length with h a measure of finite precision,
- T is the common energy/temperature of each vibrating string frequency,
- f is exterior forcing.

Oue wave equation as a blackbody model is thus defined by the pair of parameters (γ, h), assuming the coefficients of the vibrating string are normalized to 1, which can be achieved by adjusting space and time units. We shall now show that (γ, h) for a blackbody B effectively are determined by the values $(\bar{\gamma}, \bar{h})$ of a chosen reference blackbody $\bar{B}$ with the property that $\bar{\gamma}$ is maximal and $\bar{h}$ minimal.

We thus assume that $\gamma \leq \bar{\gamma}$ and $h \geq \bar{h}$, where $\bar{\gamma}$ is a maximal radiation coefficient and $\bar{h}$ a minimal precision parameter and choose the model with maximal γ and minimal h, that is $\gamma = \bar{\gamma}$ and $h = \bar{h}$, to be the reference blackbody which will be used as reference thermometer.

Consider now a blackbody body B defined by (γ, h) in radiative equilibrium with the reference blackbody $\bar{B}$ defined by $(\bar{\gamma}, \bar{h})$ by sharing a common forcing $\bar{f} = f$. Radiative equilibrium requires

$$\gamma T = \bar{\gamma}\bar{T}, \tag{13.1}$$

which determines the temperature scale for B as $T = \frac{\bar{\gamma}}{\gamma}\bar{T}$. Effectively, we may then assume that $\gamma = \bar{\gamma}$ by asking that $T = \bar{T}$ in radiative equilibrium. If we ask B to have the same cut-off as the reference $\bar{B}$, we will then also have $h = \bar{h}$.

We may thus choose the model defined by $(\bar{\gamma}, \bar{h})$ as a model of a universal blackbody with radiation only depending on temperature and frequency expressing universality of blackbody radiation: All blackbodies defined by $\gamma \leq \bar{\gamma}$ and $h \geq \bar{h}$ with $\gamma h = \bar{\gamma}\bar{h}$, will then have the same radiation spectrum given by Planck's law as $\bar{\gamma}\bar{T}\nu^2$.

By choosing $\bar{\gamma}$ maximal we ensure that a blackbody represents a reference of maximal absorption/emission to which a *greybody* with less absorption/emission will be compared, as expanded below.

We understand that the universality reflects the choice of the blackbody $\bar{B}$ as universal thermometer, which thus can be seen as a concrete model of Kirchhoff's conceptual model in the form a cavity with graphite walls.

The role of graphite walls, observed by Kirchhoff in his experiments, is to equilibrate the temperature over frequency, independent of the object placed in the cavity, which is required for universality. Kirchhoff observed that with reflecting walls this was not achieved as the radiation spectrum showed to depend on the nature of body placed in the cavity.

For an introduction to classical work with an empty cavity as an abstract universal reference blackbody, An Analysis of Universality in Blackbody Radiation by P.M. Robitaille.

Chapter 14

Radiative Heat Transfer

> Either the quantum of action was a fictional quantity, then the whole deduction of the radiation law was essentially an illusion representing only an empty play on formulas of no significance, or the derivation of the radiation law was based on sound physical conception. (Planck)

14.1 Stefan-Boltzmann for Two Blackbodies

Consider a blackbody B_1 of temperature T_1 in radiative contact with another blackbody B_2 of temperature T_2 with $T_2 > T_1$ both modeled by (11.2) and sharing a common forcing f. Consider B_2 to be a source of heat energy with the forcing f balanced by radiation from B_2 according to Planck's law, setting here for simplicity $h = 1$ so that $\nu_T = T$:

$$\overline{f_\nu^2} = \gamma T_2 \nu^2 \quad \text{for} \quad |\nu| < T_2, \quad \overline{f_\nu^2} = 0 \quad \text{else}. \tag{14.1}$$

The momentary total heating Q_{12} of B_1 by B_2 through f is given by Planck's Law as

$$\begin{aligned} Q_{12} &= \sum_{|\nu| \leq T_1} (\gamma T_2 \nu^2 - \gamma T_1 \nu^2) + \sum_{T_1 < |\nu| \leq T_2} \gamma T_2 \nu^2 \\ &\approx \frac{\gamma}{3}(T_2 T_1^3 - T_1^4) + \frac{\gamma}{3}(T_2^4 - T_2 T_1^3) = \frac{\gamma}{3}(T_2^4 - T_1^4), \end{aligned} \tag{14.2}$$

that is,

$$Q_{12} = \sigma(T_2^4 - T_1^4) \quad \text{with} \quad T_2 > T_1, \tag{14.3}$$

which expresses Stefan-Boltzmann's Law for the radiative heat transfer from a one body in radiative contact with a body of lower temperature. We see that the heat transfer has a contribution from frequencies below the cut-off T_1 for B_1 as the difference $\gamma(T_2 - T_1)\nu^2$ and one contribution from freqencies above T_1 as $\gamma T_2 \nu^2$.

We can view the Stefan-Boltzmann Law (14.3) as form of Fourier Law stating a positive rate heat transfer from a higher temperature T_2 to a lower temperature T_2. In differentiated form this law can be expressed as

$$Q_{12} \approx 4\sigma T^3 (T_2 - T_1) \quad \text{for some} \quad T_1 < T < T_2 \tag{14.4}$$

which mimics a Fourier Law expressing heat flow as being proportional to a temperature gradient.

14.2 Non-Physical Two-Way Heat Transfer

Notice the requirement in (14.3) that $T_2 > T_1$. In the literature one finds the law without this requirement in the form

$$Q_{12} = \sigma T_2^4 - \sigma T_1^4, \quad Q_{21} = \sigma T_1^4 - \sigma T_2^4 = -Q_{12} \tag{14.5}$$

where Q_{21} is the heat transfer from B_1 to B_2 as the negative of Q_{12}.

This form has led to a misinterpretation of Stefan-Boltzmann's Law as expressing heat transfer from B_2 to B_1 of size σT_2^4 balanced by a transfer $-\sigma T_1^2$ from B_1 to B_2, as if two opposing transfers of heat energy is taking place between the two bodies with their difference determining the net flow.

Such a misinterpretation was anticipated and countered in Stefan's original article [42] from 1879:

- *The absolute value the heat energy emission from a radiating body cannot be determined by experiment. An experiment can only determine the surplus of emission over absorption, with the absorption determined by the emission from the environment of the body.*

- *However, if one has a formula for the emission as a function of temperature (like Stefan-Bolzmann's Law), then the absolute value of the emission can be determined, but such a formula has only a hypothetical meaning.*

Stefan-Boltzmann's Law (14.3) thus requires $T_2 > T_1$ and does not contain two-way opposing heat transfer, only one-way heat transfer from warm to cold. Unfortunately the misinterpretation has led to a ficititious non-physical "backradiation" underlying CO_2 global warming alarmism.

Chapter 15

Greybody vs Blackbody

We now consider a greybody B defined by the wave model with (γ, h) and assume the temperature T of B is calibrated so that $T = \bar{T}$ in radiative equilibrium with the reference blackbody $\bar{B}$ with maximal γ and cut-off (minimal h). Energy balance can be expressed as

$$\gamma T = \alpha\bar{\gamma}T \tag{15.1}$$

where α is a coefficient of absorptivity of B, assuming both bodies follow Planck's Law.

We ask a blackbody to have maximal emissivity = absorptivity and we thus have $\alpha \leq 1$ and $\gamma \leq \bar{\gamma}$ reflecting that a blackbody is has maximal $\bar{\gamma}$ and cut-off.

A body B with $\gamma < \bar{\gamma}$ will thus be termed *greybody* defined by the coefficient of absorptivity

$$\alpha = \frac{\gamma}{\bar{\gamma}} < 1 \tag{15.2}$$

and will have a coefficient of emissivity $\epsilon = \alpha$.

A greybody B thus interacts through a reduced force $f = \sqrt{\alpha}\bar{f}$ with a blackbody $\bar{B}$ with full force $\bar{f}$. We thus obtain a connection through the factor $\sqrt{\alpha}$ between force interaction and absorptivity.

The spectrum of a greybody is dominated by the spectrum of a blackbody, here expressed as the coefficient $\alpha = \epsilon < 1$. A greybody at a given temperature may have a radiation spectrum of a blackbody of lower temperature as seen in the front page picture.

Chapter 16

2nd Law of Radiation

> There is only one law of Naturethe second law of thermodynamicswhich recognises a distinction between past and future more profound than the difference of plus and minus. It stands aloof from all the rest. ... It opens up a new province of knowledge, namely, the study of organisation; and it is in connection with organisation that a direction of time-flow and a distinction between doing and undoing appears for the first time. (Eddington)

> Just as the constant increase of entropy is the basic law of the universe, so it is the basic law of life to be ever more highly structured and to struggle against entropy. (Vaclav Havel)

16.1 Irreversible Heating

Radiative heating of a blackbody is an irreversible process, because the heating results from dissipation with coherent high frequency energy above cut-off being transformed into internal heat energy.

We assume that the dissipation is only active above cut-off, while the radition is active over the whole spectrum. Below cut-off radiation is a reversible process since the same spectrum is emitted as absorbed. Formally, the radiation term is dissipative and thus would be expected to transform the spectrum, and the fact that it does not is a remarkable effect to the resonance.

16.2 Mystery of 2nd Law

The 2nd Law of thermodynamics has posed a mystery to science ever since it was first formulated by Clausius in the mid 19th century, because it involve the mysterious concept of *entropy* which is postulated to never decrease by some mysterious mechanism.

In [22] I state and prove a 2nd Law of thermodynamics in terms of kinetic energy, heat energy, work and turbulent dissipation, without reference to entropy.

16.3 Stefan-Boltzmann Law as 2nd Law

Similarly, the new derivation of Planck's and Stefan-Boltzmann's laws of this book proves a 2nd law for radiative transfer between two blackbodies without any reference to entropy, which can be expressed as follows

$$Q_{12} = \sigma(T_1^4 - T_2^4) \quad \text{if} \quad T_1 > T_2 \tag{16.1}$$

where T_1 is the temperature of the blackbody 1 and T_2 that of blackbody 2, assuming that $T_1 > T_2$, and $Q_{12} > 0$ is transfer of heat energy from 1 to 2. The transfer of energy is thus from hot to cold.

The equality (16.1) is often written in the form

$$Q_{12} = \sigma T_1^4 - \sigma T_2^4 \tag{16.2}$$

without specifying that $T_1 > T_2$ and is then interpreted as expressing transfer of heat energy of size σT_1^4 from 1 to 2 and a transfer of σT_2^4 in the opposite direction from 2 to 1. But this interpretation lacks physical rationale and results from a purely formal algebraic operation of splitting the one term in (16.1) into the differenec of two terms in (16.2). This is the origin of the "backradiation" underlying climate alarmism which thus lacks physical reality.

Chapter 17

Reflection vs Blackbody Absorption/Emission

A blackbody emits what it absorbs ($f^2 \rightarrow R$), and it is thus natural to ask what makes this process different from simple reflection (e.g. $f \rightarrow -f$ with $f^2 \rightarrow f^2$)? The answer is that the mathematics/physics of blackbody radiation $f \rightarrow \ddot{u} - u'' - \gamma\dddot{u}$, is fundamenatlly different from simple reflection $f \rightarrow -f$. The string representing a blackbody is brought to vibration in resonance to forcing and the vibrating string string emits resonant radiation. Incoming waves thus are absorbed into the blackbody/string and then are emitted depending on the body temperature. In simple reflection there is no absorbing/emitting body, just a reflective surface without temperature.

Chapter 18

Blackbody as Transformer of Radiation

The Earth absorbs incident radiation from the Sun with a Planck frequency distribution characteristic of the Sun surface temperature of about 5778 K and an amplitude depending on the ratio of the Sun diameter to the distance of the Earth from the Sun. The Earth as a blackbody transforms the incoming radiation to a outgoing blackbody radiation of temperature about 288 K, so that total incoming and outgoing energy balances.

The Earth thus acts as a transformer of radiation and transforms incoming high-frequency low-amplitude radiation to outgoing low-frequency high-amplitude radiation under conservation of energy.

This means that high-frequency incoming radition is transformed into heat which shows up as low-frequency outgoing infrared radiation, so that the Earth emits more infrared radiation than it absorbs from the Sun. This increase of outgoing infrared radiation is not an effect of backradiation, since it would be present also without an atmosphere.

The spectra of the incoming blackbody radiation from the Sun and the outgoing infrared blackbody radiation from the Earth have little overlap, which means that the Earth as a blackbody transformer distributes incoming high-frequency energy so that all frequencies below cut-off obtain the same temperature. This connects to the basic assumption of statistical mechanics of *equidistribution in energy* or thermal equilibrium with one common temperature.

In the above model the absorbing blackbody inherits the equidistribution of the incoming radition (below cut-off) and thereby also emits an equidistributed spectrum. To ensure that an emitted spectrum is equidistributed even if the forcing is not, requires a mechanism driving the system towards equidistribution or thermal equilibrium.

Chapter 19

Hot Sun and Cool Earth

19.1 Emission Spectra

The amplitude of the radiation/light emitted from the surface of the Sun at 5778 K when viewed from the Earth is scaled by the viewing solid angle (scaling with the square of distance from the Sun to the Earth), while the light spectrum covering the visible spectrum centered at $0.5\,\mu m$ remains the same. The Earth emits infrared radiation (outside the visible spectrum) at an effective blackbody temperature of 255 K (at a height of 5 km), thus with almost no overlap with the incoming Sunlight spectrum. The Earth thus absorbs high-frequency reduced-amplitude radiation and emits low-frequency radiation, and thereby acts as a transformer of radiation from high to low frequency: Coherent high-frequency radiation is asborbed and dissipated into incoherent heat energy, which is then emitted as coherent low-frequency radiation.

The transformation only acts from high-frequency to low-frequency, and is an irreversible process representing a 2nd law.

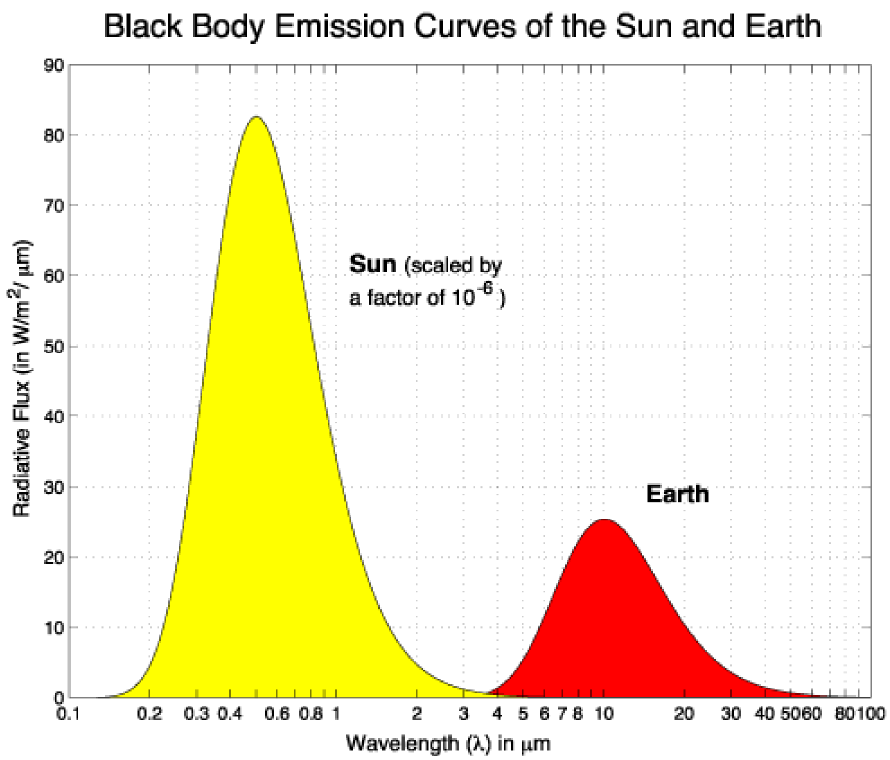

Figure 19.1: Blackbody spectrum of the Sun and the Earth.

Chapter 20

Blackbody Dynamics

20.1 Recollection of Model

We now study the dynamics of radiative transfer of heat energy between two blackbodies. We recall our model for one blackbody subject to radiative forcing as a wave equation expressing force balance:

$$U_{tt} - U_{xx} - \gamma U_{ttt} - \delta^2 U_{xxt} = f, \tag{20.1}$$

where here the subindices indicate differentiation with respect to space $latexx$ and time $latext$, and

- $U_{tt} - U_{xx}$ is out-of-equilibrium force of a vibrating string with displacement U,
- $-\gamma U_{ttt}$ is the Abraham-Lorentz (radiation reaction) force with γ a small positive parameter,
- $-\delta^2 U_{xxt}$ is a friction force replacing the radiation reaction force for frequencies larger than a cut-off frequency $\frac{T}{h}$ and then contributing to the internal energy,
- $\delta = \frac{h}{T}$ is a smallest coordination length with h a measure of finite precision,
- T is the common energy/temperature of each frequency of the vibrating string,

- f is exterior forcing.

The model is specified by the parameters γ and h. It is shown in Universality of Blackbody Radiation that all blackbodies can be assumed to have the same value of the radiation coefficient γ and the cut-off (precision h), given as the values of a chosen reference blackbody with the property that γ is maximal and h minimal.

We have shown that stationary periodic solutions U satisfy the energy balance

$$R + H = F \tag{20.2}$$

where

$$R = \int \gamma U_{tt}^2 dxdt, \quad H = \int \delta^2 U_{xt}^2 dxdt, \quad F = \int f^2 dxdt, \tag{20.3}$$

which expresses that all incident radiation F is absorbed and is either re-emitted as radiation R or stored as internal energy from heating H with a switch from R to H at the cut-off frequency. We here assume that all frequencies have the same energy $\int U_{\nu,t}^2 dxdt$, where U_ν is the amplitude of frequency ν, and we refer to the common value $\int U_{\nu,t}^2 dxdt = T$ as the temperature.

With dynamics the wave equation (1) expressing force balance is complemented by an equation for the total energy E:

$$E(t) - E(0) + R = \int_0^t \int fU dxds \tag{20.4}$$

expressing that the change $E(t) - E(0)$ is balanced by the outgoing radiation R and absorbed energy from the forcing $\int fU dxdt$, where $E = e + \epsilon$ with e the string energy and ϵ the internal energy as accumulated dissipated energy $\int \delta^2 U_{xt}^2 dx$. Equivalently, the change of the internal energy ϵ is given as $\epsilon(t) - \epsilon(0) = H$. The temperature T connects to E by $T \sim \sqrt{h}E$, assuming all frequencies U_ν of U have the same energy $U_{\nu,t}^2 = T$, because $e \sim \sum_{\nu \leq \frac{T}{h}}$ and so $U_{\nu,t}^2 = \frac{T^2}{h}$ assuming ϵ is dominated by e.

Our model thus consists of (20.1) and (20.4) combined with a mechanism for equidistribution of energy over all frequencies.

20.2 Dynamic Radiative Interaction

Let us now consider two blackbodies in radiative contact, one body B with amplitude U sharing a common forcing f with another blackbody $\bar{B}$ with amplitude $\bar{U}$, modeled by the wave equation:

$$\begin{aligned} U_{tt} - U_{xx} - \gamma U_{ttt} - \delta^2 U_{xxt} = f = \bar{U}_{tt} - \bar{U}_{xx} - \gamma \bar{U}_{ttt} - \delta^2 \bar{U}_{xxt} \\ E_t + R = \int f U dx dt, \\ \bar{E}_t + \bar{R} = \int f \bar{U} dx dt. \end{aligned} \tag{20.5}$$

This system describes the dynamic interaction of B and $\bar{B}$, with given initial values of U, U_t and E for B and the same for $\bar{B}$.

In dynamic interaction different frequencies will have different times scales and thus to maintain that all frequencies have the same temperature some mechanism to this effect will have to be adjoined to the model. We may think of this effect as a form of diffusion acting on frequencies.

We compare with the case of acoustic damping with an acoustic damping term of the form μU_t in which case all frequencies will have the same damping and thus well tempered distributions will be preserved under dynamic interaction. Recall that a piano as a blackbody with acoustic damping is isotempered in the sense that the sustain of different tones is the same.

Let us now see what the model tells in different basic cases:

20.2.1 Both below cut-off

Let us now consider the basic case of interaction with all frequencies below cut-off for both B and $\bar{B}$. The difference $W = U - \bar{U}$ then satisfies the damped wave equation

$$W_{tt} - W_{xx} - \gamma W_{ttt} = 0, \tag{20.6}$$

which upon multiplication by W_t and integration in space and time gives for $t > 0$ (modulo two terms from integration by parts in time with small effect if the time scale is not short):

$$G(t) = G(0) - \int_0^t \int \gamma W_{tt}^2 dx ds, \tag{20.7}$$

where

$$G(t) = \int \frac{1}{2}(W_t^2 + W_x^2) dx. \tag{20.8}$$

It follows that $G(t)$ decays in time which effectively means equilibration in energy with a transfer of energy from the warmer to the colder body.

The effect of the dissipative radiation term $(-\gamma U_{ttt})$ is that the difference in energy (and thus temperature) between the two bodies decreases with time: Energy is transferred from the warmer to the colder body. We have thus proved a 2nd Law as an effect of dissipative radiation.

20.2.2 One below one above cut-off

For frequencies below cut-off for B and above cut-off for $\bar{B}$, assuming B is the warmer, the model shows a transfer from R into $\bar{H}$ with a heating effect on $\bar{B}$ as a result of the energy balance $R + H = \bar{R} + \bar{H}$ with $latex H = 0$ and $latex \bar{R} = 0$.

20.2.3 Both above cut-off

This is analogous to case 1.

Chapter 21

The Photoelectric Effect

21.1 Nobel Prize to Einstein

Einstein was awarded the Nobel Prize in 1923 for "the discovery of the law of the photo-electric effect":

$$h\nu = W + P, \tag{21.1}$$

where here h is Planck's constant, $K > 0$ is the kinetic energy of an electron ejected by incident light of frequency ν on a surface and W is the energy required to release the electron from the surface. Einstein was explicitely not awarded the Prize for his derivation of law based on light as a stream of particles.

We see that Einstein's law defines a cut-off/switch at frequency $\frac{W}{h}$ below which no electrons will be ejected. We see here a connection to the switch at $\frac{T}{h}$ from radiation to internal heating in our radiation model: In both cases the energy of incident light with frequency above the cut-off is absorbed and transformed into a different form of energy.

We shall below derive the law of photoelectricity from a model analogous to the radiation model and thus counter the reservations of the Nobel Commitee.

21.2 The photoelectric effect I

The Einstein model of the photoelectric effect (studied experimentally by Hertz in 1886 and Lenard [7] in 1902), which Einstein presented on three pages in one of his five famous 1905 papers [6] earning him the Nobel Prize in 1921, has the simple form

$$K + W = h\nu,$$

where K is the kinetic energy of an electron ejected by light of frequency ν hitting a surface and $W \geq 0$ is the energy required to release the electron from the surface. In particular, there is a threshold frequency $\nu_{crit} = W/h$ below which no electrons will be emitted. Einstein motivated his model simply by viewing light of frequency ν as a stream of particle-like photons of energy $h\nu$, each of will be absorbed by an atom and eject an electron of kinetic energy $K = h\nu - W$ if $h\nu \geq W$, while it will be reflected without ejection otherwise.

The prediction that the kinetic energy K would scale linearly with the frequency (modulo the shift W) was confirmed in experiments in 1916 by Millikan in the Ryerson Laboratory at the University of Chicago (presently a cite of Finite Element Center and FEniCS). This experiment was received as a convincing proof of the existence of photons, albeit Millikan had set up the experiment in order to disprove the photon concept, which he did not believe in: "while Einstein's photoelectric equation was experimentally established... the conception of localized light-quanta out of which Einstein got his equation must still be regarded as far from being established."

Millikan's success was above all attributable to an ingenious device he termed "a machine shop in vacuo." A rotating sharp knife, controlled from outside the evacuated glass container by electromagnetic means, would clean off the surface of the metal used before exposing it to the beam of monochromatic light. The kinetic energy of the photoelectrons were found by measuring the potential energy of the electric field needed to stop them - here Millikan was able to confidently use the uniquely accurate value for the charge e of the electron he had established with his oil drop experiment.

Ironically, it was Millikan's experiment which convinced the experimentalist-inclined committee in Stockholm to give the 1921 Nobel Prize in physics to Einstein, while Millikan received it in 1923 for his work on the elementary electric charge, and the photoelectric effect.

In 1950, at age 82, Millikan conceeded in his Autobiography, in Chapter 9 entitled *The Experimental Proof of the Existence of the Photon - Einstein's Photoelectric Equation*: "The experiment proved simply and irrefutably, I thought, that the emitted electron that escapes with the energy $h\nu$ gets that

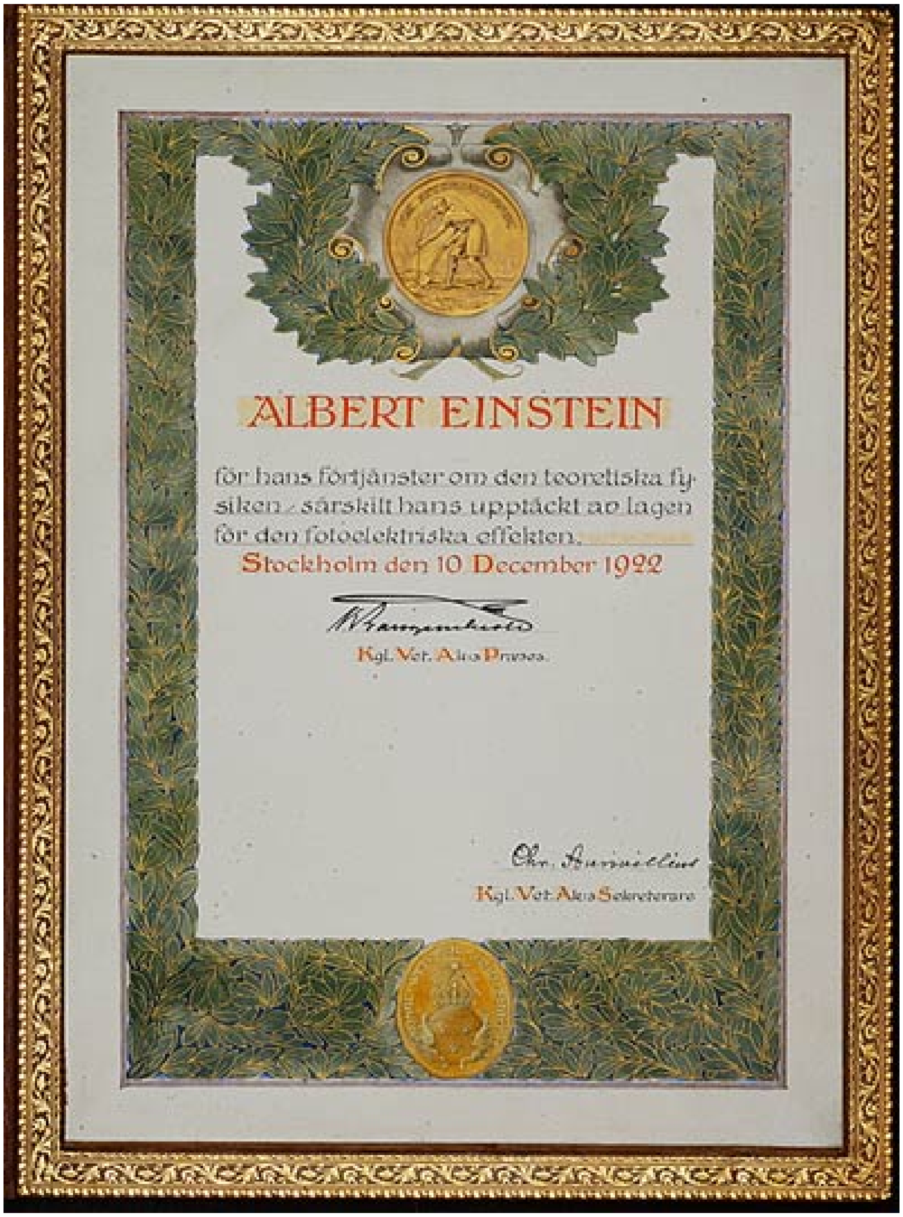

Figure 21.1: The motivation of the 1921 Nobel Prize to Einstein: For his services to theoretical physics, in particular for his **discovery** of the **law** of the photoelectric effect $K + W = h\nu$ (and in particular **not** his **derivation** based on light particles, which the Nobel Committee rejected).

energy by the direct transfer of $h\nu$ units of energy from the light to the electron, and hence scarcely permits of any other interpretation than that which Einstein had originally suggested, namely that of the semi-corpuscular or photon theory of light itself." In the end, Millikan thus seemed to have re-imagined the complex personal history of his splendid experiment to fit the simple story told in so many of our physics textbooks, but it appears that Millikan was never really convinced, maybe just getting old ...

Suppose now, following Millikan's reservations to photons, that we seek to model photoelectricity in the above wave model. This can readily be done by a frequency dependent non-linear viscosity in a model of the following form after spectral decomposition (with for simplicity $\mu = \alpha = 0$):

$$\ddot{u}_\nu + \nu^2 u_\nu - \gamma \dddot{u}_\nu + \delta^2(u_\nu)\dddot{u}_\nu = f_\nu, \qquad (21.2)$$

where the viscosity coefficient $\delta^2(u_\nu)$ is given as

$$\delta^2(u_\nu) = \alpha(T_\nu)(h\frac{|\ddot{u}_\nu|}{|\dot{u}_\nu|} - W)_+$$

with $\alpha(T_\nu)$ some positive coefficient. and $v_+ = \max(v, 0)$. The factor $\delta^2(u_\nu)$ would then correspond to the kinetic energy of ejected electrons per unit incident intensity f^2, with a threshold $h\nu - W \geq 0$ since $\frac{|\ddot{u}_\nu|}{|\dot{u}_\nu|} \approx |\nu|$.

We see that $\delta(u_\nu) > 0$ models ejection under a threshold condition of the form $\frac{|\dot{u}_\nu|}{|u_\nu|} \geq W/h$, which reflects a certain "relative sharpness" of the absorbed wave corresponding simply to its frequency. In practice, ultraviolet light of wavelength $\sim 10^{-6}$ will be able to eject electrons but not infrared light of wave-length $\sim 10^{-4}$. But even ultraviolet light has a wave-length much larger than the atomic scale, so ejection must be a phenomenon on a larger scale than atomic scale, possibly a result of superposition of large and small scale waves?

We can thus model photoelectricity as a dissipative effect acting for frequencies above a certain threshold, similar to the computational dissipation discussed above.

It thus would appear that it is possible to set up a very simple model for the photoelectric effect again without any statistics. Right?

21.3 Remark on Viscosity Models

Dissipative effects depending on high frequencies can be modeled by viscosity terms in force balance equations depending on different derivatives of the state variable. In several cases of basic importance including turbulence and radiation, the viscosity coefficients are small which results in states with a range of frequencies from small to large. In these cases mean-values of the state have a weak dependence on the absolute size of the small viscosity, which allows accurate modeling without knowing the details of the viscous effect, only that the viscosity coefficient is small. This "miracle" results from a subtle phenomenon of cancellation in fluctuating states, and allows computational simulation of e.g. turbulence without resolving the details of the turbulent flow.

21.4 The Photolelectric Effect II

As a model of the photoelectric effect consider a wave equation model of the form

$$\ddot{u} + u'' - \gamma \dddot{u} - (\delta^2(u)\dot{u}')' = f, \tag{21.3}$$

where

$$\delta^2(u)\dot{u}'(x,t) = \sum_{\nu} \alpha(T)(h\frac{|\dot{u}_\nu'(t)|}{|\dot{u}_\nu(t)|} - W)_+ \dot{u}_\nu' e^{i\nu x}.$$

Chapter 22

The Compton Effect

22.1 The Compton Effect I

The one observation believed to demonstrate the photon theory most convincingly is the effect discovered 1923, again in the Ryerson Laboratoy at the University of Chicago, by Arthur Compton (1892-1962) while investigating the scattering of X-rays. Compton observed that incoming light of frequency of a certain frequency ν could eject electrons and at the same time be scattered into light of a lower frequency $\mu < \nu$ with the change in frequency corresponding to the kinetic energy of the ejected electrons, assuming the electrons where the outmost electrons of carbon atoms with W comparatively small. This red-shift is called the Compton effect.

Can we alternatively model the Compton effect in the above model? Yes, it seems so: In the above β-model high-frequency waves are absobed and eject electrons according to the Einstein's formula, while low-frequency waves will be absorbed and radiated. Evidently, this could be viewed as a red-shift in radiated waves, if we assume multiple frequencies of incoming light.

22.2 The Compton Effect II

We could alternatively set up a model with direct shift in frequency of the form:

$$\ddot{u}_\nu + \nu^2 u_\nu + \beta(u_\nu)\dot{u}_\nu + \ddot{u}_\mu = f_\nu, \tag{22.1}$$

$$\ddot{u}_\mu + \mu^2 u_\mu - \gamma \dddot{u}_\mu + \ddot{u}_\nu = 0, \tag{22.2}$$

reflecting a “two-body problem” with two coupled bodies of different eigenfrequencies $\nu > \mu$.

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[6] A. Einstein, On a Heuristic Poimt of View Toward the Emission and Transformation of Light, Ann. Phys. 17, 132, 1905.

[7] P. Lenard, Ann. Phys. 8, 1902.

[8] ...Mechanically, the task seems impossible, and we will just have to get used to it (quanta) (Planck 1909).

[9] I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In that case, nothing remains of my entire castle in the air, gravitation theory included, and of the rest of physics. (Einstein 1954)

[10] Since the theory of general relativity implies representations of physical reality by a continuous field, the concept of particles or material points cannot have a fundamental part, nor can the concept of motion. (Einstein)

[11] You believe in the God who plays dice, and I in complete law and order in a world which objectively exists, and which I, in a wild speculative way, am tryin to capture. I hope that someone will discover a more realistic way, or rather a more tangible basis than it has been my lot to find. Even the great initial success of Quantum Theory does not make me believe in the fundamental dice-game, although I am well aware that younger collegues interpret this as a consequence of senility. No doubt the day will come when we will see those instictive attitude was the correct one. (Einstein to Born, 1944)

[12] Some physicists. among them myself, cannot believe that we must abandon, actually and forever, the idea of direct representation of physical reality in space and time; or that we must accept then the view that events in nature are analogous to a game of chance. (Einstein, On Quantum Physics, 1954)

[13] If God has made the world a perfect mechanism, He has at least conceded so much to our imperfect intellects that in order to predict little parts of it, we need not solve inumerable differential equations, but can use dice with fair success. (Born, on Quantum Physics)

[14] The theory (quantum mechanics) yields a lot, but it hardly brings us closer to the secret of the Old One. In any case I am convinced that *He* does not throw dice. (Einstein to Born 1926)

[15] A. Einstein, On a Heuristic Point of View Toward the Emission and Transformation of Light, Ann. Phys. 17, 132, 1905.

[16] Einstein: *I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In that case, nothing remains of my entire castle in the air, gravitation theory included, and of the rest of physics.* (Einstein 1954)

[17] Shut up and calculate. (Dirac on quantum mechanics)

[18] The more success the quantum theory has, the sillier it looks. (Einstein)

[19] C. Johnson, Computational Blackbody Radiation, in *Slaying the Sky Dragon: Death of the Greenhouse Gas Theory*, Stairways Press, 2010.

[20] C. Johnson, Computational Blackbody Radiation, Icarus eBooks, 2011, http://www.nada.kth.se/cgjoh/ambsblack.pdf .

[21] J. Hoffman and C. Johnson, *Computation Turbulent Incompressible Flow*, Springer 2008.

[22] J. Hoffman and C. Johnson, *Computational Thermodynamics*, http://www.nada.kth.se/cgjoh/ambsthermo.pdf

[23] Thomas Kuhn, Black-Body Theory and the Quantum Discontinuity, 1894-1912, Oxford Univ Press 1978.

[24] Millikan, R A, A Direct Photoelectric Determination of Plancks h, Physical Review 7, 355-388, 1916: "It was in 1905 that Einstein made the first coupling of photo effects and with any form of quantum theory by bringing forward the bold, not to say reckless, hypothesis of an electromagnetic light corpuscle of energy h, which energy was transferred upon absorption to an electron. This hypothesis may well be called reckless, first because an electromagnetic disturbance which remains localised in space seems a violation of the very conception of an electromagnetic disturbance, and second because it flies in the face of the thoroughly established facts of interference.... if the equation be of general validity, then it must certainly be regarded as one of the most fundamental and far reaching of the equations of physics; for it must govern the transformation of all short-wave-length electromagnetic energy into heat energy. Yet the semi-corpuscular theory by which Einstein arrived at his equation seems at present to be wholly untenable ... ...a modification of Planck's latest idea seems to me able to account for all the relations thus far known between corpuscular and ethereal radiations If any particular frequency is incident upon [a substance containing oscillators of every conceivable frequency] the oscillators in it which are in tune with the impressed waves may be assumed to absorb the incident waves until the energy content as reached a critical value when an explosion occurs and a corpuscle is shot out with an energy h .It is to be hoped that such a theory will soon be shown to be also reconcilable with the facts of black body radiation.

[25] Schrodinger, The Interpretation of Quantum Physics. Ox Bow Press, Woodbridge, CN, 1995:"What we observe as material bodies and forces are nothing but shapes and variations in the structure of space. Particles

are just schaumkommen (appearances). ... Let me say at the outset, that in this discourse, I am opposing not a few special statements of quantum physics held today (1950s), I am opposing as it were the whole of it, I am opposing its basic views that have been shaped 25 years ago, when Max Born put forward his probability interpretation, which was accepted by almost everybody.

[26] "I don't like it, and I'm sorry I ever had anything to do with it". (Erwin Schrodinger talking about Quantum Physics)

[27] Fritjof Kapra, 1975: "A careful analysis of the process of observation in atomic physics has shown that the subatomic particles have no meaning as isolated entities, but can only be understood as interconnections between the preparation of an experiment and the subsequent measurement. Quantum physics thus reveals a basic oneness of the universe. The mathematical framework of quantum theory has passed countless successful tests and is now universally accepted as a consistent and accurate description of all atomic phenomena. The verbal interpretation, on the other hand, i.e. the metaphysics of quantum physics, is on far less solid ground. In fact, in more than forty years physicists have not been able to provide a clear metaphysical model".

[28] Lamb, Willis E Jr., Antiphoton, Applied Physics B 60, 77-84 (1995)

[29] Shankland, R S, An apparent failure of the photon theory of scattering, Physical Review 49, 8-13 (1936)

[30] Stephen Hawking, 1988: "But maybe that is our mistake: maybe there are no particle positions and velocities, but only waves. It is just that we try to fit the waves to our preconceived ideas of positions and velocities. The resulting mismatch is the cause of the apparent unpredictability.

[31] Arthur C. Clarke:"If a scientist says that something is possible he is almost certainly right, but if he says that it is impossible he is probably wrong".

[32] "Schrödinger's point of view is the simplest; he thought that by his develpoment of de Broglie's wsave mechanics the whole pardoxical problem of the quanta had been settled: there are no particles, no 'quantum

jumps'– there are only waves with their well-known vibrations, characterized by integral numbers. The particles are narrow wave-packets. The objection is that one generally needs waves in spaces of many diemnsions, which are something entirely different from the waves of classical physics, and impossible to visualize" (Born in the Born-Einstein Letters)

[33] Schrödinger was, to say the least, as stubborn as Einstein in his conservative attitude towards quantum mechanics; indeed, he not only rejected the statitical interpretation but insisted that his wave mechanics meant a return to a classical way of thinking. He would not accept any objection to it, not even the most weighty one, which is that a wave in $3n$-dimensional space, such as needed to describe the n, is not a classical concept and cannot be visualized. (Born in the Born-Einstein Letters)

[34] "What wanted to say was just this: In the present circumstances the only profession I would choose would be one where earning a living had nothing to do with the search for knowledge". (Einstein's last letter to Born Jan 17 1955 shortly before his death on the 18th of April, probably referring to Born's statistical interpretation of quantum mechanics).

[35] "De Broglie, the creator of wave mechanics, accepted the results of quantum mechanics just as Schrödinger did, but not the statistical interpretation." (Born in the Born-Einstein Letters)

[36] "I cannot understand how you can combine an entirely mechanistic universe with the freedom of the ethical will". (Born in the Born-Einstein Letters)

[37] "At any moment, the knowledge of the objective world is only a crude approximation from which, by applying certain rules such as the probability of quantum mechanics, we can predict unknown (e.g. future) conditions" (Born in the Born-Einstein Letters)

[38] It seems to me that the concept of probability is terribly mishandled these days. A probabilistic assertion presupposes the full reality of its subject. No reasonable person would express a conjecture as to whether Caesar rolled a five with his dice at the Rubicon. But the quantum mechanics people sometimes act as if probabilistic statements were to be applied *just* to events whose reality is vague. (Schrödinger in a letter to Einstein 1950)

[39] Thomas Kuhn, Black-Body Theory and the Quantum Discontinuity, 1894-1912, Oxford Univ Press 1978.

[40] D. Loke, Convective and Radiative Cooling, Department of Physics, U of British Columbia, 2007, http://www.physics.ubc.ca/ lueshi/p209/term2/formal/report.ps

[41] ... those who have talked of 'chance' are the inheritors of antique superstition and ignorance...whose minds have never been illuminated by a ray of scientific thought. (T. H. Huxley)

[42] J. Stefan, Ueber die Beziehung zwischen der Warmestrahlung und der Temperatur. Wien Akad. Sitzber. Vol. 79, pp. 391-428, 1879.

[43] Abraham-Lorentz force, Wikipedia $http : //en.wikipedia.org/wiki/AbrahamLorentz_force$.

[44] H.D. Zeh, Physics Letters A 172, 189-192, 1993.