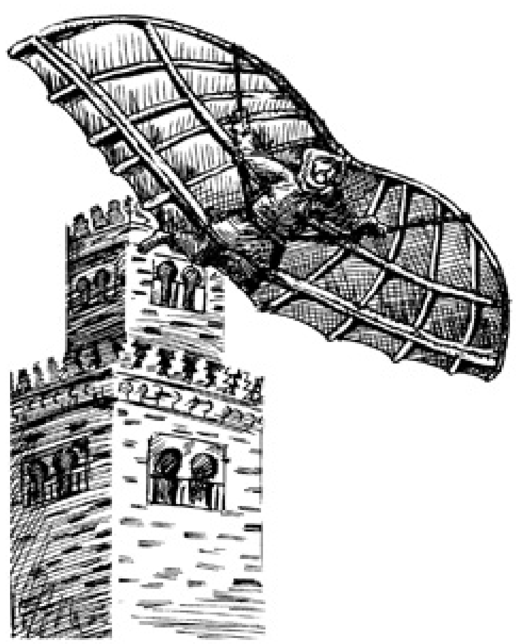
# THE SECRET OF FLIGHT (DRAFT)



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> For when propositions are denied, there is an end of them, but if they bee allowed, it requireth a new worke.

*The Essais of Sr. Francis Bacon*, London, 1612, same first page as in *Modern Developments in Fluid Dynamics* edited by S. Goldstein, Oxford University Press, 1938).

# Preface

> My two equations (the Euler equations) include not only all that has been discovered by methods very different and for the most part slightly convincing, but also all that one could desire further in this science. (Euler 1752)

> The first inquiry in the mind of the reader will probably be as to whether we know just how birds fly and what poer they consume. The answer must, unfortunately, be that we as yet know very little about it. Here is a phenomenon going on daily under our eyes, and it has not been reduced to the sway of mathematical law.... Science has been awaiting the great physicist, who, like Galileo or Newton, should bring order out of chaos in aerodynamics, and reduce its many anomalies to the rule of harmonious law. It is not impossible that when that law is formulated all the discrepancies and apparent anomalies which now appear, will be found easily explained and accounted for by one simple general cause, which has been hitherto overloooked. (Octave Chanute in *Progress in Flying Machines* 1891)

> Only a very few comprehensive presentations of the scientific fundamentals of the aerodynamics of the airplane have ever been published. (Schlichting-Truckenbrodt in *Aerodynamics of the Airplane* 1979)

This book presents a *New Theory of Flight* describing in precise mathematical terms for the first time, how the flow of air around a wing can generate a lift to drag ratio larger than 10, which makes flight possible for birds and airplanes at affordable power. The New Theory is a spin-off of the resolution of D'Alembert's Paradox of zero lift and drag of *potential flow* (stationary irrotational inviscid incompressible flow) formulated in 1752, which was published by the authors in 2008. D'Alembert's Paradox paralyzed theoretical fluid mechanics from start and the birth of modern fluid mechanics is traditionally counted as the resolution presented in 1904 by Ludwig Prandtl coronating him to *Father of Modern Fluid Mechanics* [144], an epithet asking for revision in the light of the New Theory.

The book is a continuation of our previous book *Computational Turbulent Incompressible Flow* as Vol 4 of *Applied Mathematics Body&Soul*, Springer 2007, where a first version of the New Theory of Flight was presented together with a resolution of D'Alembert's Paradox.

The official doctrine or accepted truth of the fluid mechanics community is that Prandtl resolved D'Alembert's Paradox by suggesting that drag originates from a thin boundary layer of slightly viscous flow resulting from imposing a *no-slip* boundray condition, which discriminates potential flow satisfying a *slip* boundary condition expressing zero skin friction.

The theory of flight filling text-books is a combination of (i) inviscid circulation theory by Kutta-Zhukovsky for lift and (ii) viscous boundary layer theory by Prandtl for drag and generation of circulation at a sharp trailing edge. This theory, which we refer to as the *Old Theory*, is based on potential flow modified by circulation to give physical results and possibly matched with boundary layer solutions to satisfy no-slip boundary conditions.

Our resolution of D'Alembert's Paradox shows that Prandtl's resolution does not describe correct physics, by showing that drag results from a specific separation mechanism of inviscid flow satisfying a slip boundary condition without boundary layer, described as *three-dimensional (3d) rotational slip separation*, which is generated by a basic instability of potential flow.

The New Theory of Flight shows that also the large lift of a wing results from 3d rotational slip separation, which eliminates the high pressure at the trailing edge of potential flow destroying lift generated at the leading edge.

More generally, The New Theory results from the discovery that slightly viscous bluff body flow can be described as potential flow modified by 3d slip separation. Since both potential flow and 3d slip separation can be captured by analytical mathematics, it is possible to grasp and understand main aspects of a partly turbulent flow which has been beyond comprehension.

The New Theory is based on computational solution and mathematical analysis of the Navier-Stokes equations for slightly viscous incompressible flow with slip boundary condition as a model of observed small skin friction, which according to the above vision of Euler describes subsonic aerodynamics. Computation and analysis show that solutions are not of the form postulated by the Old Theory, which thus is an unphysical theory.

Computational aerodynamics has been hampered by a Prandtl dictate to use no-slip boundary condition as the key to resolve D'Alembert's Paradox. It gives rise to thin boundary layers, which require impossible quadrillions of mesh points for computational resolution. The insight that a slip boundary condition is a physically correct boundary condition for slightly viscous flow, suddenly frees computational aerodynamics from a paralyzing spell.

The New Theory opens new possibilities of computational modeling of aerodynamics of flight and thus to innovative new design. We hope the reader after digesting the book will share our conviction that the New Theory can be seen as the dream of Euler come true, after 260 years. The book is controversial by questioning the bible of flight formulated by Kutta-Zhukovsky-Prandtl as the fathers of modern aerodynamics.

We were first asked by Dover to publish the book, but Dover then backed down apparently under pressure from the aerodynamics establishment to suppress the book. We are happy that xxxx now gives voice to new ideas.

Stockholm in November 2012, JH, JJ and CJ

This book is dedicated to the memory of Otto Lilienthal (1848-1896): *Glider King* and *Father of Flight* and author of *Birdflight as the Basis of Aviation*.


Figure 1: Otto Lilienthal (1848-1896) admired on one of his many gliding flights before the glider stalled at 15 meters above ground on August 9 1896, lost lift and crashed. Lilienthal's last words to his brother Gustav were "sacrifices must be made.".

> The employment of curved surfaces in connection with light explosion motors has produced good results; but it would be a mistake to bo satisfied with those results, and to consider that the last word has been spoken on the development of aviation. As yet, strong winds are the terror of all aviators, and to the whims of the wind have been added the whims of the motor. We have not yet succeeded in taming the wind and utilizing its wild forces. We have to continue our investigations; experiments guided by theoretical considerations will have to clear up many problems before we can claim victory over the air. (*Birdflight as the Basis of Aviation* by Otto and Gustav Lilienthal, 1891)

# Part I

# Short Story of Flight

# Chapter 1

# From Old to New Theory of Flight

> We call it theory when we know much about something but nothing works, and practice when everything works but nobody knows why. (Einstein)
>
> Of all the forces of Nature, I would think the wind contains the greatest amount of power. (Abraham Lincoln)
>
> Isn't it astonishing that all these secrets have been preserved for so many years just so we could discover them! (Orville Wright)
>
> Few physical principles have ever been explained as poorly as the mechanism of lift. ([90]).

## 1.1 What Keeps Planes in the Air?

What keeps a bird or airplane in the air? How can the flow of air around a wing generate large *lift* $L$ (balancing gravitation) at small *drag* $D$ (requiring forward thrust) with a *lift to drag ratio* $\frac{L}{D}$ ranging from $10$ for short thick wings to $70$ for the long thin wings of extreme gliders, which allows flying at affordable power for both birds and airplanes? How can an albatross glide 50 meters in still air upon losing 1 meter in altitude, with apparently $\frac{L}{D} = 50$?

Somehow evolution over millions of years has designed birds so that they can fly by their own muscle power. The dream of human-powered flight came true in 1977 on $60\,m^2$ wings generating a lift of $100\,kp$ at a thrust of $5\,kp$ (thus with $\frac{L}{D} = 20$) at a speed of $5\,m/s$ supplied by a $25\,kp\,m/s = \frac{1}{3}\,hp$ human powered pedal propeller as shown in Fig. 1.2.

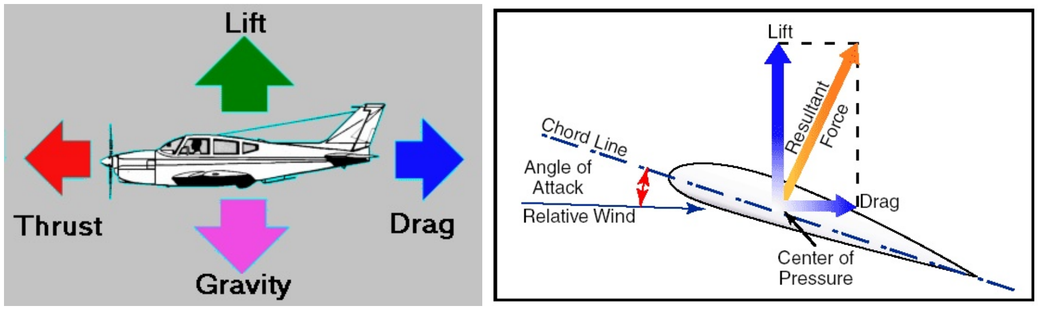



Figure 1.1: Left: Lift $L$ balancing gravity and drag $D$ balanced by thrust. Right: Forces on an airplane wing (section) from the flow of air around the wing. How is it possible that $\frac{L}{D} > 10$? Notice the shape of the wing section with a rounded *leading edge* meeting to flow and a more or less sharp *trailing edge* where the flow is leaving the wing. Notice the *angle of attack* or tilting of the wing with respect to incoming horisontal flow of air.

Figure 1.2: The Gossamer Condor constructed by Paul MacCready (wing area of 60 $m^2$ and weight of 32 $kg$) flew the first figure-eight, a distance of $2.172$ meters winning the first Kremer prize of $50.000$ pounds offered by the industrialist Henry Kremer in 1959.

The era of engine powered human flight was initiated on December 17 1903 by Wilbur and Orville Wright, who managed to get their $12\,hp$ *Flyer* off ground in controled flight lasting 12 seconds, with Orville winning the bet to be pilot. Like the birds, Orville and Wilbur did not have to really understand *why* what they managed to do was possible; they just managed somehow to do it by trial and error, building on the careful studies of bird wings by Lilienthal from the 1890s. Orville and Wilbur had computed that with $\frac{L}{D} = 10$ and a velocity of 10 meter per second, 4 effective $hp$ would get the 300 kg of the *Flyer* off ground at an effective thrust of 30 kp, and it worked!

Charles Lindberg crossed the Atlantic in 1927 at a speed of 50 m/s in his 2000 kg *Spirit of St Louis* at an effective engine thrust of 150 kp (with $\frac{L}{D} = 2000/150 \approx 13$) from $100\,hp$.

A $400\,ton$ Airbus 340 crusies at maximal speed of $900\,km/h$ with a wing lift of (about) $1\,ton/m^2$ and $66.000\,hp$ engine thrust of 20 tons with $\frac{L}{D} = 20$. The plane takes off at $300\,km/h$ after accelerating during 35 seconds at a maximal thrust of 100 tons from 4 engines.

Figure 1.3: How can a 560 ton Airbus380 take off?

How is this possible? Is it a miracle? Is there a theory of flight and what does it say? What is the dependence of lift and drag on wing form, wing area, angle of attack (the tilting of the wing in the flow direction) and speed? Can engineers compute the distribution of forces on an Airbus 380 during take-off and landing using mathematics and computers, or is model testing in wind tunnels the only way to figure out if a new design will work? Let's see what media says.

New York Times informs us in 2003 [130] under the slightly scaring headline *STAYING ALOFT: What Does Keep Them Up There?*:

- *To those who fear flying, it is probably disconcerting that physicists and aeronautical engineers still passionately debate the fundamental issue underlying this endeavor: what keeps planes in the air?*

The authority NASA dismisses on its website [117] all popular science theories for lift, including your favorite one, as being incorrect, but refrains from presenting any theory claimed to be correct and ends without helping the curious with: *To truly understand the details of the generation of lift, one has to have a good working knowledge of the Euler Equations*.

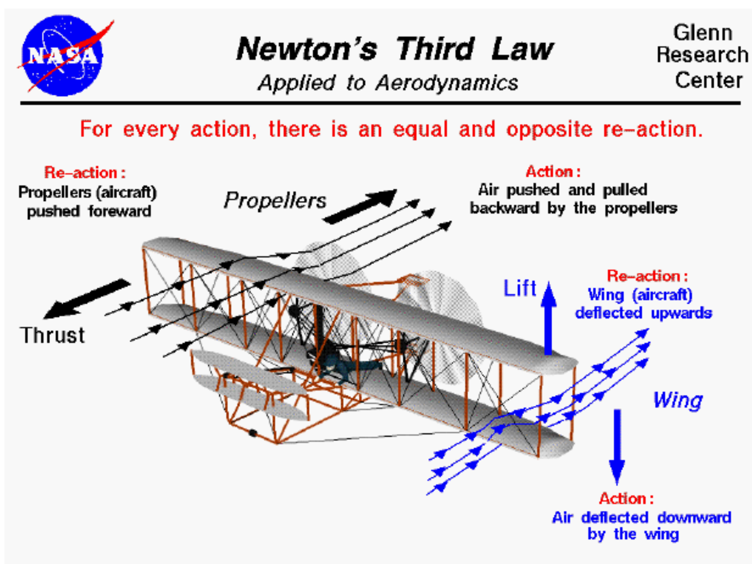



Figure 1.4: Tautological explanation of the flight of *The Flyer* by NASA: There is upward lift on the wing from the air as a reaction to a downward push on the air from the wing.

How can that be? Is there no answer to give a curious child or someone with fear of flying? No, not according to *Airfoil Lifting Force: Misconceptions in K-6 Textbooks* [91]:

- *How do airplane wings really work? Amazingly enough, this question is still argued in many places, from elementary school classrooms all the way up to major pilot schools, and even in the engineering departments of major aircraft companies. This is unexpected, since we would assume that aircraft physics was completely explored early this century. Obviously the answers must be spelled out in detail in numerous old dusty aerodynamics texts. However, this is not quite the case. Those old texts contain the details of the math, but it's the interpretation of the math that causes the controversy. There is an ongoing Religious War over both the way we should understand the functioning of wings, and over the way we should explain them in children's textbooks.*

Science is about understanding and understanding flight is to be able to explain from the principles of fluid mechanics how a wing can generate large lift at small drag. Is it possible that NASA cannot explain what keeps planes in the air? Yes, it is possible: birds fly without explaining anything. Is it not only possible but even true that NASA does not know? We leave this question to the reader to answer after digesting the book.

## 1.2 Basic Lift and Drag Formula

The lift $L$ and drag $D$ of a wing scales with the wing area $A$, the air density $\rho$ and the square of the velocity $V$ according to the basic formula

$$L = C_L A \frac{1}{2}\rho V^2, \;\; D = C_D A \frac{1}{2}\rho V^2 \tag{1.1}$$

where $C_L$ is a *lift coefficient* and $C_D$ a *drag coefficient* characteristic of the wing form. The formulas (1.1) can be derived by dimensional analysis from the Euler equations, and thus must have been understood in principle already by Euler. The density of air at sea level is $1.2\, kg/m^3$ and at 1500 meters $1.0\, kg/m^3$. Below we normally set $\rho = 1$.

Lilienthal found experimentally that for arched wings with a camber of $\frac{1}{12}$, $C_L \approx 0.1\alpha$, where $\alpha$ is the *angle of attack* of the wing up to a maximal angle of $15 - 20$ degrees before *stall*, with thus a maximal $C_L \approx 1.5 - 2$, which describes common wings as shown in Fig. 1.5.

Lilienthal also determined $C_D$ experimentally from his many gliding flights and found $\frac{L}{D} = \frac{C_L}{C_D} = 10 - 15$ for larger angles of attack. For common wings, experiments show $C_D = 0.02 + 0.001\alpha^2$ which gives a maximal $\frac{L}{D} = 10 - 15$ for $\alpha = 2 - 12$, see Fig. 1.5.

Lilienthal computed the power required to carry $100\,kg$ through air of unit density at a speed of $10\,m/s$ and thrust of $5\,kp$, with thus $\frac{L}{D} = 20$, to be $50\,kpm/s = \frac{2}{3}\,hp$. With $C_L = 1.5$ the basic formula gave a wing area of $14\,m^2$. The computation made Lilienthal believe in human flight, but lacking a light weight engine he had to restrict himself to gliding flight.

One may compare with the data for the Gossamer Condor with a half the speed $(5\,ms)$, half the power $(\frac{1}{3}\,hp)$ and 4 times as large wing area $(60\,m^2)$.

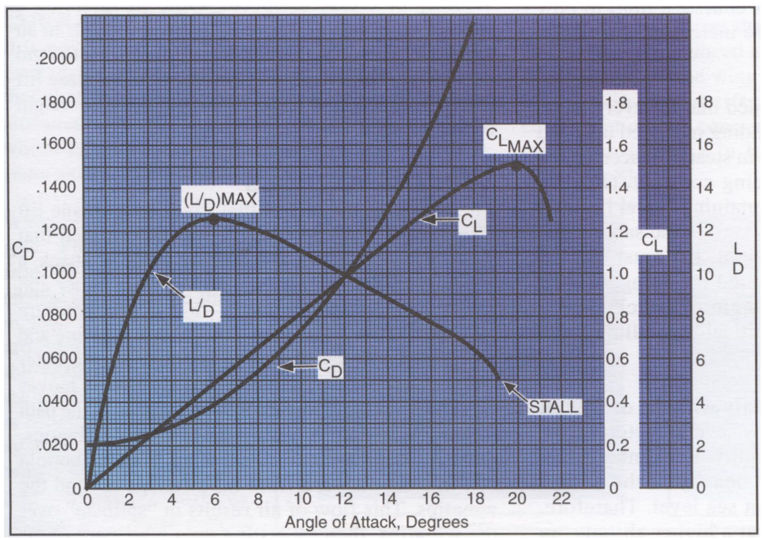



Figure 1.5: Lift and drag coefficients $C_L$ and $C_D$, and lift to drag ratio $\frac{L}{D} = \frac{C_L}{C_D}$ for a typical wing, as functions of the angle of attack.

## 1.3 Basic Objective of Theory of Flight

We formulate as a basic objective of a theory of flight to explain in mathematical terms, experimental observations for common wings of:

1. $\frac{L}{D} = 10 - 50$,
2. $C_L \approx 0.1\alpha$ and $C_D \approx 0.02 + 0.001\alpha^2$ up to stall at $\alpha \approx 15 - 20$ degrees.

The wider objective concerns mathematical modeling of the full aerodynamics of real airplanes and flyers including stall and extreme dynamics.

## 1.4 Text Book Theory of Flight

Text books tell a different story than New York Times and NASA, by claiming: Yes, there is a solid mathematical theory of flight explaining what keeps planes in the air, which was developed 100 years ago by

- German physicist *Ludwig Prandtl*, named *Father of Modern Fluid Mechanics*,
- German mathematician *Martin Kutta*,
- Russian mathematician *Nikolay Zhukovsky*, named *Father of Russian Aviation*.

These fathers suggested in 1904 that both lift and drag of a wing result from an effect of *viscosity* retarding fluid particles in a thin boundary layer from free stream to zero *no-slip* relative velocity on the wing surface, capable of changing the global features of the flow from the

- zero lift and drag of idealized unphysical *inviscid laminar potential flow*,

to the

- large lift and small drag of real physical *slightly viscous turbulent flow*.

The unphysical feature of inviscid laminar potential flow of having zero drag and lift, making flight in particular impossible, named *D'Alembert's Paradox* formulated in 1752, was thus claimed by Prandtl in 1904 to be a result of allowing fluid particles to glide or *slip* on the wing surface without friction. Prandtl thus suggested that drag was caused by a *no-slip* boundary layer with fluid particles sticking to the surface. Kutta and Zhukovsky complemented by obtaining lift by modifying potential flow with a flow circulating around the wing section and thus obtained lift from *circulation*, assuming that the circulation somehow was created by the boundary layer.

This became the basic postulate of modern fluid mechanics: In mathematical modeling of fluid flow it is necessary to use a *no-slip boundary condition* accompanied by a *viscous boundary layer*: Both lift and drag originate from the boundary layer; without a boundary layer there will be no lift and no drag. The current text book theory of flight is basically a 100 year old Kutta-Zhukovsky-Prandtl theory with Kutta-Zhukovsky supplying lift (without drag) and Prandtl drag (without lift).

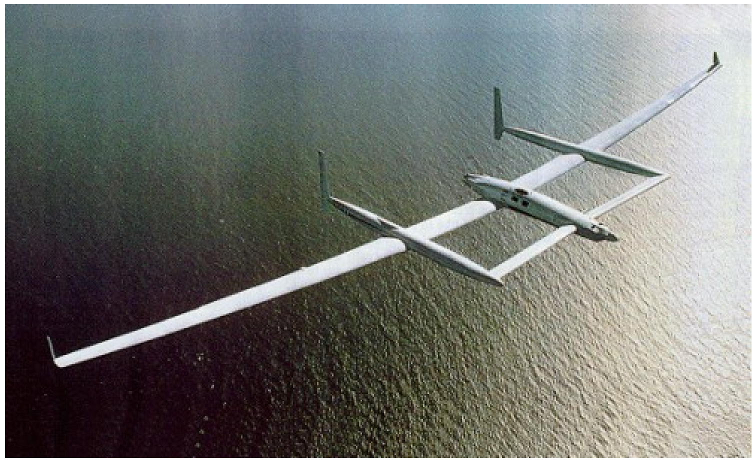


Figure 1.6: The Rutan Model 76 Voyager designed by Burt Rutan was the first aircraft to fly around the world without stopping or refueling, piloted by Dick Rutan and Jeana Yeager in 1986. The Voyager covered the 40.212 km round the globe in about 216 hours at a maximal speeed of 196 km per hour. The Voyager had $\frac{L}{D} = 32$ and a take-off weight of $73\%$ fuel.

Prandtl used a no-slip boundary condition in 1904 to handle D'Alembert's Paradox, which had paralyzed theoretical fluid mechanics from start, but in doing so Prandtl sent fluid mechanics from an 18th century paralysis of zero lift and drag into a 20th and even 21st century computational mathematics impossibility of resolving very thin boundary layers.

## 1.5 New Theory of Flight

In this book we present a mathematical theory of flight based on computing and analyzing turbulent solutions of the incompressible Navier-Stokes equations for slightly viscous flow subject to a slip boundary condition. We thus break the no-slip spell of Prandtl and we do this by a new resolution of D'Alembert's Paradox identifying the unphysical aspect of potential flow to be *instability* rather than violation of no-slip, understanding that slip correctly models the very small *skin friction* of slightly viscous flow.

We focus on *subsonic flight* on fixed wings at speeds well below the speed of sound as the most basic, mysterious and elegant form of flight practiced in *gliding flight* by birds and gliders, propeller airplanes and jet airplanes in take-off and

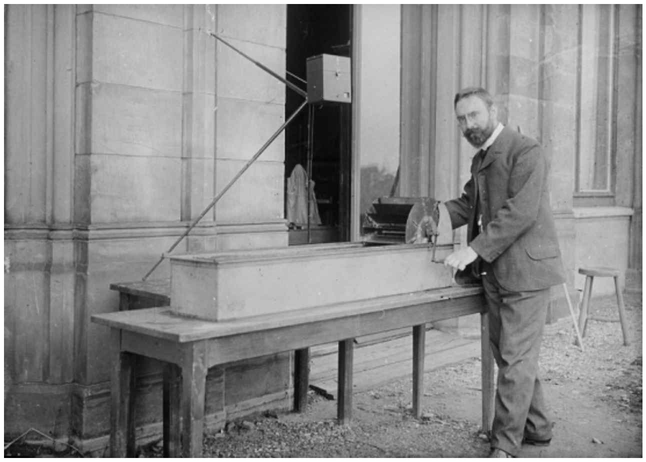


Figure 1.7: Prandtl formulating the credo of modern fluid dynamics in 1904: *I have now set myself the task to investigate systematically the laws of motion of a fluid whose viscosity is assumed to be very small...A very satisfactory explanation of the physical process in the boundary layer [Grenz-schicht] between a fluid and a solid body could be obtained by the hypothesis of an adhesion of the fluid to the walls, that is, by the hypothesis of a zero relative velocity between fluid and wall.*.

landing, in which case the flow of air around the wings is nearly *incompressible*.

To fly at supersonic speed like a jet driven rocket in compressible flow is not gliding flight and no aerodynamics wonder. But like an albatross stay gliding for days without flapping the wings, is a miracle, and this is the miracle we will explain. Not by reference to exotic difficult mathematics which nobody can understand and which does not capture physics, but by exhibiting certain fundamental mathematical aspects of the physics of slightly viscous incompressible flow, which can be grasped without any tricky physics or mathematics.

We will thus identify the primary mechanism for the generation of substantial lift at the expense of small drag of a wing, and discover it to be fundamentally different from the classical circulation theory of lift by Kutta-Zhukovsky and the boundary layer separation theory of drag by Prandtl.

The essence of the new flight theory can be grasped from Fig. 1.8 depicting the following crucial features of the pressure distribution around a section of a wing with (H) indicating high pressure and (L) low pressure:

- Left: Potential flow with leading edge positive lift from (H) on lower wing surface and (L) on upper, cancelled by trailing edge negative lift from (L)

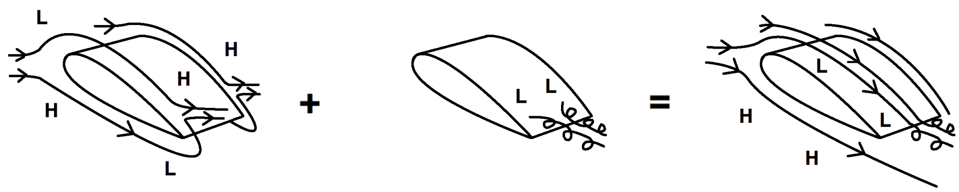



Figure 1.8: Correct explanation of lift by perturbation of potential flow (left) at separation from physical low-pressure turbulent counter-rotating rolls (middle) into 3d rotational slip separation changing the pressure and velocity at the trailing edge into a flow with downwash and lift (right).

on lower and (H) on upper surface, giving zero lift. Potential flow does not cause *downwash* redirecting the flow downwards and thus no lift by reaction.

- Middle: Instability at trailing edge separation with low pressure swirling flow generating *3d ( three-dimensional) rotational slip separation*.
- Right: The resulting flow as potential flow modified by 3d rotational separation at the trailing edge at background pressure without (H) and (L), thus with substantial lift from (H) and (L) at the leading edge, accompanied by downwash redirecting incoming flow downwards.

The reality of 3d rotational slip separation at the trailing edge is shown in Fig. 1.9 by traces of ink ejected on the upper wing surface from an experiment in the Dryden water channel.

The rest of the book can be seen as an expansion and explanation of this picture with the following key questions to be answered:

- Why does the air stick to the upper wing surface without separation and thus cause lift by low-pressure suction?
- Why does the air not turn around the trailing edge like potential flow destroying lift?

We shall see that the miracle of gliding flight results from a happy combination of all of the following features of subsonic flow of air around a wing:

1. nearly incompressible with velocity well below the speed of sound,
2. sufficiently small viscosity, sufficient speed and size of the wing,

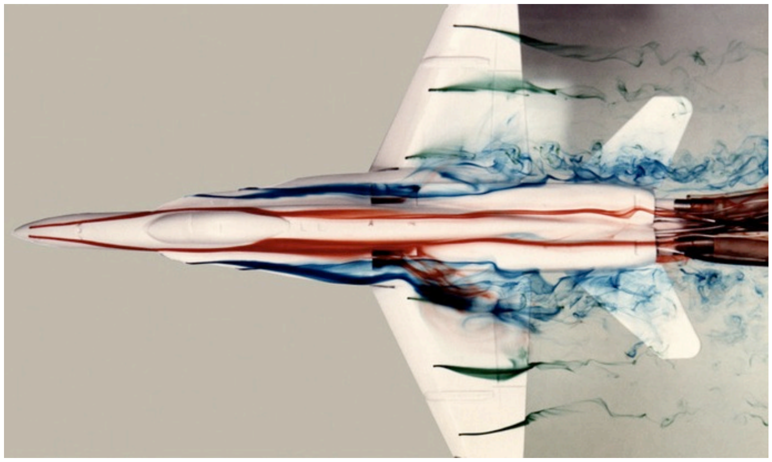


Figure 1.9: The flow around an airplane is visualized in the Dryden water tunnel by emission of dye of different colors from different points on the airplane surface. Notice in particular the trace of the dye ejected on top of the wing follwing a streamline without rotation until the trailing edge where rolls of swirling flow are generated, as a characteristic of 3d rotational slip separation.

3. 3d rotational slip separation at the trailing edge.

The requirement 2 can be expressed as a requirement of a sufficiently large *Reynolds number* $Re = \frac{UL}{\nu}$, where $U$ is a characteristic fluid velocity ($10 - 100\,m/s$), $L$ a characteristic length scale ($1 - 10\,m$) and $\nu$ the (kinematic) viscosity (about $10^{-5}$ for air).

Experiments recorded in [121] show that the lift to drag ratio $\frac{L}{D}$ of a typical wing sharply increases from about 2 to 20 as a result of decreasing drag and increasing lift, as $Re$ increases beyond a *critical Reynolds number* $Re_{crit} \approx 0.5 \times 10^6$. The drag reduction is in the literature described as *drag crisis* reflecting that it is poorly explained by Prandtl's theory. The increase of the lift to drag ratio with a factor 10 is what makes gliding flight possible, and can be seen as a form of miracle as any factor 10 above normality. This book presents the mathematics of this miracle, thereby turning it into science.

As a consequence, gliding flight in syrup is not possible (because of too large viscosity) and neither in two space dimensions or 2d (because the 3d separation is missing). We shall see that the classical flight theory of Kutta-Zhukovsky-Prandtl violates 2 and 3: Prandtl talks about a viscous flow, and Kutta-Zhukovsky about 2d flow.

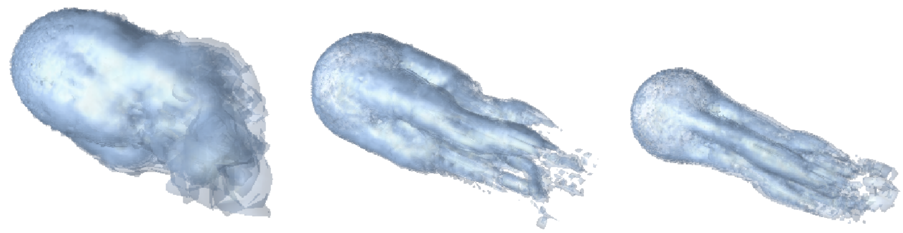


Figure 1.10: Illustration of the drag crisis in the flow around a sphere as the Reynolds number $Re$ passes $Re_{crit} \approx 0.5 \times 10^6$, with delayed separation, decreasing wake and reduction of drag (coefficient) from $0.5$ to $0.2$. Notice the separation at the crest for $Re < Re_{crit}$ with large drag and and the structure of the wake flow for $Re > Re_{crit}$ with four counter-rotating rolls of swirling flow resulting from 3d rotational slip separation or elegant separation.

The result is that the flight theory of Kutta-Zhukovsky-Prandtl does not describe reality. Aviation developed despite theory and not with the help of theory, just like real life is made possible by making a clear distinction from theoretical religious scholastics. To say one thing and do another may be a necessity of life including religion and politics, but is not the scientific method.

## 1.6 3d Rotational Separation: Elegant

We emphasize that a crucial element of the New Theory is

- **3d rotational slip separation**,

to be described and studied in detail below, which makes it possible for the flow to leave the wing without the pressure rise of non-physical 2d potential flow separation destroying the lift created by suction over the crest of the wing.

The 3d rotational slip separation pattern is similar to the elegant swirling motion of the hand used by noble men at the royal court when backwards leaving from an audience with the king.

## 1.7 Elegant Separation: Minimal Stagnation

We show in Part IV that irrotational potential flow separation is unstable because of opposing flow retardation and therefore changes into more stable (or quasi-stable) 3d rotational separation, with the unstable retardation to stagnation of potential flow being replaced by more stable rotating transversally accelerating flow.

It is thus possible to see potential flow separation as maximally unstable by maximal retardation/stagnation and 3d rotational separation as minimally unstable by minimal retardation/stagnation and as such the configuration chosen by the real flow. Elegant separation can therefore be characterized by minimal retardation/stagnation as ”effortless” separation under small pressure gradients.

On the other hand, potential flow attachment is quasi-stable because the retardation is caused by the body and not by opposing flow, and thus the real flow is similar to potential flow in attachement and beyond until separation, as discussed in Chapter 7 and analyzed in Part IV.

## 1.8 Bluff Body Flow: Computable Understandable

The key to unlock the Secret of Flight is the discovery that slightly viscous bluff body flow in general, including the flow around a wing, can be described as

- **potential flow modified by 3d rotational slip separation**,

which can be seen as a summary of this book. This means that slightly viscous bluff body flow, although partly turbulent, is found to have a large scale structure determining large scale mean values like lift and drag, which can been captured by solving the Navier-Stokes equations with slip using millions of mesh points in *Large Eddy Simulation (LES)* without resort to turbulence models. Slightly viscous turbulent bluff body flow is thus discovered to be both computable and understandable.

## 1.9 From Symmetric to Unsymmetric Separation

The lift formula $C_L \sim \alpha$ with lift proportional to the angle of attack $\alpha$ can be understood as an effect of unsymmetric attachment-separation in real flow with 3d rotational separation resulting in downwash, to be compared with zero lift of the symmetrical attachment-separation of potential flow without downwash, as illustrated in Fig. 1.11. Since the downwash in real flow appears to be proportional to $\alpha$, it is natural to expect lift to scale the same way, which is also what is observed. On the other hand, we may expect drag to change little for small angles of attack and thus have a quadratic dependence on $\alpha$, as observed. The basic objective of

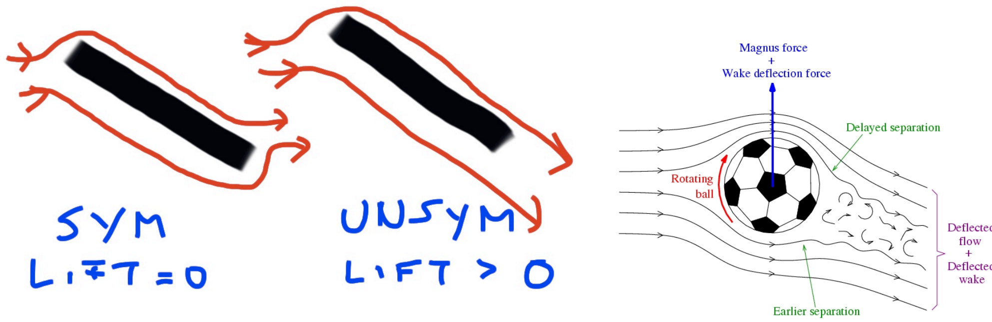



Figure 1.11: Left: Potential flow around flat plate with symmetric attachment-separation without lift and downwash. Middle: Real flow with unsymmetric attachment-separation with positive lift and downwash. Right: Unsymmetric separation of rotating ball creating the lift force of the *Magnus effect*.

showing that $C_L = 0.1\alpha$ and $C_D = 0.02 + 0.001\alpha^2$, thus may seem intuitively reasonable and thus potentially possible to rationalize by mathematical analysis.

## 1.10 From Old Theory to New Theory

We will refer the the text-book theory since 100 years of the Kutta-Zhukovsky-Prandtl theory as the *Old Theory of Flight* and our new theory as the *New Theory of Flight*. The full picture includes both the Old and New Theory and a comparison between the two. To fully understand why the New Theory is correct, requires (from a historical perspective) understanding why the Old Theory is incorrect. In short, the Old Theory is incorrect because it is a 2d theory which does not describe actual full 3d physics, and the New Theory is correct because it describes actual full 3d physics.

## 1.11 Butterfly in Brazil and Tornado in Texas

How can you prove that a butterfly in the Amazonas cannot set off a Tornado in Texas? Well, take away the butterfly and notice that tornados anyway develop.

We shall use the same strategy to show that lift and drag of a wing do not emanate from any boundary layer: In computational simulation we eliminate the boundary layer by using a slip boundary condition, and we find that the computed flow and lift and drag fit very well with experimental observation.


Figure 1.12: Tornados in Texas are not caused by butterflies in Brazil.

## 1.12 Lift and Drag without Boundary Layer

We thus show that Prandtl's explanation is insufficient by computationally solving the Navier-Stokes equations with slip boundary condition, which eliminates the boundary layer, and observing turbulent solutions with lift and drag in accordance with observation. We thus observe tornados without any butterfly: Lift and drag do not originate from a thin boundary layer. The central thesis of Prandtl underlying modern fluid mechanics cannot be correct. As a consequence much of the material propagated in text books of fluid mechanics needs to be fundamentally revised.

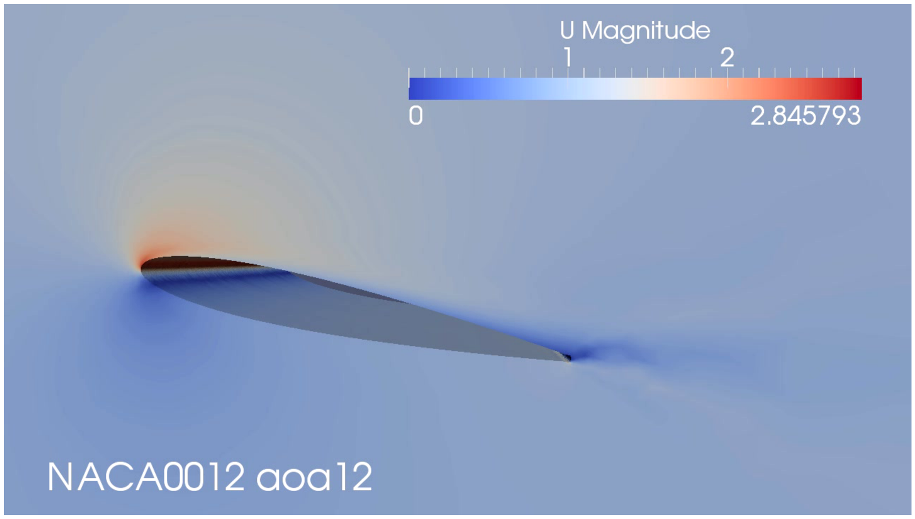



Figure 1.13: Lift and drag of a wing are not caused by a thin boundary layer.

## 1.13 Why Slip is a Physical Boundary Condition

The slip boundary condition models the small boundary friction force of slightly viscous flow, understanding that Navier-Stokes equations can be solved with (small) friction force boundary conditions or no-slip velocity boundary conditions, and that the former choice is computationally more advantageous since there is no no-slip boundary layer to resolve. We will show that this allows accurate computation of both lift and drag for complex geometries using millions of mesh point, which is possible today, instead of the quadrillions required for boundary layer resolution, which is way beyond the capacity of forseeable computers.

## 1.14 From Unphysical 2d model to Physical 3d Model

The classical Kutta-Zhukowsky circulation theory for lift is a 2d theory over a section of a long wing, as well as Prandtl's boundary layer theory for drag. The Old Theory is thus essentially a 2d theory, although Prandtl developed a version with some 3d aspects. The New Theory is based on solving the Navier-Stokes equations in 3d and shows that the generation of both lift and drag of a wing is a full 3d phenomenon, which cannot be captured in a 2d based physical model. The Old Theory is thus a mathematical theory which does not describe actual physics and thus is unphysical. This is acknowledged in text-book presentations of the

Old Theory using phrases such as ”the real flow can be represented (or viewed) as 2d potential flow augmented by 2d circulation” thus admitting that the real flow is not really of this form, while seeking justification by arguing that ”all models are models”.

## 1.15 Flight and Turbulence

The reason it has taken so long time to develop a theory of flight describing the real physics, is that the flow of air around a wing is partly turbulent, and turbulence has been a main unresolved mystery of continuum mechanics: The complex and seemingly chaotic nature of 3d turbulent flow defies description by analytical mathematics.

But today computers are powerful enough to allow computational simulation of 3d turbulent flow which opens to demystify turbulence and thereby demystify flight. In [103] we demonstrated the new possibilties of both simulating and understanding turbulence and thus opened to a new theory of flight. The central experience is that turbulent flow can be simulated without resolving the physicallly smallest scales, which allows accurate prediction of mean-value outputs such as lift and drag with today affordable computer power.

Turbulent flow can be seen as Natures way of handling an intrinsic instability of laminar slightly viscous flow with the fluid constantly seeking a more stable configuration without ever succeeding and thus fluctuating on a range of scales from large scale more or less organized motion to smaller scale fully chaotic motion. 3d rotational slip separation represents a large scale quasi-stable flow, which reflects a basic instability of potential flow at separation and thus can be captured analytically into an understandable feature of turbulent flow.

Zero lift and drag potential flow around a wing thus develops into a flow which at the trailing edge of the wing separates into turbulent flow which has big lift and small drag. Without turbulence it would be impossible to fly.

Turbulence consumes energy by internal friction or viscous dissipation into heat and thus flying requires input of energy, from flapping wings, propellers or jet engine or from upwinds for powerless gliders.

Turbulence is 3d phenomenon, which makes also flying a 3d phenomenon. The classical flight theories of Kutta-Zhukovsky-Prandtl are 2d and do not describe real 3d flight.

Figure 1.14: Turbulent flow depicted by Da Vinci. This picture properly viewed reveals the secret of flight.

## 1.16 Compute-Analyze-Understand

This book is based on computing solutions to the incompressible Navier-Stokes equations with slip using a stabilized finite element method with automatic a posteriori error control in mean-value outputs such as lift and drag. The error control is based on duality measuring output sensitivity with respect to resolution of the Navier-Stokes equations in the fluid domain and the boundary conditions. This is a form of *Direct Numerical Simulation DNS* without user-specified turbulence or wall model made possible by using slip as a simple wall model and finite element stabilization as automatic turbulence model.

Computed solutions thus can be guaranteed to give correct output information (up to a tolerance depending on the computational work) and thereby offer a digital fluids laboratory or wind tunnel. This book shows that the secret of flight is revealed by combining the experimental facility of the digital laboratory with a bit of mathematical analysis. The secret revealed shows the ingenious design of a wing generating large lift at small drag, which was somehow first discovered by Nature itself and then copied by man.

## 1.17 Computation vs Experiments

In Fig. 1.16 we show computed lift and drag (coefficients) $C_L$ and $C_D$ of a long NACA012 wing under increasing angles of attack from cruising over take-off/landing to stall, obtained by solving the Navier-Stokes equations using automatically adapted meshes with less than $10^6$ mesh points (blue curve) compared to measured values in wind tunnel experiments. The computational values lie within the range of the experimental values and thus evidently captures reality.

The message is that it is possible to compute the lift and drag of an airplane in the whole range of angles of attack, from small angles of stationary cruising at high speed, to large angles close to stall in the dynamics of start and landing at low speed. This is a happy new message, since state-of-the-art tells [59] that 50 years of doubled computer power every 18 months according to Moore's Law, are needed to make computation of lift and drag of an airplane possible by solving the Navier-Stokes equations.

We hope this gives the reader motivation to continue reading to discover how a wing generates large lift at small drag, the mystery of flight.

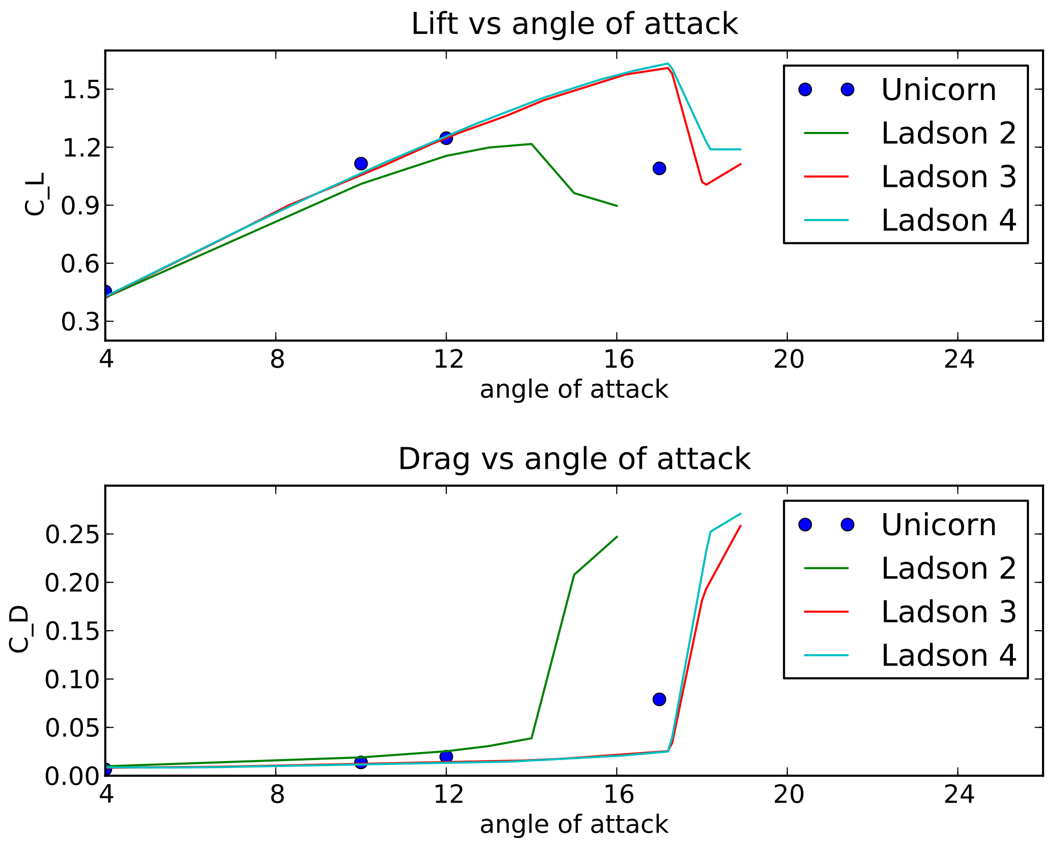



Figure 1.15: Evidence that computation of lift and drag coefficients $C_L$ and $C_D$ of a wing is possible from small angles of attack at crusing to large angles at start and landing: The blue curve shows computed coefficients by solving the Navier-Stokes equations by Unicorn [141] compared to different wind tunnel experiments by (Gregory/O'Reilly and) Ladson [143].

## 1.18 Aerodynamics as Navier-Stokes Solutions

Fluid mechanics is well described by the Navier-Stokes equations expressing conservation of mass, momentum and energy, but there are two basic issues to handle: The viscosity of the fluid is needed as input and the equations in general have turbulent solutions defying analytical description and thus have to be solved computationally using computers.

In aerodynamics or fluid dynamics of air with small viscosity, these issues come together in a fortunate way: The precise value of the small viscosity shows to be largely irrelevant as concerns macroscopic quantities such as lift and drag, and the turbulent solutions always appearing in slightly viscous flow, show to be computable.

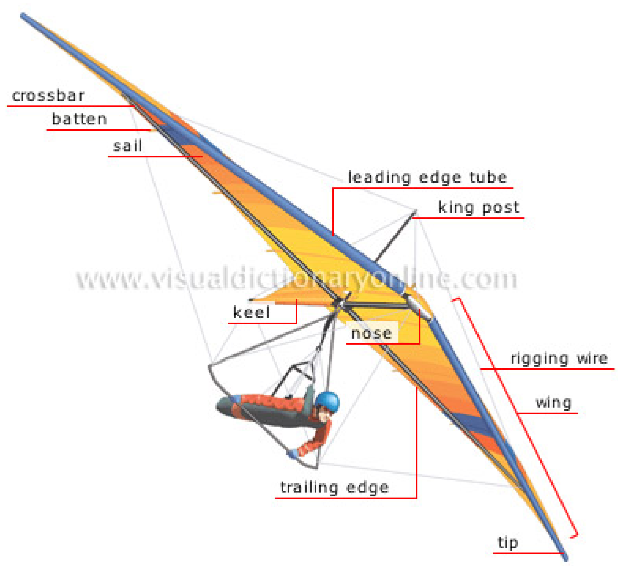



Figure 1.16: Modern hang glider with $\frac{L}{D} = 15 - 20$ and $C_L \approx 0.1\alpha$.

We shall find that there is a catch here, which we will adress shortly, but fortunately a catch which can be overcome, because in Einstein's words:

- *Subtle is the Lord, but malicious He is not.*

This means that the secret of flight can be uncovered by solving the Navier-Stokes equations and analyzing the computed solutions. We thus have at our disposal

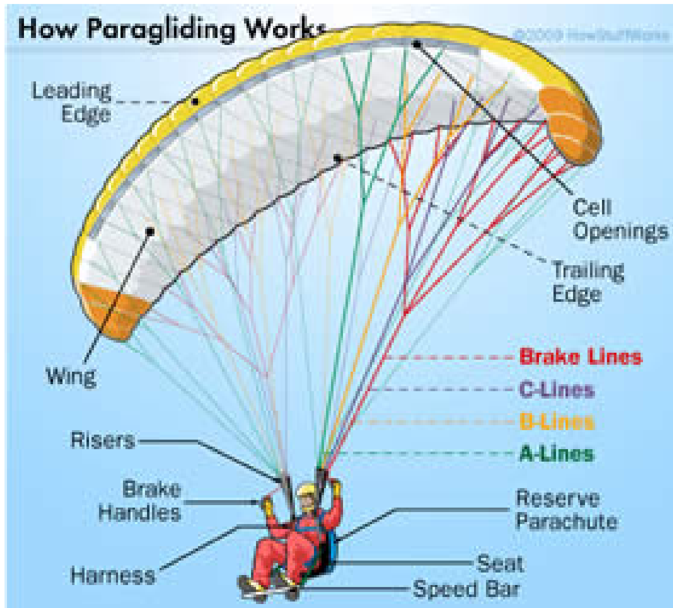



Figure 1.17: Modern paraglider with $\frac{L}{D} \approx 10$ and $C_L \approx 0.1\alpha$.

a complete fluid mechanics laboratory where we can study every aspect of the flow around a wing, or an entire airplane in the critical dynamics of take-off and landing, and also the flapping flight of bird. Nature can hide its secrets in real analog form, but not in digital simulation. This leads to the program of this book:

- Solve the Navier-Stokes equations computationally.
- Study the computed turbulent solutions.
- Discover the mechanism creating large lift at small drag.
- Formalize the discovery into a new understandable theory of flight captured by analytical mathematics.

## 1.19 Flight Control

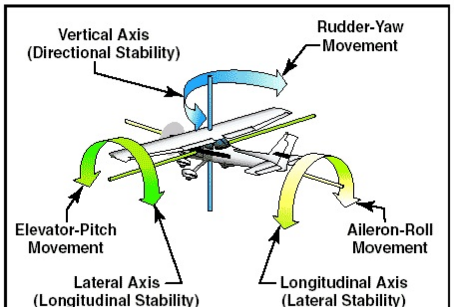
The motion of an airplane is controled by

- roll: longitudinal axis: lateral stability: ailerons,

- yaw: vertical axis: directional stability: rudder,
- yaw: vertical axis: directional stability: spoilers,
- pitch: lateral axis: longitudinal stability: elevators,
- thrust: throttle,

as illustrated in Figs 1.18 and 1.19. The vertical motion is controled by pitch and speed through elevator and throttle and the direction of horisontal motion is controled by ailerons, spoilers and rudder. Turning is made by tilting the airplane (banking) using the ailerons to give different lift on the wings, which gives a sideway force as a horisontal componenet of the lift. The two ailerons are typically interconnected so that one goes down when the other goes up: the down-going aileron increases the lift on its wing while the up-going aileron reduces the lift on its wing, producing a rolling moment about the aircraft's longitudinal axis.

The banking is combined with change of direction by the rudder to keep the nose in the direction of motion, to avoid yaw.




| PRIMARY CONTROL SURFACE | AIRPLANE MOVEMENT | AXES OF ROTATION | TYPE OF STABILITY |
|---|---|---|---|
| Aileron | Roll | Longitudinal | Lateral |
| Elevator/ Stabilator | Pitch | Lateral | Longitudinal |
| Rudder | Yaw | Vertical | Directional |

Figure 1.18: Basic mechanisms for control of lateral, longitudinal and directional stability.

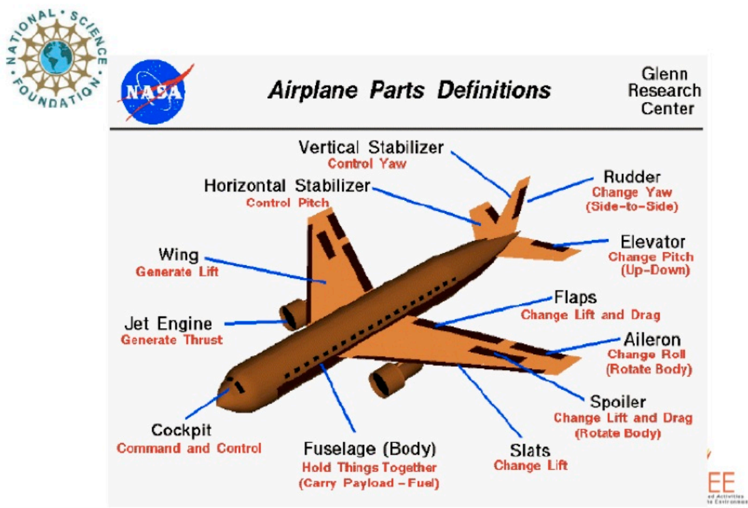



Figure 1.19: The motion of an airplane is controled by ailerions, rudder, elevators and spoilers.

## 1.20 Flight Simulator based on New Theory of Flight

Computational solution of Navier-Stokes/slip/FSI allows direct simulation of flight stability under actions of ailerons, spoilers, rudder and elevators. This open to constructing a new generation of flight simulator offering realistic simulation of flight under exterme conditions as a new tool for pilot training.

The Wright brothers used variable warping of the wings to maintain lateral stability in their Flyer, instead of ailerons which were patented in 1868 by the English inventor Matthew Piers Watt Boulton but did come into use until 1908 when the U.S: inventor and busineman Glenn Curtiss flew an aileron-controlled aircraft.

For longitudinal stability the Flyer had a wing (canard) mounted ahead of the main wings, instead of the conventional tail wing mounted after, which gave an inherently unstable flight which could only be controled by the Wright brothers. The Wright brothers gave up the canard in later designs.

> The course of the flight up and down was exceedingly erratic, partly due to the irregularity of the air, and partly to lack of experience in handling this machine. The control of the front rudder was difficult on account of its being balanced too near the center...(Orville Wright)

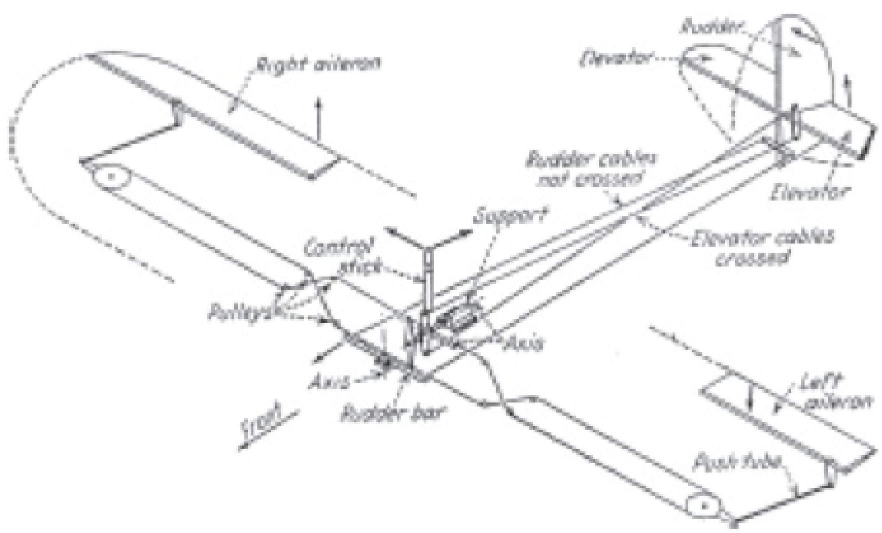



**Figure 1.3** Diagrammatic sketch of a simple airplane control system. When the controls are moved as shown by arrows on the stick and rudder bar, the surfaces move as shown by the arrows. (From Chatfield, Taylor, and Ober, *The Airplane and Its Engine*, McGraw-Hill, 1936)

Figure 1.20: A simple control system showing the effect of the steering pin on ailerons and rudder in sideways motion, and on elevator in motion forward-backward.

Let us hope that the advent of a successful flying machine, now only dimly foreseen and nevertheless thought to be possible, will bring nothing but good into the world; that it shall abridge distance, make all parts of the globe accessible, bring men into closer relation with each other, advance civilization, and hasten the promised era in which there shall be nothing but peace and good-will among all men. (Octave Chanute)

# Chapter 2

# Short History of Aviation

> I feel perfectly confident, however, that this noble art will soon be brought home to man's general convenience, and that we shall be able to transport ourselves and families, and their goods and chattels, more securely by air than by water, and with a velocity of from 20 to 100 miles per hour. (George Cayley 1809)

> During the summers of 1891 and 1892, Mr Lilienthal of Berlin has been gliding downward through the air, almost every Sunday and sometimes on weekdays, upon an aeroplane with which he expects eventually to imitate the soaring of the birds, when he has learned to manage it safely. (Octave Chanute)

> If birds can glide for long periods of time, then why cant I ? (Orville Wright)

> Age appears to be best in four things; old wood best to burn, old wine to drink, old friends to trust, and old authors to read. (Francis Bacon)

## 2.1 Leonardo da Vinci, Newton and d'Alembert

Is it conceivable that with proper mathematics, humans would have been flying, at least gliders (without engine), several hundred years before this actually came true in the late 19th century? Well, let's face some facts.

The idea of flying, like the birds, goes back at least to Greek mythology about the inventor and master craftsman Deadalus, who built wings for himself and his son Icarus in order to escape from imprisonment in the Labyrinth of Knossos on the island of Crete.

Leonardo da Vinci made impressive and comprehensive investigations into aerodynamics collected into his *Codex on the Flight of Birds* from 1505, and designed a large variety of devices for muscle-powered human flight. After extensive testing da Vinci concluded that even if both arms and legs got involved through elaborate mechanics, human power was insufficient to get off the ground.

Newton confirmed these experiences by calculating the lift of a tilted flat plate, representing a wing, in a horisontal stream of "air particles" hitting the plate from below, to be proportional to the sine-squared of the angle of attack, about 6 times too small.

Newton's result was further supported by D'Alembert's Paradox predicting that both the drag and the lift of a body traveling through air would be close to zero, clearly at variance with many early observations of birds flying long distances even without flapping their wings. d'Alembert built his computations of drag (and lift) on particular solutions to the Euler equations referred to as *potential solutions*, with the velocity given as the gradient of a potential satisfying Laplace's equation. But nobody could come up with any kind of resolution of the paradox before Ludwig Prandtl (1875-1953), called the Father of Modern Fluid Dynamics, in a short note from 1904 suggested a resolution based on boundary layer effects from vanishingly small viscosity, which still today remains the accepted resolution of the paradox. As already indicated, we shall below present computational evidence that Prandtl's resolution is not credible and instead put forward a new scientifically more satisfactory resolution.

## 2.2 Cayley and Lilienthal

Despite the pessimistic predictions by Newton and d'Alembert, the 29 years old engineer George Cayley (uncle of the mathematician Arthur Cayley) in 1799 sketched the by now familiar configuration of an airplane with fixed cambered wings and aft horisontal and vertical tails, and also investigated the characteristics of airfoils using a whirling arm apparatus. Cayley outlined his ideas about the principles of flying in *On Aerial Navigation* (1809). But Cayley did not produce any mathematical description of the motion of an aircraft and thus had no quantitative basis for designing airplanes. In 1849 Cayley built a large glider, along the lines of his 1799 design, and tested the device with a 10-year old boy aboard. The glider carried the boy aloft on at least one short flight.

The next major step was taken by the German engineer Otto Lilienthal, who made careful experiments on the lift and drag of wings of different shapes and designed various gliders, and himself made 2000 more or less successful flights starting from a little hill, see Fig 2.1, before he broke his neck in 1896 after the glider had stalled 15 meter above ground.

Otto Lilienthal reported his investigations made together with his brother Gustav during 1866 - 1889 in *Birdflight as the Basis of Aviation, a Contribution towards a System of Aviation, compiled from the Results of Numerous Experiments made by Otto and Gustav Lilienthal* published in 1891, predicting that human flight should be possible with a suitable light weight engine and even by sole human power of $\frac{1}{3}\,hp$.


Figure 2.1: A copy of Otto Lilienthal and one of his gliders. Note the similarity with a modern hang glider.

## 2.3 Kutta, Zhukovsky and the Wright Brothers

Stimulated by Lilienthal's successful flights and his widely spread book *Bird Flight as the Basis of Aviation* from 1899, the mathematician Martin Kutta (1867-1944) in his thesis from 1902 modified the erronous classical potential flow so-

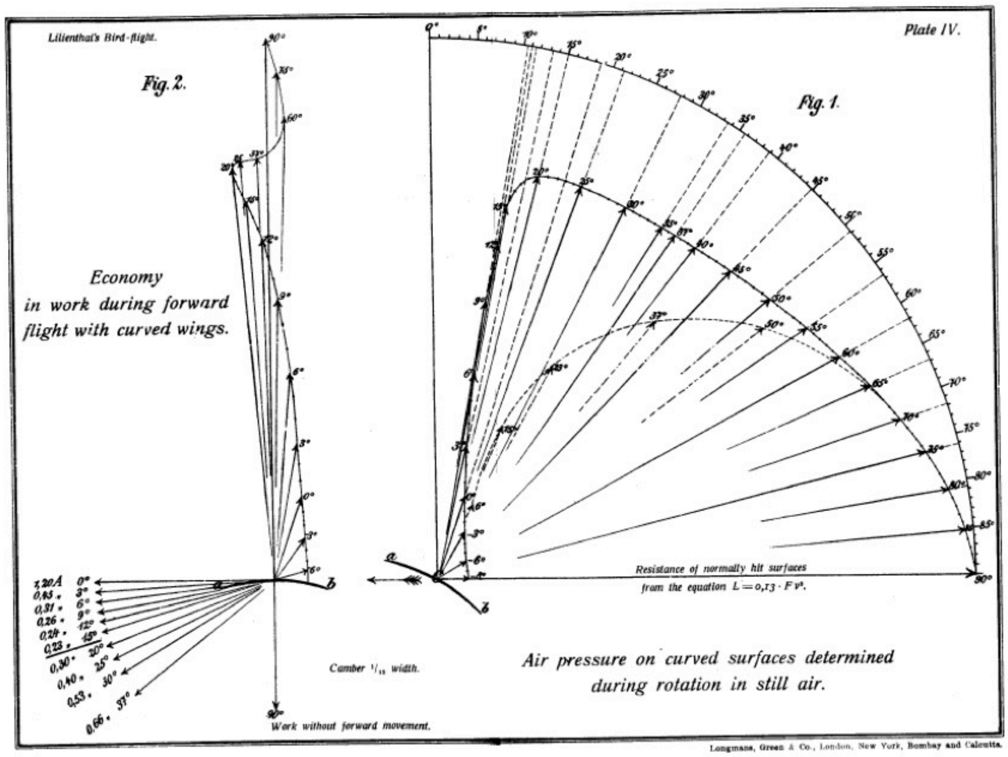
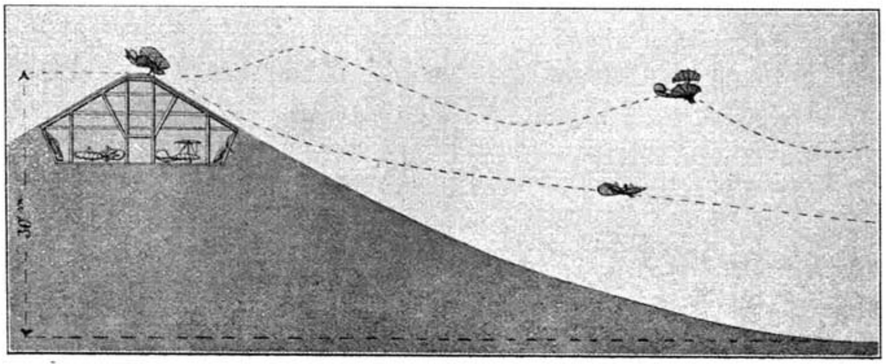



Figure 2.2: Table IV in *Birdflight as the Basis of Aviation* by Otto and Gustav Lilienthal records measurement by of the lift and drag of an arched wing with a camber of $\frac{1}{12}$ for angles of attack from $0$ to $90$ degrees in the right graph, in a so-called *drag polar* showing lift on the vertical axis and drag on the horisontal axis, with $C_L \approx 0.1\alpha$ up to beginning stall at $15$ degrees. This conforms with basic experimental experience of the lift of airplane wings, also captured in the formula $C_L = \frac{2\pi^2}{180}\alpha \approx 0.1\alpha$ of classical circulation theory. The lift and drag curve is the middle curve in the right graph showing the lift and drag as fraction of a drag (coefficient of) $2.6$ represented by the outer quarter circle. The smaller half circle curve shows the much smaller lift a plane wing.



**Schematische Zeichnung des Lilienthalschen Abflughügels**
Der Abflug erfolgt oben von dem zur Aufbewahrung der Maschinen in den Hügel eingebauten Schuppen. Die obere Linie zeigt einen Gleitflug, bei dem durch aufsteigende Luftströmungen der Flieger gelegentlich wieder gehoben wird.

Figure 2.3: Gliding flights from an artificial hill by Otto Lilienthal in gusty head wind and in calm air showing $\frac{L}{D} = 10 - 15$.

lution by including a new term corresponding to a rotating flow around the wing with the strength of the vortex determined so that the combined flow velocity became zero at the trailing edge of the wing. This *Kutta condition* reflected the observation of Lilienthal that the flow should come off the wing smoothly, at least for small angles of attack. The strength of the vortex was equal to the *circulation* around the wing of the velocity, which was also equal to the lift. Kutta could this way predict the lift of various airfoils with a precision of practical interest. But the calculation assumed the flow to be fully two-dimensional and the wings to be very long and became inaccurate for shorter wings and large angles of attack.

The first successful powered piloted controled flight was performed by the brothers Orville and Wilbur Wright on December 17 1903 on the windy fields of Kitty Hawk, North Carolina, with Orville winning the bet to be the pilot of the *Flyer* and Wilbur watching on ground, see Fig 43.1. In the words of the Wright brothers from Century Magazine, September 1908: “The flight lasted only twelve seconds, a flight very modest compared with that of birds, but it was, nevertheless, the first in the history of the world in which a machine carrying a man had raised itself by its own power into the air in free flight, had sailed forward on a level course without reduction of speed, and had finally landed without being wrecked. The second and third flights were a little longer, and the fourth lasted fifty-nine seconds, covering a distance of 852 feet over the ground against a twenty-mile wind."

## 2.4 The Modern Era of Aviation

The Flyer's controled flight in 1903 signfies the start of the modern era of aviation: The required effective engine power of about 4 hp per 100 kg with $\frac{L}{D} = 10$ and speed of $30\, m/s$ was available and the Wright brothers had shown that flight could be controled (longitudinally by a forward canard and laterally by wing warping as discussed below).

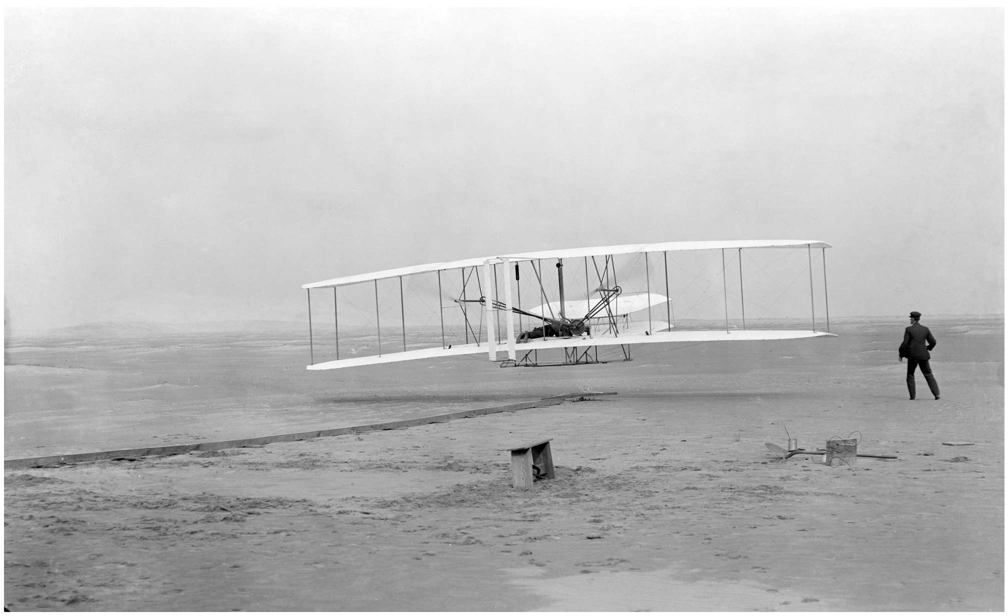


Figure 2.4: Orville Wright (1871-1948) and Wilbur Wright (1867-1912) and the lift-off at Kitty Hawk, North Carolina, the 17th December 1903: The start of the modern era of aviation.

The mathematician Nikolai Zhukovsky (1847-1921), called the Father of Russian Aviation, in 1906 independently derived the same mathematics for computing lift as Kutta, after having observed several of Lilienthal's flights, which he presented before the Society of Friends of the Natural Sciences in Moscow as: "The most important invention of recent years in the area of aviation is the flying machine of the German engineer Otto Lilienthal". Zhukovsky also purchased one of the eight gliders which Lilienthal sold to members of the public.

Kutta and Zhukovsky thus could modify the mathemathical potential theory of lift of a wing to give reasonable results, but of course could not give anything but a very heuristic justification of their Kutta-Zhukovsky condition of zero velocity at the trailing edge of the wing, and could not treat realistic wings in three dimensions. Further, their modified potential solutions were not turbulent at all, so their calculations would seem merely like happy coincidences, knowing ahead the correct answer to obtain.

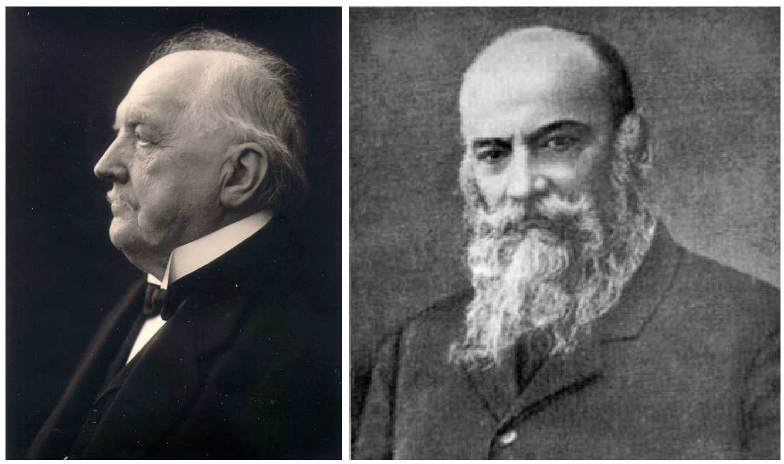


Figure 2.5: Martin Kutta (1867-1944) and Nikolai Egorovich Zhukovsky (1847-1921): Fathers of aerodynamics of flight.

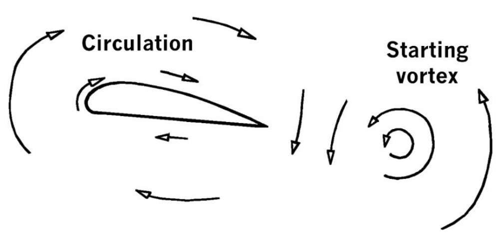



Figure 2.6: Text book description of Kutta-Zhukovsky circulation theory with a sharp trailing edge generating a starting vortex separating from the sharp trailing edge thereby generating a compensating circulation around the wing section removing the singularity created by the sharp trailing edge and thus creating lift by high speed and low pressure above the wing.

Today computational methods open new possibilities of solving the equations for fluid flow using the computational power of modern computers. Thus, for the first time the mathematical fluid models of Euler and Navier-Stokes may come to a real use, which opens new revolutionary possibilities of computational simulation and prediction of fluid flow in science and technology. The range of possible applications is incredibly rich! For example, it is now becoming possible to simulate the turbulent flow around an entire aircraft and thus systematically investigate questions of stability and control, which caused severe head-ache for the Wright brothers, as well as the designers of the modern Swedish jet fighter JAS39 Gripen. Actually, both the 1903 Wright *Flyer* airplane, with a forward canard instead of an aft tail, and the JAS39 Gripen are unstable and require careful control to fly. The instability of the fighter is intentional allowing quick turns, but the Wrights later replaced the canard with the conventional aft tail to improve stability. The stability of an airplane is similar to that of a boat, with the important design feature being the relative position of the center of gravity and the center of the forces from the flow of air (center of buoyancy for a boat), with the center of gravity ahead (below) giving stability.

It is remarkable that 400 years passed between Leonardo da Vinci's investigations and the largely similar ones by Lilienthal. Why did it take so long time from almost success to success? Can we blame the erronous mathematics of Newton and d'Alembert for the delay? Or was the reason that the (secret) writings of da Vinci were made public with a delay of 300 years? We leave the question open.

## 2.5 Bairstow vs Glauert/Prandtl/Lanchester

*The Enigma of the Aerofoil* by David Bloor describes the dispute in England after the 1st World War between Leonard Bairstow representing "inside" Cambridge mathematics and "outside" engineering represented by the circulation theory first put forward by Frederick Lanchester but killed by Lord Rayleigh and instead picked up by Prandtl and then reimported to England by Hermann Glauert, concerning the fundamental problem of aerodynamics of flight with the following highlight:

- *The International Air Congress for the year 1923 was held in London. It provided a further occasion for assessing the advances that had been made in aeronautics during the war years and for addressing unresolved problems. It was a highly visible platform on which the supporters and opponents of the circulatory theory could express their opinions and, in some cases, air their grievances.*

- *In the morning session of Wednesday, June 27, there were three speakers: Leonard Bairstow, Hermann Glauert, and Archibald Low. The first to speak was Bairstow, whose talk was titled The Fundamentals of Fluid Motion in Relation to Aeronautics. Bairstow was explicit: his aim was nothing less than the mathematical deduction of all the main facts about a wing from (Navier-)Stokes equations and the known boundary conditions:.*

- *The range of these equations covers all those problems in which viscosity and compressibility are taken into account, and from them should follow all the consequences which we know as lift, drag etc. by mathematical argument and without recourse to experiment. Such a theory is fundamental.*

- *The boundary conditions were empirical matters, but thereafter everything should follow deductively: lift, drag, changes in center of pressure, the onset of turbulent flow and stalling characteristics, along with a host of other results.*

In short, Bairstow stated that theoretical aerodynamics could be reduced to solving the Navier-Stokes equations for slightly viscous flow using mathematics. Bairstow thus aimed at reducing aerodynamics to mathematics, in the same way as celestial mechanics by Newton was reduced to solving the equations of motion for a set of n bodies subject to gravitational attraction, referred to as the n-body problem.

Newton showed that the equations of motion could be solved analytically for a sun with one planet (n=2) and by followers using perturbation techniques for a planetary system like ours (n=10), which made Newton immensely famous.

But Bairstow (and nobody else at his time) could solve the Navier-Stokes equations in any case of interest of aerodynamics, which made his grand plan useless. Bairstow thus had nothing to match Glauert's *The Elements of the Aerofoil and Air Screw Theory* published in 1926 based on circulation theory for inviscid flow with a fix-up in the form of the Kutta condition at the trailing edge, which gave some results and set the text book standard into our time. This

Today it is possible to solve the Navier-Stokes equations using computers and Bairstow's grand plan can finally be realized. This shifts the weight back to fundamentals without fix-up boosted by computational, and circulation theory with fix-up has no longer any role to play, as explained in detail in this book.

Figure 2.7: Sir Leonard Bairstow (1880-1963).

## 2.6 State-of-the-Art in England 1920

Bairstow describes in his Applied Aerodynamics the state-of-the-art of the aerodynamics of flight in England in 1920 as follows:

- *The most prominent important parts of an aeroplane are the wings, and their function is the supporting of the aeroplane against gravitational attraction. The force on the wings arises from motion through the air, and is accompanied by a downward motion of the air over which the wings have passed. The principle of dynamic support in a fluid has beed called the "sacrificial" principle (by Lord Rayleigh, I believe), broadly expresses the fact that if you do not wish to fall yourself you must make something else fall, in this case air.*

- *...if we knew the exact motion of the air round the wing the upward force could be calculated. The problem is, however, too difficult for the present state of mathematical knowledge, and our information is almost entirely based on the results of tests on models of wings in an artificial air current.*

Bairstow's explanation of lift trivially shifted the question of how a wing generates lift to the question how a wing generates downwash, which he did not answer, and thus his theory was empty and could no challenge the Kutta-Zhukovsky-Prandtl theory, which was not empty in the same sense, only unphysical and thus incorrect.

## 2.7 First Study of Stability of Flight

Bairstow main contribution was a study, extending earlier work by G. H. Bryan in *Stability in Aviation* from 1911, of longitudinal and lateral stablity of flight performed by analytical rigid body dynamics assuming the forces acting on the plane to be known, with similarities to mathematical analysis of ship stability.

Inherent longitudinal stability without need of control surfaces is achieved if the center of gravity is positioned ahead of the center of lift forces. On the other hand, later stability requires active control of the difference of lift force between the two wings usually realized through the use coupled ailerons (wing flaps): With one flap up and the other down the wings get different lift and thus can counteract rotation around the longitudinal axis.

As noted above, the Wright Flyer was inherently unstable and required skilfull active control by a forward canard, instead of a tail for longitudinal stability and active coupled control of the wing warping for lateral stability. The Swedish fighter JAS39 Gripen has center of gravity behind the center of pressure and requires sophisticated active control by little wings ahead of the main wings similar to the canard of the Flyer.

We shall see that the computational technique underlying the New Theory allows direct dynamic simulation of the flight of an airplane under the control of wing flaps and rudder and thus gives direct assessment of stability without complicated analytical rigid body dynamics which can only give a very crude answer.

## 2.8 The Paradox of Circulation Theory

The New Theory of Flight shows that the classical circulation theory of lift by Kutta-Zhukovsky is unphysical and thus incorrect, which is also reflected by the fact that it builds on a paradox:

Figure 2.8: The Swedish JAS39 Gripen after crash from longitudinal instability and landing. Note the forward canards supposed to keep the inherently unstable plane in stable flight by rapid active computer control.

- Kutta-Zhukovsky lifting flow around a 2d airfoil is obtained by augmenting non-lifting potential flow by large scale circulation around the airfoil.

- The sharp trailing edge of an airfoil is supposed to be necessary for generation of lift.

- The lift is proportional to the circulation, and the circulation is determined so as eliminate the singularity of potential flow arising from the sharp trailing edge.

- Lift is thus paradoxically obtained by elimination of what has been introduced, that is by eliminating the singularity introduced by the sharp trailing edge. This is like obtaining an effect by first introducing something and then removing it, as if (+ 1 - 1) could be greater than zero.

The paradox is supposedly resolved by claiming that the singularity from the sharp trailing edge is eliminated by effects of viscosity. The New Theory shows that lift is generated, not by a sharp trailing edge, but by wings with smoothly rounded trailing edges with diameter up to 10 percent of the chord length. The above JAS39 Gripen picture thus illustrates the present status of circulation theory.

## 2.9 Timeline of Old Theory of Flight

John D. Anderson gives in *A History of Aerodynamics and its Impact on Flying Machines* a time line for theoretical aerodynamics including:

- 350 B.C. Aristotle describes a model for a continuum and suggests that a body moving through a continuum encounters resistance.
- 250 B. C. Archimedes suggests that a fluid is set into motion by the existence of a pressure difference exerted on the fluid.
- 1523 Da Vinci formulates a first version of Bernoulli's principle with ”thinner air” at lower pressure above a wing than below in his *Codex of Bird Flight* as a first explanation of flight, 300-400 years ahead of time.
- 1600 Galieo is the first to understand that aerodynamic resistance varies directly as the density of the fluid.
- 1673 Edme Mariotte, in Paris, states that aerodynamic resistance varies as the square of the velocity.
- 1687 Newton proves his sine-squared law for the lift of a tilted plate showing much too small lift to make flight possible, as a spin-off of Proposition 34 of Book II of *Principa Mathematica* on the drag of a sphere.
- 1738 Daniel Bernoulli states in *Hydrodynamica* a version of Bernoulli's Law.
- 1744 D'Alembert discovers the paradox of zero drag of potential flow, along with Euler.
- 1752 Euler formulates the Euler equations for inviscid incompressible fluid flow expressing conservation of mass and momentum in the Euler coordinates of a fixed reference coordinate system.
- 1752 New proof by d'Alembert's of his paradox.
- 1789 Laplace's equation for potential flow.
- 1799-1810 Cayley initiates applied aerodynamics.
- 1840 Formulation of Navier-Stokes equations for viscous flow.

- 1892 Lanchester connects lift to circulation assuming that the lift of the Magnus effect of a rotating cylinder comes from circulation.
- 1903 First powered flight by the Wright brothers.
- 1904 Prandtl suggests that drag originates from a thin boundarylayer caused by a no-slip boundary condition.
- 1906 Circulation theory of lift is developed by Kutta and Zhukovsky.
- 1915 Prandtl presents his lifting line theory with input from Lanchester and Kutta-Zhukovsky.
- 1922 Thin-airfoil theory by Max Munck.
- 1962 Computational Fluid Dynamics CFD starts to develop with panel methods for potential flow and solution methods for the Navier-Stokes equations with no-slip boundary condition for low Reynolds numbers, combined with turbulence models and wall models for large Reynolds numbers. Direct Numerical Simulation DNS considered impossible because of thin boundary layers from no-slip boundary condition.
- 2008 DNS is shown to be possible using stabilized finite element methods with slip boundary condition, which leads to resolution of D'Alembert's Paradox and revelation of the Secret of Flight.

The Old Theory of Flight was formulated hundred years ago by Kutta-Zhukovsky-Prandtl and is still the theory of flight presented in text books. The fact that the theory has not evolved over a century is not to be taken as a sign that it was correct from the beginning, but rather the opposite, that it was a dead-end ad hoc non-physical theory which could not be improved.

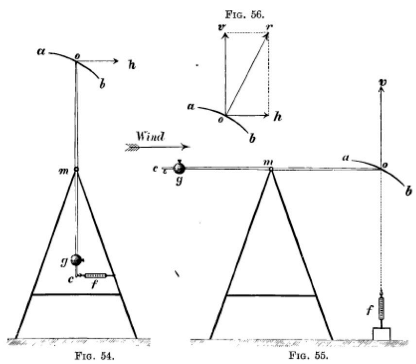



Figure 2.9: The experimental apparatus used by the Lilienthal brothers to measure drag and lift of an arched surface.

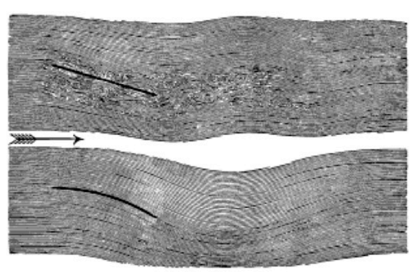



Figure 2.10: Lilienthal's conception of the flow around a plane wing compared to an arched wing with more downwash.

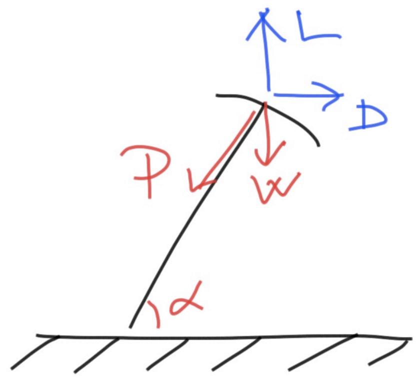



Figure 2.11: The lift $L$ and drag $D$ of a kite can be determined by measuring the pull $P$, the angle $\alpha$ and the weight $W$ of the kite.

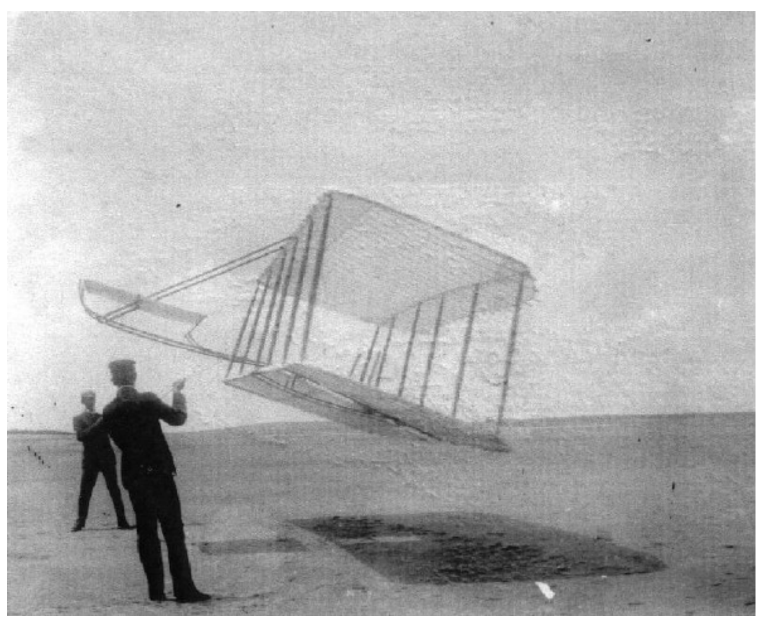


Figure 2.12: The Wright brothers measuring lift and drag of an early version of the Flyer flown as a kite.

For some years I have been afflicted with the belief that flight is possible. My disease has increased in severity and I feeel that it will soom cost me an increased amount of money if not my life. I have been trying to arange my life in such a way that I can devote my entire time for a few months to experiments in this field. (Wilbur Wright to Chanute in 1900)

Who is there who, at such times at least, does not deplore the inability of man to indulge in voluntary flight and to unfold wings as effectively as birds do, in order to give the highest expression to his desire for migration? Are we still to be debarred from calling this art our own, and are we only to look up longingly to inferior creatures who describe their beautiful paths in the blue of the sky? Is this painful consideration to be still further intensified by the conviction that we shall never be able to discover the flying methods of the birds? Or will it be within the scope of the human mind to fathom those means which will be a substitute for what Nature has denied us? (*Birdflight as the Basis of Aviation* )

# Chapter 3

# New Perspective on History

> I have not the smallest molecule of faith in aerial navigation other than ballooning or of expectation of good results from any of the trial we hear of. (Lord Kelvin refusing to join the Aeronautical Society of London)

> The stability or instability of a few basic flows was conjectured, debated, and sometimes proved in the nineteenth century. Motivations varied from turbulence observed in real flows to permanence expected in hydrodynamic theories of matter. Contemporary mathematics often failed to provide rigorous answers, and personal intuitions sometimes gave wrong results. Yet some of the basic ideas and methods of the modern theory of hydrodynamic instability occurred to the elite of British and German mathematical physics, including Stokes, Kelvin, Helmholtz, and Rayleigh. This usually happened by reflecting on concrete specific problems, with a striking variety of investigative styles. (Darrigol 2002)

> A sudden bold and unexpected question doth many times surprise a man and lay him open. (Francis Bacon)

## 3.1 The Official Doctrine

The official doctrine or accepted truth of the fluid mechanics community is that Prandtl resolved D'Alembert's Paradox of zero drag and lift of inviscid potential flow satisfying a slip boundary condition, by suggesting that drag and lift originate from a thin boundary layer of slightly viscous flow resulting from imposing a no-slip boundary condition. Our new resolution of D'Alembert's Paradox shows that drag and lift instead arise from 3d rotational slip separation developing from a basic instability of potential flow at separation. We thus observe drag and lift in accordance with observation without presence of boundary layer from no-slip boundary condition, and conclude that Prandtl's basic idea was and is wrong.

## 3.2 How to View History of Hydrodynamics

It is instructive to read the *History of Hydrodynamics from the Bernoullis to Prandtl* by O. Darrigol, with a shift of perspective from the official doctrine that Prandtl resolved D'Alembert's Paradox and thus explained drag and lift of a wing, to the new perspective presented in this book. This fundamentally changes the story as indicated in comments to key quotes from the book and New Short History:

**Darrigol**: *What distinguishes the history of hydrodynamics from that of other physical theories is not so much the tremendous effect of challenges from phenomenal world, but rather it is the slowness with which these challeneges were met. Nearly two centuries elapsed between the first formulation of the fundamental equations of the theory (Euler equations for inviscid incompressible flow) and the deductions of laws of resistance in the most important case of large Reynolds numbers...The reason for this extraordinary delay are easily identified a posteriori. They are the ... nonlinear character of the fundamental equations...Moreover, instability often deprives the few known exact solutions of any physical relevance....almost every theoretical description of a natural or manmade flow involves instabilities...These difficulties have barred progress along purely mathematical lines. They have also made physical intuition a poor guide, and a source of numerous paradoxes.....Hydrodynamicists therefore sought inspiration in concrete phenomena. Challenged to understand and act in the real world, they developed a few innovative strategies. One was to modify the fundamental equations, introducing for instance Navier's viscous term. Another was to give up the continuity of the solutions of Euler's equations, and to study the evolution of the resulting singularities. Helmholtz pursued this approach without leaving the realm of the perfect fluid.*

**Comment**: We see how Darrigol struggles to rationalize the "slowness" of the development of hydrodynamics to be a result of the "nonlinear character of the fundamental equations" and lack of "physical relevance of the few known exact solutions" which combined with "instabilities of natural and manmade flow...barred progress along purely mathematical lines". In other words, Darrigol describes a complete failure of theoretical hydrodynamics. The correct mathematical question of instability was formulated but was not followed up and replaced by a non-mathematical search of "inspiration in concrete phenomena".

**Darrigol**: *None of these strategies sufficed to fully master the real flow for which they were intended. Prandtl's ultimate success depended on combining them within the asymptotic framework of high Reynolds numbers /quasi-inviscid flow) and large aspect ratios (quasi-2d-flow). The role of small viscosity, Prandtl reasoned, is to produce boundary layers of high shear, and vortex sheets to which Helmholtz's theory of vortex motion may be applied in a second step. Vortex sheets are always unsatable, and boundary layers ofteh are so. Thg instabilities lead to turbulence... When separation occurs, the hydrodynamicist is left with Columbus's egg, unless strong resistance is desired, in which case he can appeal to model measurements combined with similitude arguments*.
**Comment**: This is the official mantra presented as an Egg of Columbus, that is, a solution which is not a true solution but only a trick (to make an unstable egg stable).

**Darrigol**: *The evolution from paper theory to an engineering tool thus depended on transgression of the limits between academic hydrodynamics and applied hydrodynamics. The utilitarian spirit of Victorian science, the Polythechnique ideal of a theory-based engineering, a touch of Helmholtz's eclectic genius, and the Gttingen pursuit of applied mathematic, all contributed to the fruitful blurring of borders between physics and engineering. The "sagacious geometers" who answered d'Alembert's ancient call for a solution to his resistance paradox all visited the real worlds of flow*.
**Comment**: The official picture is presented as a mish-mash of theory (mathematics) and practice (real world engineering flow) with a touch of genius and utilitarian spirit.

**Darrigol**: *In the 1890s, interest in flying contraptions grew tremendously, partly as a consequence of Otto Lilienthal's invention of the man-carrying glider in 1889. The prospects of building a motor-powered, piloted airplane seemd high in some engineering quarters. The materialized in 1903 when Wilbur and Orville Wright flew the first machine of that kind. Theory played almost no role in this spectacular success....The contemporary flight frenzy prompted theoretical comments and reflections from flat rejection to elaborate support. ... In summary, Rayleigh, Lamb and Kelvin knew too much fluid mechanics to imagine that circulation around wings was the main cause of lift. The two men who independently hit upon this idea lacked training in theoretical physics. One of the them was an engineer (Lanchester), and the other was a young mathematician (Kutta).*

**Comment**: Lanchester's circulation theory was not credible to people like Rayleigh, Lamb and Kelvin with training in theoretical physics in the late 19th century, simply because it did not make sense, but the critique was overpowered by the circulation theory of Kutta-Zhukovsky-Prandtl during and after the 1st World War.

**Darrigol**: *By the end of the 1st World War, Prandtl and his collaborators could legitimately claim a mathematical, quantitative solution to the wing problem. The leftover was to justify the various approximations that Prandtl had introduced at various steps of the reasoning. Although Prandtl's justifications for these assumptions lacked rigor, experiments performed during the war in the Gttingen wind tunnel vindicated them. Post-war British and American experiments further confirmed Prandtl's theory....After some hesitation on the British side, by the mid 1920s, it became routine...Under the stimulus of the rising field of aeronautics and with the strong support of Göttingen institutions, Prandtl's group put an end to the engineer's legitimate distrust of the theoretical predictions of fluid mechanics.*
**Comment**: Kutta circulation theory is a non-physical mathematical theory, as well as Prandtl's extended lifting-line version of circulation theory boosted with a boundary layer theory. Prandtl's institutions boomed under the German preparations for the 2nd World War in the 1920-30s and lack of correct theory was compensated by powerful wind tunnels and steel.

**New Short History of Hydrodynamics**: Theoretical hydromechanics is based on the Euler/Navier-Stokes equations formulated in 1752/1840, but since the equations could not be solved analytically, the theory was empty until computational solution became possible in the 21st century, which for the first time opened to a fruitful interplay between theory and practice.

> With each advent of spring, when the air is alive with innumerable happy creatures; when the storks on their arrival at their old northern resorts fold up the imposing flying apparatus which has carried them thousands of miles, lay back their heads and announce their arrival by joyously rattling their beaks; when the swallows have made their entry and hurry through our streets and pass our windows in sailing flight; when the lark appears as a dot in the ether and manifests its joy of existence by its song; then a certain desire takes possession of man. He longs to soar upward and to glide, free as the bird, over smiling fields, leafy woods and mirrorlike lakes, and so enjoy the varying landscape as fully as only a bird can do. (*Birdflight as the Basis of Aviation*)

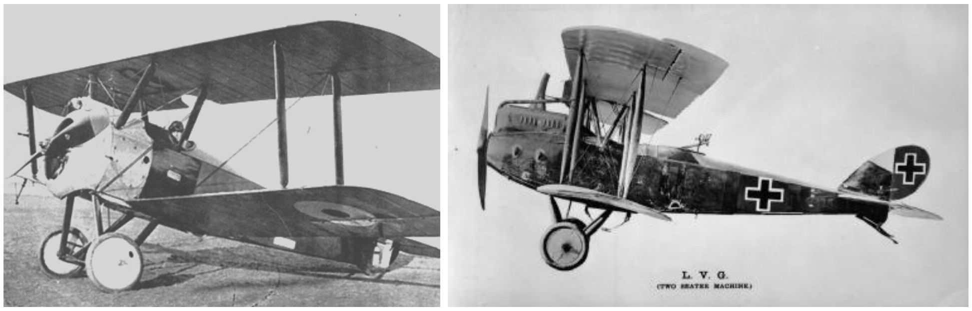



Figure 3.1: Left: An agile, highly maneuverable biplane, the Sopwith Camel accounted for more aerial victories than any other Allied aircraft during World War I. Right: Fokker DVII Scout Single seat biplane scout with Mercedes engine. The DVII posed a serious threat to Allied air supremacy in 1918 with its manoeuvrability in combat.

> When we observe the awkward first flying attempts of young storks, how they drop their beaks and legs and execute the most curious movements with the neck in order to re-establish their equilibrium, we conclude that such unskilled flight must be extremely easy, and we are tempted to construct a pair of wings for experimental flight. Following the young bird's progress after the lapse of but a few days, we feel encouraged to emulate its example. (*Birdflight as the Basis of Aviation* )

# Chapter 4

# Euler and Navier-Stokes Equations

> However sublime are the researches on fluids which we owe to Messrs Bernoulli, Clairaut and d'Alembert, they flow so naturally from my two general formulae that one cannot sufficiently admire this accord of their profound meditations with the simplicity of the principles from which I have drawn my two equations...they include not only all that has been discovered by methods very different and for the most part slightly convincing...but also all that one could desire further in this science. (Euler 1752)

> There may even be no stable steady mode of motion possible, in which case the fluid would continue perpetually eddying.(Stokes 1867)

> Stokes did not attempt a mathematical investigation of the stability of flow. (Darrigol [139])

> Truth is so hard to tell, it sometimes needs fiction to make it plausible. (Francis Bacon)

## 4.1 Model of Fluid Mechanics

The basic mathematical model of fluid mechanics (or fluid dynamics) consists of *Navier-Stokes equations* expressing *conservation of mass, momentum and energy* of a *viscous fluid*. Theoretical fluid mechanics can be viewed as the science of computing and analyzing solutions of the Navier-Stokes equations.

Water has a very small compressibility and air at speeds well below the speed of sound is nearly incompressible. The fluid mechanics of water and air at subsonic speeds is thus captured by the *incompressible Navier-Stokes equations* with the *Reynolds number* $Re = \frac{UL}{\nu}$ as an important flow characteristic, where $U$ is a characterisitic flow speed, $L$ a characteristic length scale and $\nu$ the kinematic viscosity. For water $\nu \approx 10^{-6}$ and for air $\nu \approx 10^{-5}$. The typical Reynolds number of an airplane may be $10^7$, of larger birds $10^5$ and of insects $10^3$, while formally the *incompressible Euler equations* would describe flow at vanishing viscosity or infinite Reynolds number.

In the incompressible Navier-Stokes equations the equation expressing conservation of total energy as the sum of kinetic energy and heat energy, is decoupled from the equations expressing conservation of mass and momentum, and mass conservation can be reduced to requiring fluid velocities to be incompressible.

The incompressible Navier-Stokes equations thus can be viewed to consist of two equations in the fluid velocity $u(x,t)$ and pressure $p(x,t)$ as functions of a space coordinate $x$ in a fixed (Eulerian) reference system and a time coordinate $t$, expressing

- **Newton's 2nd Law (conservation of momentum)**,
- **incompressibility (conservation of mass)**,

which in standard mathematical notation take the following (deceptively simple) form in the domain occupied by the fluid:

$$\begin{aligned} \frac{Du}{Dt} + \nabla p - \nu\Delta u &= 0, \\ \nabla \cdot u &= 0, \end{aligned} \tag{4.1}$$

where $\frac{Du}{Dt} = \frac{\partial u}{\partial t} + (u \cdot \nabla)u$ is fluid particle acceleration at $(x,t)$, $\nu \geq 0$ is viscosity, the density is normalized to 1 and we consider the basic case without exterior forcing. Here $-\nabla p$ is the pressure force and $\nu\Delta u$ is the viscous force, and $\nu = 0$ gives the inviscid Euler equations.

Flight at speeds up to $300\, km/h$ of airplanes and larger birds is described by solutions of the incompressible Navier-Stokes equations at a Reynolds number of size $10^6$ or larger, which with normalization to $U = L = 1$ translates to small viscosity $\nu$ of size $10^{-6}$ or smaller.

We shall find that the Euler case with $\nu = 0$ is *illposed* or *not-wellposed*, in the sense that solutions including potential solutions to the Euler equations, are unstable at separation and thus there lack physical significance. But we shall find that slightly viscous bluff body flow such as the flow around a wing, can be described as potential flow modified at separation into quasi-stable partially turbulent real flow. Potential flow thus captures aspects of real flow before separation and analytical solution formulas for potential flow can aid understanding of these aspects, as shown in Chapter 7.

We thus always assume the viscosity $\nu$ is small strictly positive and we thus always consider the Navier-Stokes equations with the Euler equations as a formal limit case without true physical significance.

## 4.2 Boundary Conditions

The Navier-Stokes equations in the fluid domain are complemented by *boundary conditions* on the boundary of the fluid domain prescribing velocities or forces or combinations thereof. The following conditions prescribed on the boundary of a solid object moving through a fluid have particular significance:

- **no-slip**: normal and tangential velocities put to zero,
- **slip**: normal velocity and tangential forces put to zero.

From mathematical point of view we have a real choice to use slip or no-slip, and we use this freedom to choose slip as a model of the small tangential force observed for slightly viscous flow. We thus combine the Navier-Stokes equations in the interior of the fluid domain with small positive viscosity, with slip boundary conditions corresponding to vanishing viscosity.

We compare with Prandtl's argument that only no-slip is allowed because fluid particles have to stick to the boundary in the case of positive viscosity in the fluid. But this is an ad hoc assumption without sound physical basis, because it is conceivable that fluid particles can glide along the boundary without friction even if there is interior friction between fluid particles. Prandtl thus claims to know that fluid particles cannot glide on a boundary, but this only an hoc assumption which does not have to be true. The exact nature of the (quantum mechanical) interaction between fluid particles and solid boundary may be beyond inspection, but in the setting of the Navier-Stokes equations, this is irrelevant: If measurements show that the tangential force is very small on the boundary, this information is enough to make it possible to combine Navier-Stokes equations in the fluid with slip boundary condition, irrespective of the exact nature of the quantum mechanics of the fluid-boundary interaction.

Prandtl used the no-slip condition to solve d'Alembert's Paradox by discriminating potential flow satisfying slip, but then introduced thin boundary layers which made computational solution of the Navier-Stokes equations impossible. In our new resolution of d'Alembert's Paradox potential solutions are instead discriminated because of instability, which allows slip boundary condition without boundary layer making computational solution possible, thus fundamentally changing the game.

Note that the slip boundary condition connects to the so-called *discontinuity theory* of Helmholtz in the sense that the non-zero tangential velocity allowed by slip can be seen as a sudden jump or discontinuity from the zero tangential velocity of no-slip. However, while discontinuity theory focussed of interior discontinuities, which could not really be observed presumably by inherent instability, the slip discontinuity could be observed as a sudden jump across a very thin boundary layer. In the controversy between Bairstow and Glauert, the discontinuity theory represented by England did not resist the effective advocasy of circulation theory.

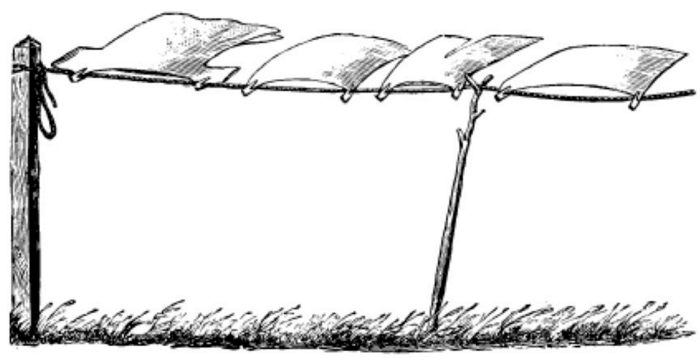


FIG. 64.

Figure 4.1: The linen which is placed on a rope in order to dry, as well as any flag which flies from a horizontal pole, demonstrate the strong lift which curved surfaces experience in wind, and which pushes them even above the horizontal position. (*Birdflight as the Basis of Aviation*)

> We are, therefore, forced to the conclusion that the only possibility of attaining efficient human flight lies in the exact imitation of birdflight with regard to the aerodynamic condition, because this is probably the sole method which permits of free, rapid flight, with a minimum of effort. (*Birdflight as the Basis of Aviation* )

# Chapter 5

# D'Alembert's Paradox

> It appears to me very probable that the spreading out motion of the fluid, which is supposed to take place behind the middle of the sphere or cylinder, though dynamically possible, nay, the only motion dynamically possible when the conditions which have been supposed are accurately satisfied, is unstable; so that the slightest cause produces a disturbance in the fluid, which accumulates as the solid moves on, till the motion is quite changed. Common observation seems to show that, when a solid moves rapidly through a fluid at some distance below the surface, it leaves behind it a succession of eddies in the fluid. (Stokes)

> They are ill discoverers that think there is no land, when they can see nothing but sea. (Francis Bacon)

## 5.1 D'Alembert and Euler and Potential Flow

Working on a 1749 Prize Problem of the Berlin Academy on flow drag, the great mathematician d'Alembert was led to the following contradiction referred to as *D'Alembert's Paradox* [75, 76, 77, 78, 83] between observation and theoretical prediction:

- *It seems to me that the theory (potential flow), developed in all possible rigor, gives, at least in several cases, a strictly vanishing resistance, a singular paradox which I leave to future Geometers to elucidate*.

The even greater mathematician *Leonard Euler*(1707-1783) had come to same conclusion of zero drag of potential flow in his work on gunnery [79] from 1745 based on the observation that in potential flow the high pressure forming in front of the body is balanced by an equally high pressure in the back as shown in Fig. (7.1 below), in the case of a boat moving through water expressed as

- *...the boat would be slowed down at the prow as much as it would be pushed at the poop...*

This connects to the idea Aristotle's idea of motion adopted by da Vinci as a form of peristaltic motion (see Part V), with a body moving through a fluid without resistance as if following Newton's 1st Law stating rectilinear motion at constant velocity in the absence of forcing. Here, *potential flow* is defined as a formal solution of the Euler equations in the form of (i) incompressible, (ii) inviscid (iii) irrotational and (iv) stationary (time-independent) flow, with the flow velocity $u$ given as the gradient $u = \nabla\varphi$ of a stationary potential function $\varphi$ satisfying Laplace's equation $\Delta\varphi = 0$.

More generally, D'Alembert's Paradox concerns the contradiction between observations of substantial drag/lift of a body moving through a slightly viscous incompressible fluid such as air and water, with the mathematical prediction of zero drag/lift of potential flow defined as inviscid, incompressible, irrotational and stationary flow.

Evidently, flying is incompatible with potential flow, and in order to explain flight D'Alembert's Paradox had to be resolved. But d'Alembert couldn't do it and all the great mathematical brains of the 18th and 19th century stumbled on it: Nobody could see that any of the assumptions (i)-(iv) were wrong and the paradox remained unsolved.

We show below that the true reason that potential flow with zero drag/lift is never observed, is that it is a formal solution of the Euler equations which is unstable and thus not a physically meaningful solution required to be stable or *wellposed*, that is, change little under small perturbations. It took 256 years for this idea to descend.

## 5.2 What's Wrong with the Potential Solution?

D'Alembert left his paradox to "future Geometers": Evidently something was wrong with the potential solution as an approximate solution to the Navier-Stokes equations. Since the Navier-Stokes equations cannot be questioned as expressing Newton's 2nd law and incompressibility, there are only two possibilities:

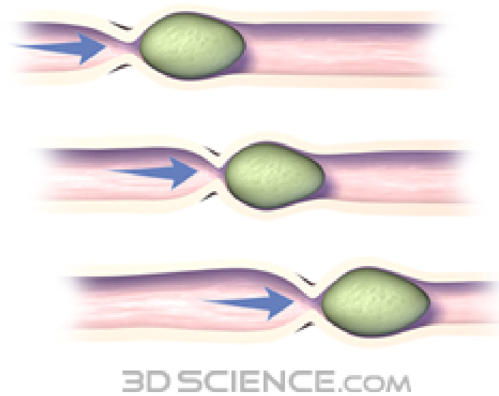



Figure 5.1: D'Alembert contemplating his paradox in 1749 maybe with an idea of peristaltic motion and Fig. (7.1) .

- (A) The potential solution is not a correct approximate solution of the Navier-Stokes equations in the fluid domain.
- (B) The potential solution does not satisfy the correct boundary condition between fluid and solid.

The mathematicians of the 18th century could not resolve the paradox, neither could the 19th century masters including Navier and Stokes. When powered flight showed to be possible at the turn to the 20th century the paradox simply had to be resolved one way or the other. The emerging modern man preparing to take control over Nature, required rational science and mathematics and paradoxes and contradictions could no longer be tolerated.

## 5.3 Prandtl's Resolution in 1904

It was the young physicist Ludwig Prandtl who in an 8-page article in 1904 gave birth to modern fluid mechanics suggesting that the unphysical feature of the potential solution was (B): The potential solution satisfies a *slip* boundary condition allowing fluid particles to glide along the boundary without friction, while in a real fluid with positive viscosity even very small, fluid particles would have to stick to the boundary and thus form a thin *boundary layer* where the fluid speed would increase rapidly from zero at the solid boundary to the free stream value in the fluid. In other words, real physical flow would satisfy a *no-slip* condition with zero velocity on the boundary, which discriminated the potential solution as non-physical and resolved the paradox.

## 5.4 The Mantra of Modern Fluid Mechanics

This became the mantra of modern fluid mechanics: Both lift and drag originate from thin boundary layers casued by a no-slip boundary condition. The mantra saved theoretical fluid mechanics through the first half of the 20th century, but instead killed computational fluid dynamics emerging with the computer by asking for resolution of very thin boundary layers using quadrillions of mesh points.

## 5.5 New Resolution 2008

In 2008 we published a resolution of D'Alembert's Paradox [104], 256 years after its formulation, which identified instead (A) as the unphysical feature: A zero-drag potential solution is unstable as an approximate solution of the Navier-Stokes equations with a slip boundary condition, and because it is unstable it changes under infinitesimal perturbations into a more stable turbulent solution with substantial drag.

We identified the primary instability which gives the main features of the turbulent solution with lift and drag. We thus observed and explained the emergence of lift and drag of a wing and showed both to be accuractely computable by solving the Navier-Stokes equations with a slip boundary condition without boundary layers using some hundred thousand mesh points.

We thus computed and explained lift and drag without resorting to any effects of boundary layers and thus in particular concluded that Prandtl's mantra of modern fluid mechanics has little to do with reality. The consequences are far-reaching.

The new flight theory to be presented can be shown to be correct as far as the Navier-Stokes equations describe fluid mechanics, because the new flight theory directly reflects properties of Navier-Stokes solutions.

By studying computed Navier-Stokes solutions, which Prandtl could not do, we discover that Prandtl's resolution of D'Alembert's Paradox does not correctly describe the essential physics, nor does the flight theory conceived by the father of modern fluid mechanics. A study of the new flight theory thus naturally starts with a study of the new resolution of D'Alembert's Paradox.

## 5.6 Suppression of Birkhoff's Innocent Question

The mathematician Garret Birkhoff at Harvard asked in his *Hydrodynamics* first published in 1950, the seemingly innocent question if there is any time-independent inviscid flow, including potential flow, which is stable, but was met with so harsh criticism by J. Stoker at Courant Institute that Birkhoff deleted the question in the second edition of his book and never wrote anything more on hydrodynamics.

But Birkhoff's question was very motivated and the active suppression delayed the progress of computational fluid dynamics by several decades. Suppression of correct science is often more harmful than the promotion of incorrect science. Unfortunately, Birkhoff did not live to see that after all he was right and Stoker wrong.

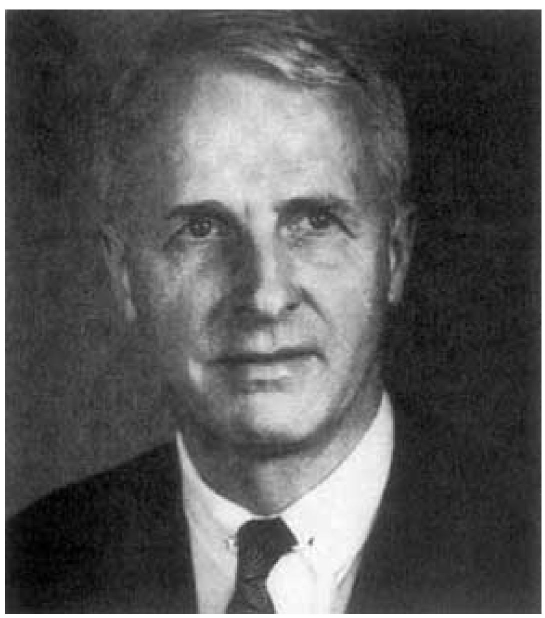


Figure 5.2: Garret Birkhoff in 1950: *Is there any potential flow which is stable?*

# Chapter 6

# From Circular Cylinder to Wing

> Isn't it astonishing that all these secrets have been preserved for so many years just so we could discover them! (Orville Wright)

> The most salient characteristics of *wing form* are common to birds of widely different species and habit of life. In spite of variations in detailand in general proportions, there is a certain uniformity of design and construction that cannot fail to impress even the most superficial observer. (Lanchester 1907)

## 6.1 One Way to Construct a Wing

A wing can be constructed by letting two circular cylinders stretch a fabric to form the wing surface with leading and trailing edges being formed by half cylinder surfaces as depicted in Fig. 6.1.

## 6.2 The Princeton Sailwing

The *Princeton sailwing* [140] is intended to provide a light-weight, low-cost lifting surface suitable for a number of low-speed applications. It consists of a leading-edge spar with attached ribs which (ideally) form a rigid framework supporting a trailing-edge cable in tension. A non-porous, non-stretchable cloth membrane, usually dacron, is wrapped around the leading-edge and attached to the trailing-edge forming the upper and lower sail surfaces. The aerodynamic efficiency of the sailwing can approach that of a hard wing.

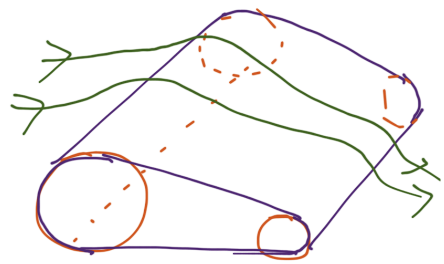


Figure 6.1: Wing as two circular cylinders stretching a fabric forming the wing surface.

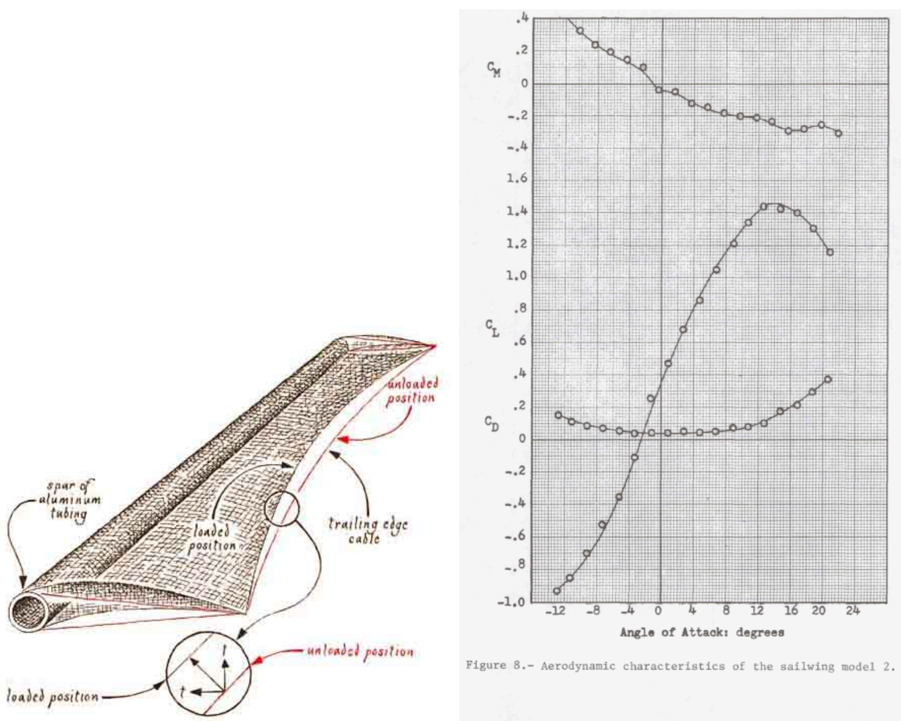



Figure 6.2: Wing as membrane stretched over leading edge cylinder and trailing edge chord with lift and drag (and moment) coefficients as functions of angle of attack (right).

## 6.3 What to Look For

We are thus led to study the flow around a circular cylinder with particular attention to the attachment in the front and separation in the back with:

1. attachment of the flow at the frontal part of the cylinder as attachment at the leading edge of a wing,
2. flow over the top of the cylinder as flow over the crest of a wing,
3. separation in the back of the cylinder as separation at the trailing edge of a wing.

We shall then see that understanding the flow around a circular cylinder opens to understanding the flow around a wing and thus to revealing the miracle of flight.

We start by considering potential flow around a circular cylinder and discover that it is unstable at separation and thus cannot be observed as a physical flow. We discover that the instability of potential flow transforms the flow into real physical flow with a more stable separation pattern described as *3d rotational separation* (in contrast to *2d irrotational separation* of potential flow), as depicted in the Fig. 1.8 and reproduced here:

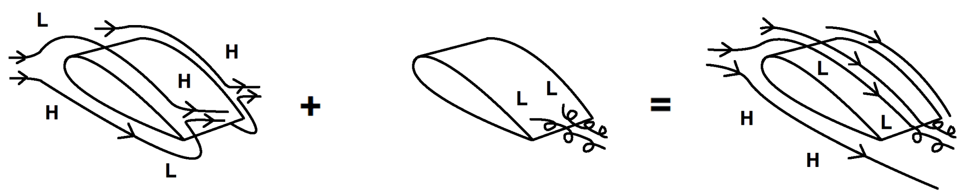



Figure 6.3: Correct explanation of lift as potential flow with 2d irrotational separation without lift and downwash (left) modified by instability at separation (middle) into 3d rotational separation (right) with downwash and lift, with the instability arising from opposing flow developing swirling flow without the high pressure of potential flow at separation.

> The science of flight cannot, as yet, be considered a separate profession, and it does not yet include a series of exponents worthy of absolute confidence; this is due to the existing uncertainty and want of system, no firm basis having been laid down, from which every investigator must start. (*Birdflight as the Basis of Aviation* )

# Chapter 7

# Potential Flow

> If we worked on the assumption that what is accepted as true really is true, then there would be little hope for advance. (Orville Wright)
>
> The forms of flow that result from the assumption of continuity and the equations of motion (potential flow), bear in general but scnat resemblance to those that obtain in practice (D'Alembert's Paradox), and it is not altogether easy to account for the cause of the failure. (Lanchester in Aerodynamics 1907)
>
> Truth emerges more readily from error than from confusion. (Francis Bacon)

## 7.1 Circular Cylinder vs Wing

Fig. 7.1 shows the pressure (left) and velocity/streamlines (right) in a section of potential flow (from left to right) around a circular cylinder: We see that the flow is fully symmetric so that the pictures would be the same if instead the flow was from right to left. We see that the pressure is high (red) in the front of the cylinder, but also in the back giving zero drag (Alembert's paradox), while the pressure is low on top and bottom (blue) giving also zero lift. We see that the velocity is low (blue) in the high pressure zones and high (red) in the low pressure zones, in accordance with *Bernoulli's Law* to be discussed below.

With connection to the peristaltic motion in Fig. 5.1, we see the streamlines apparently "squeezing" the cylinder in the back thus (mysteriously) "pushing" the cylinder to overcome the high pressure in the front allowing the cylinder to move through the fluid without drag: The flow opens in the front and closes in the back with zero net drag. Simple, but puzzling.

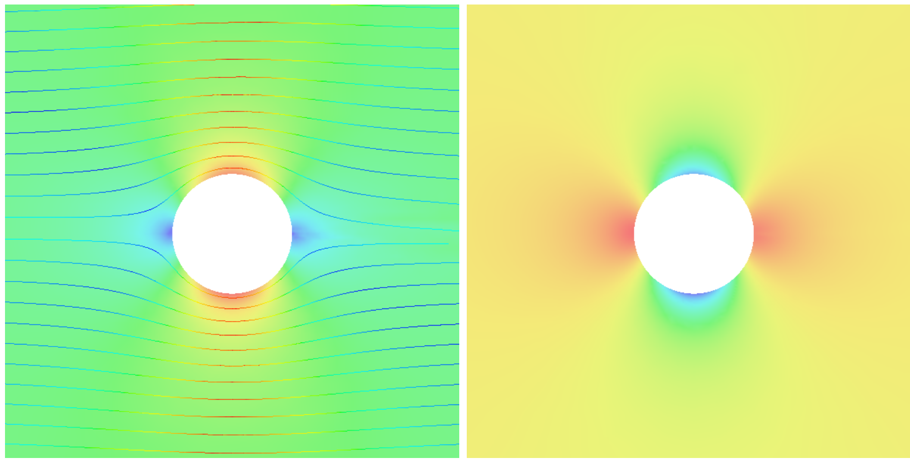


Figure 7.1: Section of potential flow around circular cylinder: velocity left and pressure right. Notice full symmetry with the same solution independent of the direction of the flow from left-to-right and from right-to-left, with attachment and separation pattern the same.

In Fig. 7.2 we display the principal feature of the pressure distribution of potential flow around a circular cylinder and a wing, in both cases with zero lift and drag. For a wing see that the leading edge generates lift and drag which is matched by high pressure on top and low pressure below the trailing edge, to give potential flow net zero lift and drag.

We shall find that the real flow around a wing is not potential flow, but potential flow with a modification at the trailing edge as 3d rotational slip separation, which eliminates the high and low pressure and thus gives substantial lift at small drag from the leading edge.

## 7.2 Analytical Solution for Circular Cylinder

We now give the analytical formula for potential flow around a circular cylinder of unit radius with axis along the $x_3$-axis in 3d space with coordinates $x = (x_1, x_2, x_3)$, assuming the flow velocity is $(1, 0, 0)$ at infinity in each $x_2x_3$-plane.

It is not necessary at this point to follow the details of the mathematical analysis, but it is quite simple and surely illuminating, so it is worth some effort to digest. The analytical mathematical description of the flow opens to an analysis and understanding of the flow according to the principle that understanding in physics

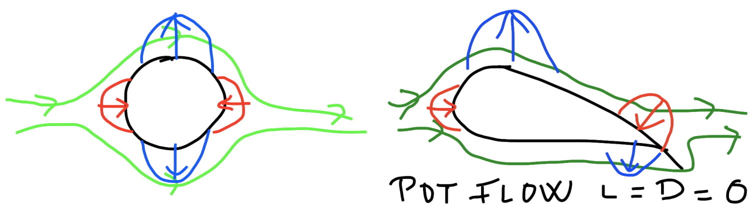



Figure 7.2: Pressure distribution of potential flow around circular cylinder and wing with zero lift and drag.

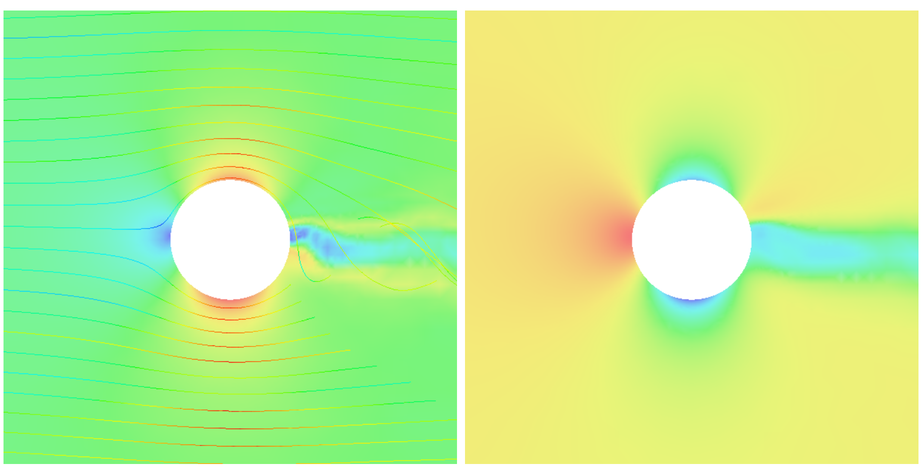


Figure 7.3: Turbulent flow past a cylinder; velocity (left) and pressure (right). Notice the low pressure wake of strong streamwise vorticity generating drag.

means understanding of an analytical mathematical model of the phenomenon in question.

In polar coordinates $(r, \theta)$ in a plane orthogonal to the cylinder axis, the flow is given by the potential function

$$\varphi(r,\theta) = (r + \frac{1}{r})\cos(\theta) \tag{7.1}$$

with corresponding velocity components

$$\begin{aligned} u_r &\equiv \frac{\partial \varphi}{\partial r} = (1 - \frac{1}{r^2})\cos(\theta), \\ u_s &\equiv \frac{1}{r}\frac{\partial \varphi}{\partial \theta} = -(1 + \frac{1}{r^2})\sin(\theta), \end{aligned} \tag{7.2}$$

with streamlines being level lines of the conjugate potential function

$$\psi \equiv (r - \frac{1}{r})\sin(\theta. \tag{7.3}$$

Potential flow is constant in the direction of the cylinder axis with velocity $(u_r, u_s) = (1, 0)$ for $r$ large, is fully symmetric with zero drag/lift, attaches and separates at the lines of stagnation $(r, \theta) = (1, \pi)$ in the front and $(r, \theta) = (1, 0)$ in the back.

By Bernoulli's principle the pressure is given by

$$p = -\frac{1}{2r^4} + \frac{1}{r^2}\cos(2\theta) \tag{7.4}$$

when normalized to vanish at infinity. We see that the negative pressure on top and bottom (-3/2) is 3 times as big in magnitude as the high pressure at the front (+1/2). We see that the pressure switches sign for $\theta = 30$.

We also compute

$$\begin{aligned} \frac{\partial p}{\partial \theta} &= -\frac{2}{r^2}\sin(2\theta), \\ \frac{\partial p}{\partial r} &= \frac{2}{r^3}(\frac{1}{r^2} - \cos(2\theta)), \end{aligned} \tag{7.5}$$

and discover an adverse pressure gradient in the back with unstable retarding flow as illustrated in Fig. 7.4. We may further compute

$$\frac{\partial \psi}{\partial r} = (1 + \frac{1}{r^2})\sin(\theta),$$

expressing that the flow is increasingly compressed before the crest and then correspondingly expanded after the crest.

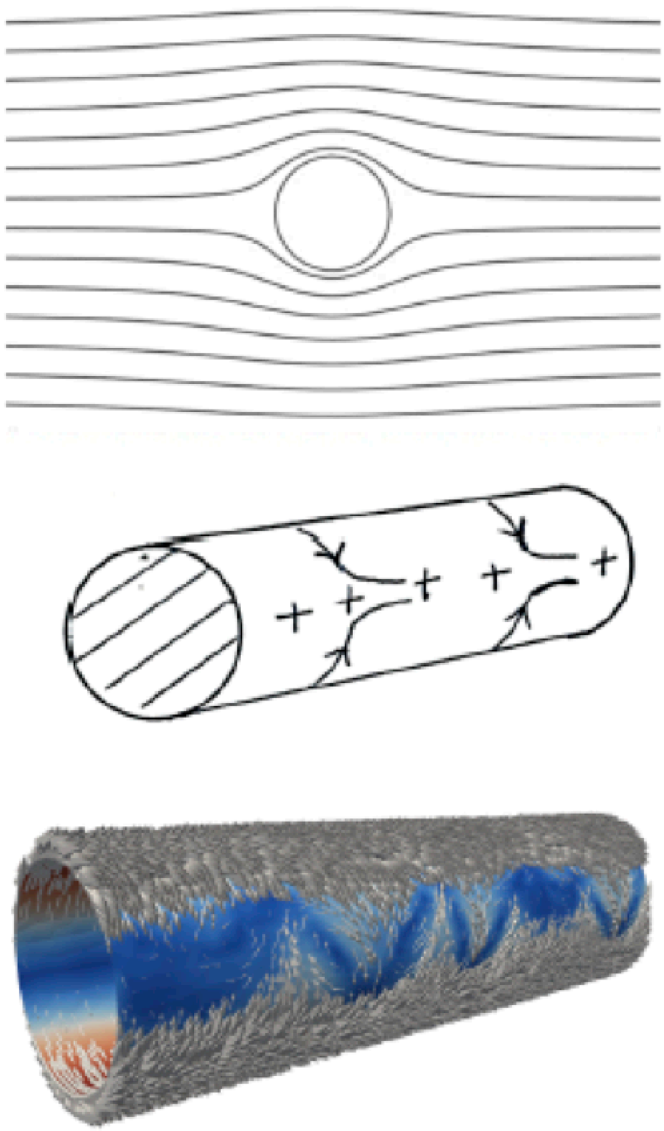


Figure 7.4: Instability of potential flow at separation.

## 7.3 Non-Separation of Potential Flow

The normal pressure gradient on the boundary

$$\frac{\partial p}{\partial r} = 4\sin^2(\theta) \geq 0$$

is precisely the force required to accelerate fluid particles with speed $2|\sin(\theta)|$ to follow the circular boundary without separation, by satisfying the condition of non-separation on a curve with curvature $R$, as identified already by Euler,

$$\frac{\partial p}{\partial n} = \frac{U^2}{R}, \tag{7.6}$$

where $U$ is the flow speed and $R = 1$ the radius of curvature of the boundary (positive for a concave fluid domain thus positive for the cylinder). The relation (7.6) is Newton's law expressing that fluid particles gliding along the boundary must be accellerated in the normal direction by the normal pressure gradient force in order to follow the curvature of the boundary. More generally, (7.6) is the criterion for *non-separation*: Fluid particles will stay close to the boundary as long as (7.6) is satisfied, while if

$$\frac{\partial p}{\partial n} < \frac{U^2}{R}, \tag{7.7}$$

then fluid particles will separate away from the boundary tangentially. In particular, as we will see below, laminar flow separates on the crest or top/bottom of the cylinder, since the normal pressure gradient is small in a laminar boundary layer with no-slip boundary condition [124, 134].

## 7.4 2d Stable Attachment 3d Unstable Separation

Potential flow around a circular cylinder shows exponentially unstable 2d irrotational separation and quasi-stable 2d attachment, and is a useful model for both attachment at the leading edge of a wing and separation at a rounded trailing edge.

The separation of the flow from the cylinder in the rear is of crucial importance: Potential flow separation can be described as 2d irrotational slip separation with *line stagnation* (along a line on the cylinder surface parallel to the cylinder axis), noting that separation and attachment have the same pattern in potential flow, as illustrated in Fig. 7.5. In D'Alembert's Paradox we show that potential flow is exponentially unstable at separation (but not at attachment) and therefore develops into a different quasi-stable separation pattern as 3d rotational slip separation with *point stagnation*, in which the high pressure of 2d irrotational slip separation with line stagnation is replaced by free stream pressure with non-zero drag as illustrated Fig. 7.4.

We understand that the role of the high pressure at attachment is to redirect the flow around the cylinder, and likewise the role of the high pressure of potential flow at separation to to redirect the flow into horisontal motion.

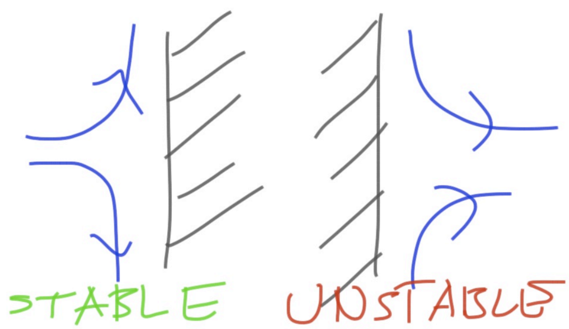



Figure 7.5: The essence of stability of slightly viscous flow: stable attachment and unstable separation, according to mathematical analysis given below: Elementary but profound!

## 7.5 Lift of Half Cylinder

It is instructive to compute the lift of potential flow over a half cylinder (of unit radius) glued to a half plane: By symmetry the pressure is the same as for a full cylinder, that is on the cylinder surface the pressure $P$ is given by

$$P = -\frac{1}{2} + \cos(2\theta) \quad \text{for } 0 \leq \theta \leq \pi, \tag{7.8}$$

and thus the lift force $L$ is given by

$$L = \int_0^{\pi} (-\frac{1}{2} + \cos(2\theta)) \sin(\theta)\, d\theta = 1.67, \tag{7.9}$$

which gives a *lift coefficient* $C_L \equiv \frac{F}{2\rho V^2 A} \approx 1.67$, where $F$ is the lift force, $V = 1$ the free stream velocity, $\rho \approx 1$ is the density of air and $A = 2$ is the planform width of the cylinder. We may compare with the maximal $C_L$ of about 1.5 from measurements as shown in Fig. 7.7.

## 7.6 Drag from Frontal Part of Half Cylinder

We also compute a drag coefficient $C_D$ from the frontal part high pressure

$$C_D = \int_0^{\frac{\pi}{3}} (-\frac{1}{2} + \cos(2\theta)) \cos(\theta)\, d\theta = \frac{1}{6}, \tag{7.10}$$

suggesting that $\frac{L}{D} \approx 10$, modulo rear part drag.

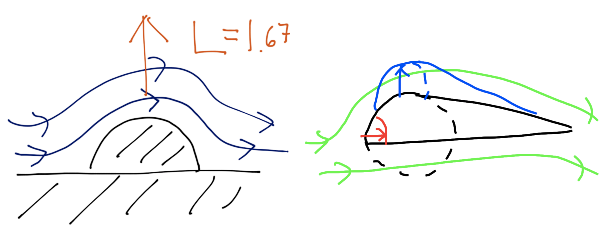



Figure 7.6: Potential flow above a half cylinder with $C_L = 1.67$ and wing as deformed half cylinder.

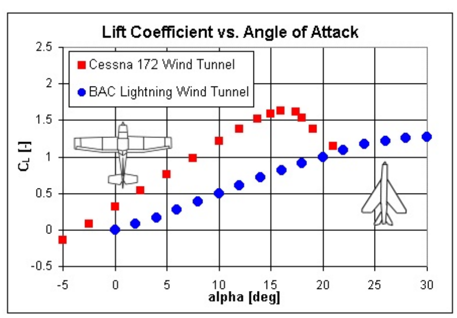



Figure 7.7: Observed maximal lift coefficient $C_L$.

# Chapter 8

# Real Flow: Circular Cylinder

> We were lucky enough to grow up in an environment where there was always much encouragement to children to pursue intellectual interests; to investigate what ever aroused curiosity. (Orville Wright)
>
> There are no atheists on turbulent airplanes. (Erica Jong in Fear of Flying)
>
> Aye, 'twill not be so easy, To mate the wings of mind with material wings. (Goethe's Faust)
>
> The subtlety of nature is greater many times over than the subtlety of the senses and understanding. (Francis Bacon)

## 8.1 3d Rotational Slip Separation

The instability of potential flow around a circular cylinder at separation changes the flow into a more stable real flow pattern characterized by *3d rotational slip separation with point stagnation* with the high pressure of potential flow being reduced to the background pressure by swirling flow as illustrated in Fig. 8.1.

The instability of potential flow at separation results from the *retardation* of the flow by the *opposing flows* from above and below behind the cylinder, which can be understood to be exponentially unstable in velocity by inspecting the analytical formula for potential flow, as will be shown below. The instability is manifested by an oscillating pressure with high pressure zones redirecting the flow thus replacing unstable high pressure opposing flow by quasi-stable oscillating pressure swirling flow allowing elegant separation at nominal mean pressure.

As noted above, the retardation of the flow at attachment in the front is more stable since the retardation is caused by the cylinder front surface and not by opposing flow, as shown in more detail in Chapter 25.

## 8.2 Drag

Assuming that 3d rotational separation gives a small contribution to drag, we can estimate the drag to result from the frontal high pressure of potential flow, which we just computed to be $\frac{1}{3}$, twice that of a half cylinder, which matches measurements for Reynolds numbers larger than $5 \cdot 10^6$.

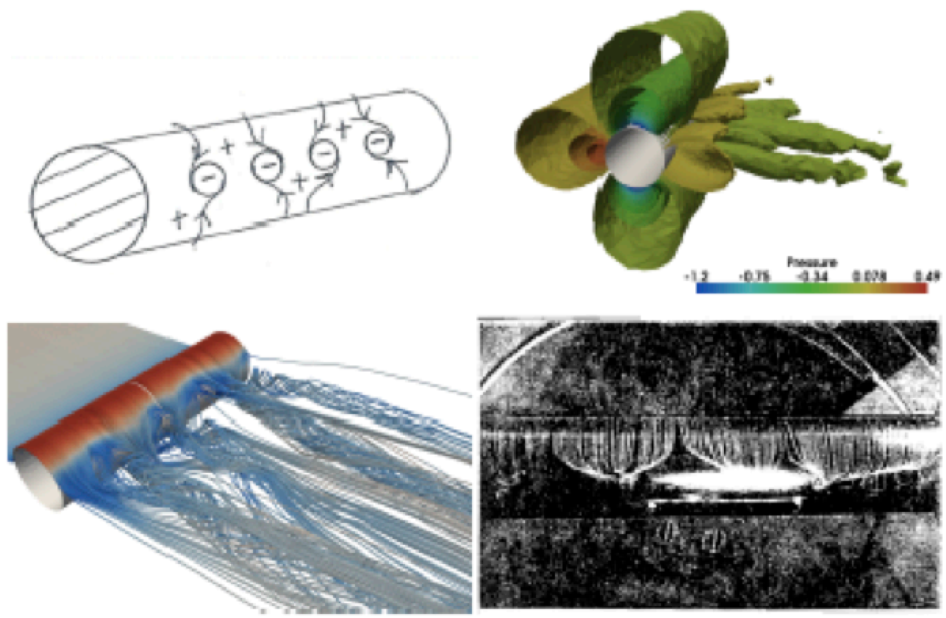



Figure 8.1: Real flow around a circular cylinder with 3d rotational slip separation structured as an array of counter-rotating rolls of swirling flow attaching to low-pressure regions around stagnation points separated by high-pressure regions deviating opposing flows meeting in the back.

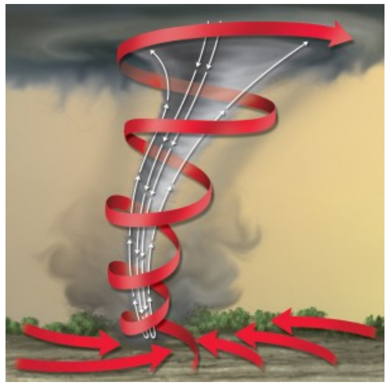


Figure 8.2: Rotational slip separation generates swirling flow similar to a tornado created by opposing flow generated by low presssure rising hot air. The flow of water into a bathtub drain shows a similar swirling motion.

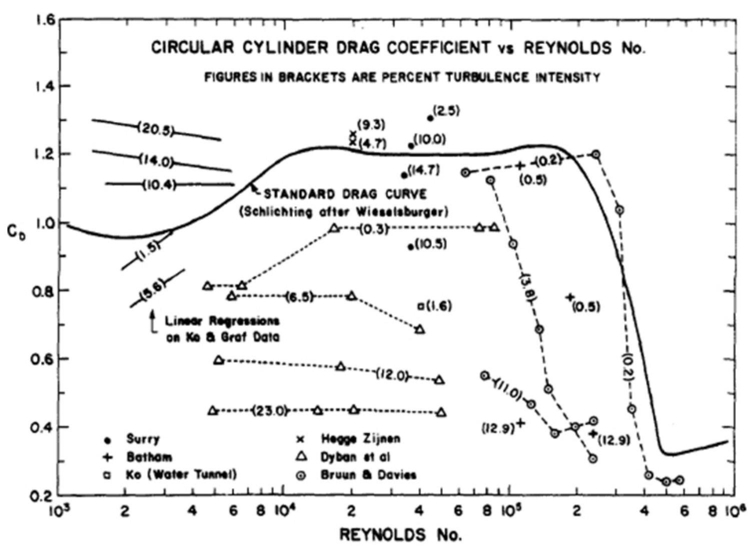



Figure 8.3: Measured drag of circular cylinder as function of Reynolds number. Notice the drop from $1.2$ to $0.4$ for Reynolds number bigger than $5 \cdot 10^6$.

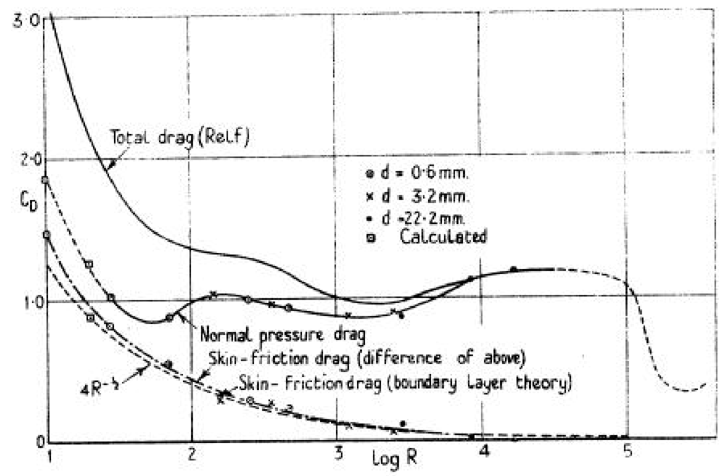



FIG. 154.

Figure 8.4: Total drag as pressure drag plus skin friction drag for a circular cylinder as functions of Reynolds number. Notice the neglible contribution from skin friction for larger Reynolds numbers.

> It will, indeed, be no easy matter to construct a useful wing for man, built upon the lines of the natural wing and endowed with all the dynamically economical properties of the latter; and it will be even a more difficult task to master the wind, that erratic force which so often destroys our handiwork, with those material wings which nature has not made part of our own body. But we must admit the possibility that continued investigation and experience will bring us ever nearer to that solemn moment, when the first man will rise from earth by means of wings, if only for a few seconds, and marks that historical moment which heralds the inauguration of a new era in our civilization. (*Birdflight as the Basis of Avaition*)

# Chapter 9

# Real Flow: Wing

> Before the date of the recent additions to the mathematical theory relating to discontinuous motion (largely initiated by Helmholtz himself), it might almost have been said that the hydrodynamic theory of the text-book had nothing to do with the motions of any known liquid or gas.
>
> The immediate function of the *sectional form* of the aerofoil is to receive a current of air in upward motion and impart to it a downward velocity.
>
> In all real fluids the influence of viscosity accounts for the departure from the theoretical Eulerian form of flow (potential flow), and the departure is *greater* the *less the viscosity*. (Lanchester in *Aerodynamics* 1907)

## 9.1 From Potential Flow to Real Flow

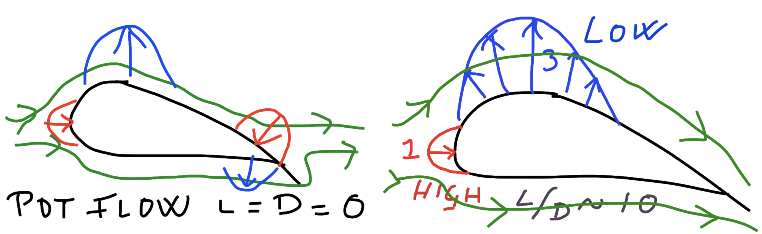



Figure 9.1: Real flow around a wing (right) with high and low pressure zones at trailing edge of potential flow (left) eliminated by 3d rotational separation.

The real flow around a wing can be described as potential flow modified by 3d rotational separation into a flow without pressure rise or drop at the trailing edge, thus with substantial lift and small drag from the leading edge with $\frac{L}{D} > 10$, as illustrated above. Computational results from solving the Navier-Stokes equations with slip are shown in Fig. 9.2.

## 9.2 What to Observe

Notice in particular that the pressure at the

- trailing edge is equal to the background pressure here normalized to zero,
- upper part of the trailing edge is small (suction),
- lower part of the trailing edge is high.

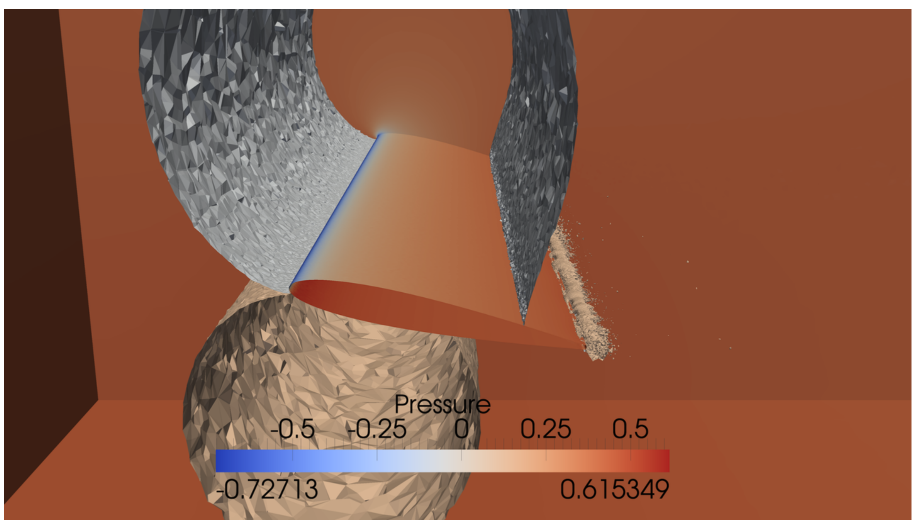



Figure 9.2: Pressure distribution at $\alpha = 12$.

The flow around a wing can roughly be thought of as the flow around the upper frontal quarter of the cylinder which generates drag and lift, with the rest of the flow as around the upper half of the cylinder extended back to a trailing edge as indicated in Fig. 7.6.

The secret of flight is hidden in answers to the following two questions:

- Q1: Why does the flow not separate on the crest of the wing but follows the downward direction of the upper wing surface and thus creates downwash?
- Q2: How can the flow can separate so smoothy from the trailing edge without high/low pressure?

We shall see that answers are given by our analysis of the flow around a circular cylinder as potential flow (answers Q1) modified by 3d rotational separation (answers Q2).

You cannot fly with circular cylinder wings, you can fly with effort on upper half cylinder wings, and you can fly with elegance on a wing formed by two circular cylinders as indicated in Fig. 6.1.

## 9.3 What the New Theory Offers

A common reaction when confronted with the New Theory is the following: Well, a New Theory may be fine, but engineers have been able to construct airplanes without theory or using the Old Theory, and so what is the use of the New Theory? We may answer that the New Theory offers two basic capabilities:

1. Observation: computational solution of the Navier-Stokes equations describing aerodynamics.
2. Understanding: analysis of solutions identifying main characteristics.

The New Theory opens to more efficient and more safe air transportation, by offering a virtual computational wind tunnel in which virtually any question concerning the performance of a flying object can be answered, combined with conceptual analyses opening to innovative new design. This is much very more than what is offered by the Old Theory, which basically consists of a collection of ad hoc methods for computing certain quantities related to simple models problems. The heavy reliance on wind tunnel experimrents all through the development of modern aviation signifies the lack of reliable computation for realistic problems.

> The experiments which we described in the foregoing section demonstrate that certain properties of curved surfaces with regard to wind pressure make actual sailing in the air a possibility. The sailing bird, a kite liberated from its string, are things of reality and not of the imagination.

From the above considerations we are justified in stating that if we call the laws of air resistance the general foundations of aviation, the knowledge of the laws relating to the air resistance of curved surfaces, similar to birds' wings, actually forms the basis for every effective research into actual flight. It is just as thankless to calculate the purely theoretical value of pressures on curved surfaces as it was for plane surfaces. It is true that we may obtain a number of interesting theoretical facts, and indeed the dynamics of curved surfaces in air are more amenable to correct theoretical treatment than those of planes obliquely moved ; but it is obvious that in practice things are not so simple...For the determination of air pressures on curved wings under different inclinations, we must have recourse to experiment; only actual measurements of forces can give us useful figures for the explanation of birdflight and for aviation. (*Birdflight as the Basis of Aviation* )

# Chapter 10

# Mathematical Miracle of Flight

> If a man's wit be wandering, let him study the mathematics. (Francis Bacon)
>
> ...do steady flow ever occur in nature, or have we been pursuing fantasy all along? If steady flows do occur, which ones occur? Are they stable, or will a small perturbation of the ow cause it to drift to another steady solution, or even an unsteady one? The answer to none of these questions is known. (Marvin Shinbrot in *Lectures on Fluid Mechanics* 1970)
>
> What quality of air surrounds the birds as they fly? The air surrounding birds is thinner above than the ordinary thinness of the other air, as it is accordingly thicker below. And it is as much thinner behind than above, accordingly as the movement of the bird is faster in the forward direction than in comparison to the direction of the wings toward the ground. (da Vinci in *Paris Manuscript E*)

Computing turbulent solutions of the Navier-Stokes equations led us to a resolution of D'Alembert's Paradox, which turned out to reveal the miracle of flight. To understand flight means to identify the relevant mathematical aspects of solutions to the Navier-Stokes equations, which are:

- *Non-separation of potential flow* before trailing edge creating suction on the upper surface of wing and downwash.
- *3d rotational slip separation* at the trailing edge without destruction of lift.

This principle is pictured in the by now familiar principal picture Fig. 10.1 supported by computation in Figs. 10.2 and 10.3. We see the zero lift/drag of potential flow being modified by 3d rotational slip separation in to flow with large lift and small drag, as described in a detailed analysis below.

The enigma of flight is thus to explain why the air flow separates from the upper wing surface at the *trailing edge*, and not before, with the flow after separation being redirected downwards depending onthe angle of attack recalling Fig. (1.16). We will reveal the secret to be an effect of a fortunate combination of features of *slightly viscous incompressible flow* including a crucial *instability mechanism of potential flow at separation* (with connections to the swirling flow down a bathtub drain).

We show that this mechanism of lift and drag is operational for angles of attack smaller than a critical value of $15-20$ degrees depending on the shape of the wing, for which the flow stalls and separates from the upper wing surface well before the trailing edge with a quick increase of drag and reduction of $\frac{L}{D}$ into flight failure. In this book you will meet a new theory which you will discover to be both correct and computable.

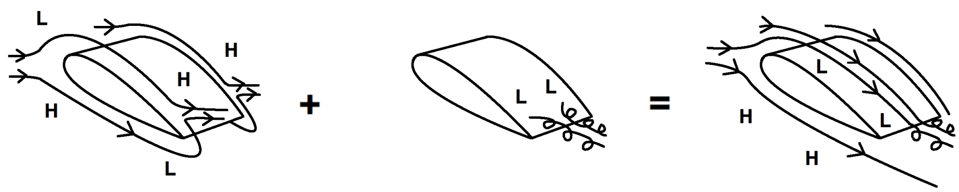



Figure 10.1: Correct explanation of lift by real flow as potential flow (left) modified by 3d rotational slip separation (right) arising from instability of potential flow at separation generating counter-rotating rolls (middle), allowing the flow to leave the trailing edge smoothly with downwash.

We show that lifting flow results from an instability at rear separation of potential flow generating counter-rotating low-pressure rolls of streamwise vorticity inititated as surface vorticity resulting from meeting opposing flows. This mechanism is entirely different from the mechanism based on global circulation of Kutta-Zhukovsky theory. We show that the new theory allows accurate computation of lift, drag and twisting moments of an entire airplane using a few millions of mesh-points, instead of the impossible quadrillions of mesh-points required according to state-of-the-art following Prandtl's dictate of resolution of very thin boundary layers connected with no-slip velocity boundary conditions.

> Finally we endeavoured to prove, by actual experiments, that the true secret of birdflight was the curvature of the bird's wing, which accounted for the

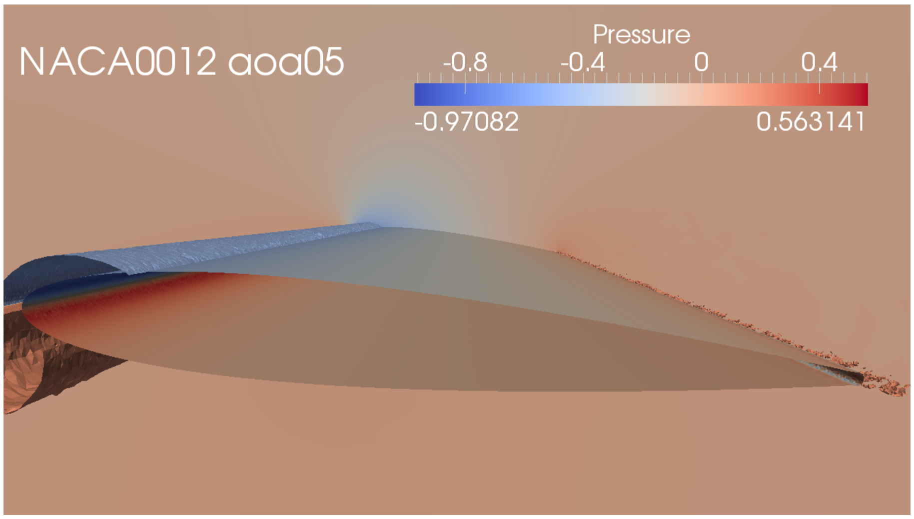



Figure 10.2: Pressure distribution with big leading edge lift (suction above and push below) and small drag at $\alpha = 5$. Notice that the high pressure at the trailing edge of potential flow is missing thus maintaining big lift.

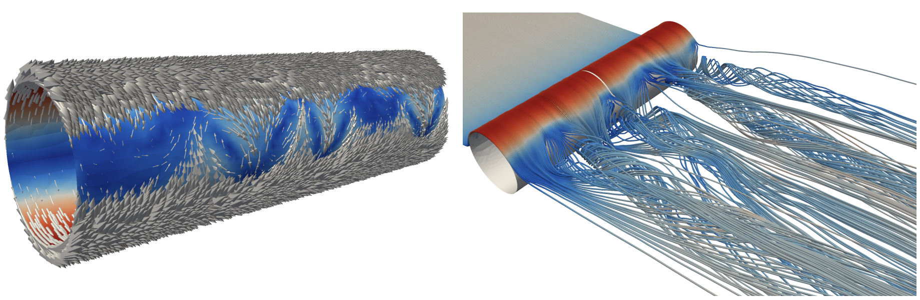


Figure 10.3: Turbulent separation in flow around a circular cylinder from surface vorticity forming counter-rotating low-pressure swirling flow, illustrating 3d rotational slip separation at the trailing edge of a wing [101].

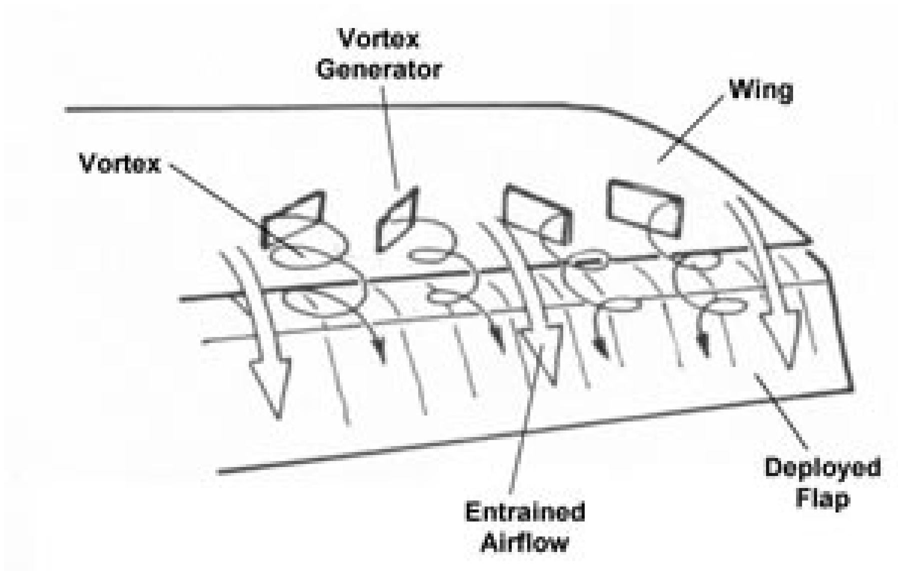



Figure 10.4: Vortex generators guiding 3d rotational separation and thus delaying separation on upper wing surface and stall.

> natural, small effort required for forward flight, and which, together with the peculiar lifting effects of wind, explained the ability of birds to "sail."
>
> ... we should not take the insect world as our model for flying, but, on the contrary, we must study the large flyers, for whom the ratio between wing area and weight approaches as nearly as possible those conditions which are necessary for human flight. Our attention has been drawn to the shape of the wings, and we all know that birds' wings are not plane, but somewhat curved. The question therefore arises whether this fact is capable of explaining the small amount of energy necessary for natural flight, and in how far wings other than planes are of assistance in reducing the work of flying. Theoretical predictions do not seem to be of much use in this respect, except by referring us back again and again to nature and to the exact imitation of a bird's wing. (*Birdflight as the Basis of Aviation*)

# Chapter 11

# Mathematical Miracle of Sailing

> By denying scientific principles, one may maintain any paradox. (Galileo Galilei)

> Truth is a good dog; but always beware of barking too close to the heels of an error, lest you get your brains kicked out. (Francis Bacon)

## 11.1 Sail and Keel Act Like Wings

Both the sail and keel of a sailing boat under tacking against the wind, act like wings generating lift and drag, as illustrated in Fig. 11.1. But the action, geometrical shape and angle of attack of the sail and the keel are different. The effective angle of attack of a sail is typically 15-20 degrees and that of a keel 5-10 degrees, for reasons which we now give.

## 11.2 Basic Action

The boat is pulled forward by the sail, assuming for simplicity that the beam is parallel to the direction of the boat at a minimal tacking angle, by the component $L\sin(15)$ of the lift $L$, as above assumed to be perpendicular to the effective wind direction, but also by the following contributions from the drag assumed to be parallel to the effective wind direction: The negative drag on the leeeward side at the leading edge close to the mast gives a positive pull which largely compensates for the positive drag from the rear leeward side, while there is less positive drag from the windward side of the sail as compared to a wing profile, because of the

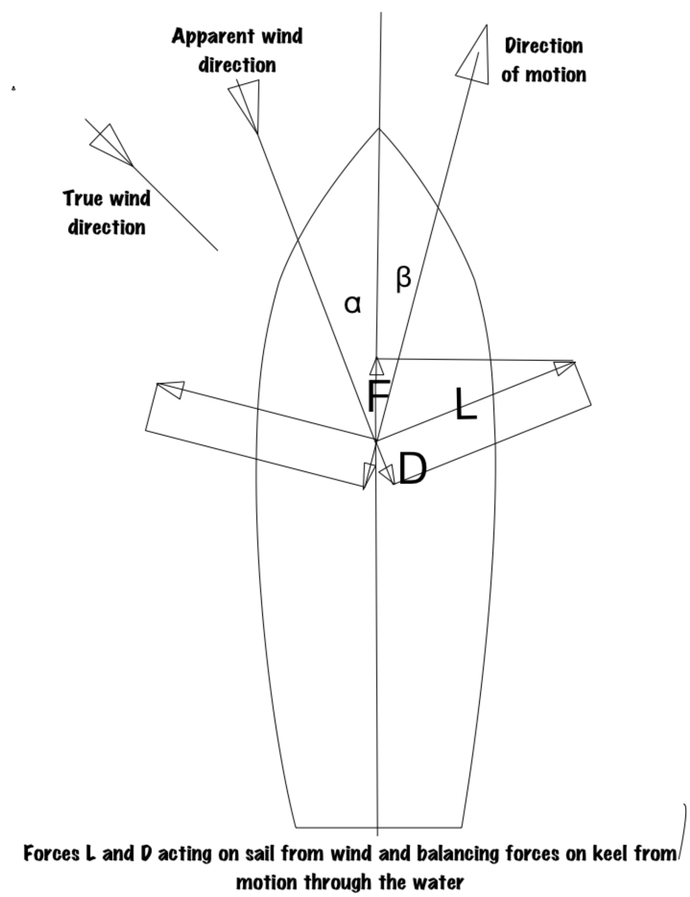



Figure 11.1: Lift $L$ and drag $D$ from sail at beating with drive $F = \sin(\alpha)L$ at angle of attack $\alpha$, balanced by lift and drag from the keel at the angle of attack $\beta$ (plus drag from the hull). Notice that the wind approaches the sail from the left, while the water approaches the keel from the right, giving opposite lift forces.

difference in shape. The result is a forward pull $\approx \sin(15)L \approx 0.2L$ combined with a side (heeling) force $\approx L\cos(15) \approx L$, which tilts the boat and needs to be balanced by lift from the the keel in the opposite direction. Assuming the lift/drag ratio for the keel is 13, the forward pull is then reduced to $\approx (0.2 - 1/13)L \approx 0.1L$, which can be used to overcome the drag from the hull minus the keel.

Figure 11.2: Seeking to balance the heeling on a beat.

## 11.3 Sail vs Wing

The shape of a sail is different from that of a wing which gives smaller drag from the windward side and thus improved forward pull, while the keel has the shape of a symmetrical wing and acts like a wing. A sail with aoa $15 - 20$ degrees gives maximal pull forward at maximal heeling/lift with contribution also from the rear part of the sail, like for a wing just before stall, while the drag is smaller than for a wing at 15-20 degrees aoa (for which the lift/drag ratio is 4-3), with the motivation given above. The lift/drag curve for a sail is thus different from that of wing with lift/drag ratio at aoa 15-20 much larger for a sail. On the other hand, a keel with aoa 5-10 degrees has a lift/drag ratio about 13. A sail at aoa 15-20 thus gives maximal pull at strong heeling force and small drag, which together with a keel at aoa 5-10 with strong lift and small drag, makes an efficient combination.

This explains why modern designs combine a deep narrow keel acting efficiently for small aoa, with a broader sail acting efficiently at a larger aoa.

In Chapter 6 we saw that a sail can be used as a wing, and it is possible to use a symmetrical wing as a sail as shown in the next section.

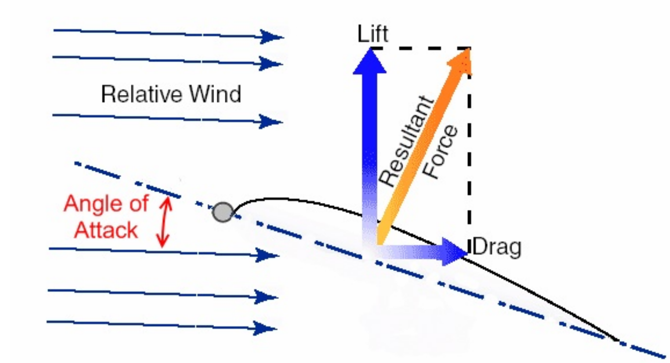



Figure 11.3: Lift $L$ and drag $D$ forces on a sail.

## 11.4 Americas Cup Wing-Sail

AC45 is a forerunner to the next generation of Americas Cup boats: A wing-sailed catamaran designed for speed over 30 mph and close racing, see Figs. 11.5 and 11.6. Vestas Sailrocket 2 became the fastest sailboat on planet reaching a peak speed of 62 knots on Nov 12 2012, see Fig. 11.6.

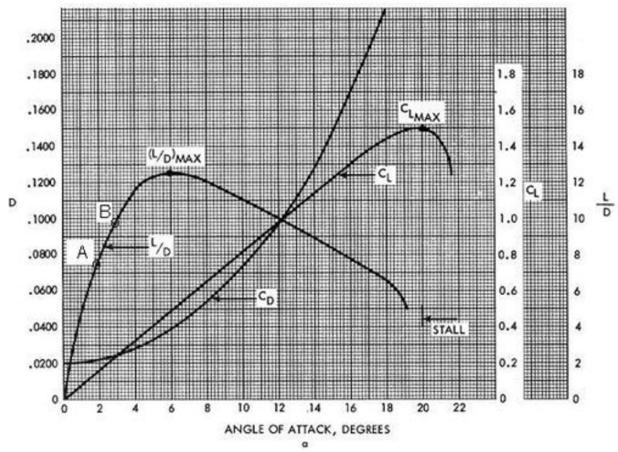



Figure 11.4: The secret of sailing: Lift $L$, drag $D$ and lift-to-drag $\frac{L}{D}$ for different (apparent) angles of attack. Note that $\frac{L}{D} \approx 6$ at maximal lift at an angle of attack of 20 degrees, while maximum $\frac{L}{D} \approx 13$ is obtained at an angle of attack of 6 degrees.

Figure 11.5: AC 45 wing sail.

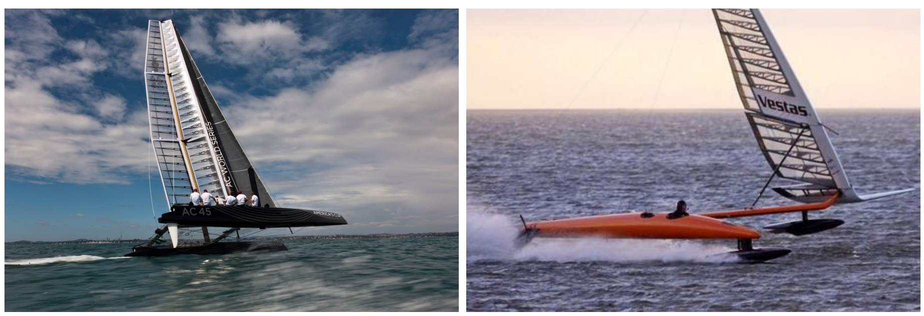



Figure 11.6: AC 45 and Vestas Sailrocket 2.

# Chapter 12

# Real and Virtual Wind Tunnels

> Turbulence is the most important unsolved problem of classical physics. (Richard Feynman)
>
> I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic. (Horace Lamb)
>
> Studies perfect nature and are perfected still by experience. (Francis Bacon)

The difficulty of computational mathematical simulation in aerodynamics led to the construction of wind tunnels for experimental testing. Prandtl based his leadership in aerodynamics on the closed-circuit $2\,m \times 2\ m$ wind tunnel he constructed at his Institute for Technical Physics in Göttingen in 1908 with a second-generation completed in 1916 to serve the war effort.

The Full-Scale 30-by 60-Foot Wind Tunnel at NASA's Langley Research Center completed in 1931 was a double-return atmospheric pressure tunnel with two fans powered by 4,000 hp electric motors, capable of testing aircraft with spans of $12\,m$ air at speeds up to $190\,km/h$.

The tunnel was used to test virtually every high-performance aircraft used by the United States in World War II. For much of the war, when it was operational 24 hours a day, seven days a week, the full-scale tunnel was the only tunnel in the free world large enough to perform these tests.

The wind tunnel was in use through the 2000s, modified to allow new testing procedures, such as free-flight and high angle of attack, but demolition of the tunnel began in 2010 with the fan blades being salvaged for display. The era of the physical wind tunnel is now over and the era of the virtual computational wind tunnel has begun.


Figure 12.1: Langley full scale wind tunnel 1931: Entrance and exit cones with people showing thye dimensions.

# Chapter 13

# Data

> Birds raise their opened wings with greater facility than they lower them. ...because the wing is convex on top and concave below, so that the air can more conveniently escape from the percussion of the wings in their rising than in their lowering, where in the air included within the concavity tends to become more condensed than to escape. (da Vinci in *Paris Manucript E*)

## 13.1 Classical Data

We present below experimental data from Goldstein: *Modern Developments in Fluid Dynamics* including comparisons with predictions by classical circulation theory. We shall below return to these observations in the light of the New Theory.

## 13.2 Boeing 787-8 Dreamliner

- Overall $\frac{L}{D} = 20.84$
- Takeoff $C_L max = 1.91$
- Takeoff $V2 = 167\, keas$ (climb out speed) ($knots,\ 1\, knot \approx 1.8 km/h$ )
- $\frac{L}{D} = 14.15$ at 2nd segment.
- Takeoff Thrust/Weight = 0.280
- Landing $C_L max = 2.66$
- Landing Approach Speed $133\, keas$

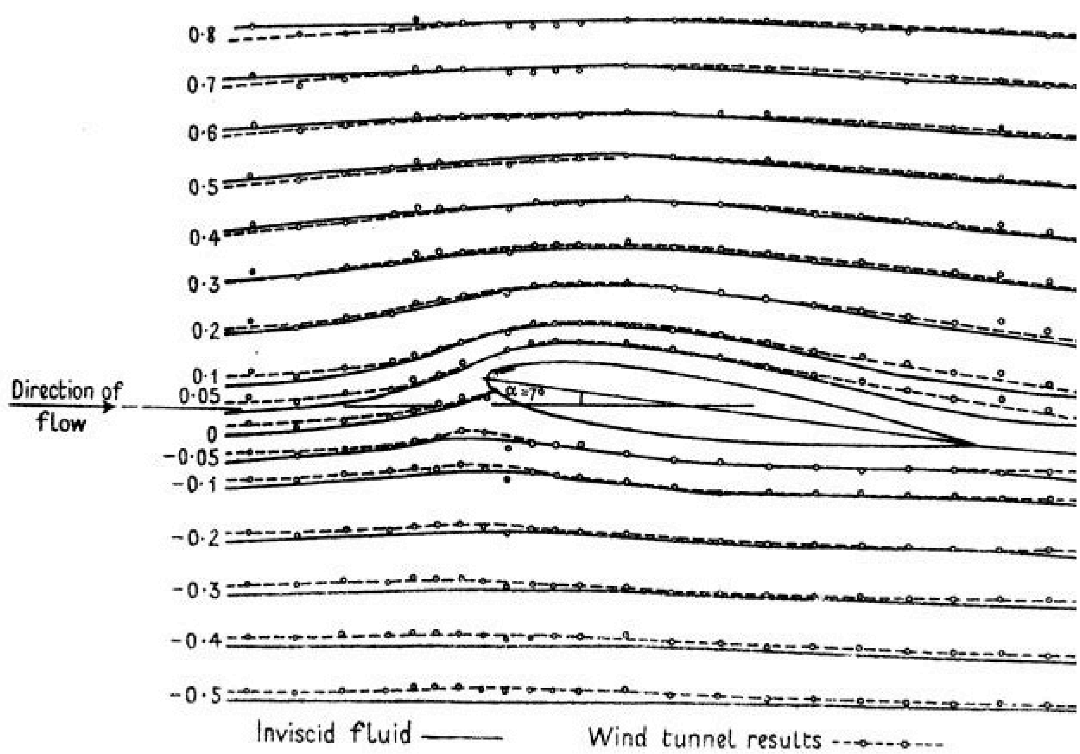



Figure 13.1: Circulation theory velocity around an airfoil compared with observation. Notice the unphysical inflow velocity of circulation theory.

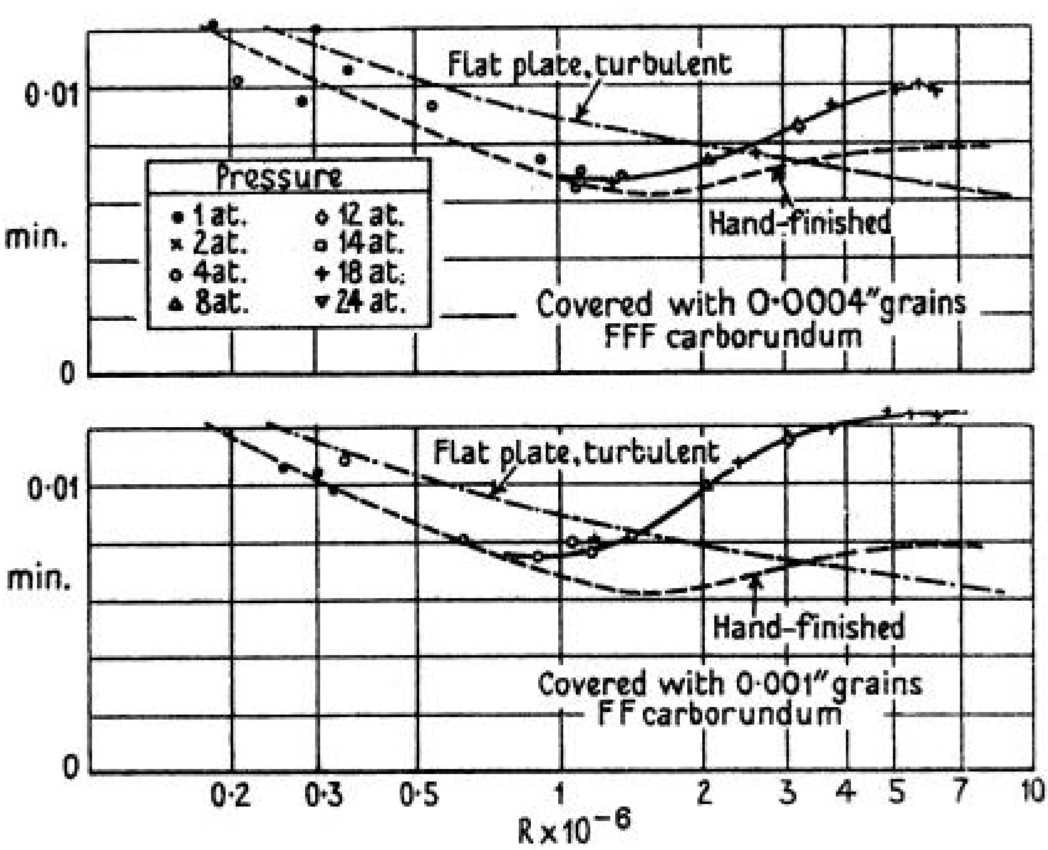



FIG. 190.

Figure 13.2: Drag coefficient $c_D$ for different surface roughness as function of Reynolds number.

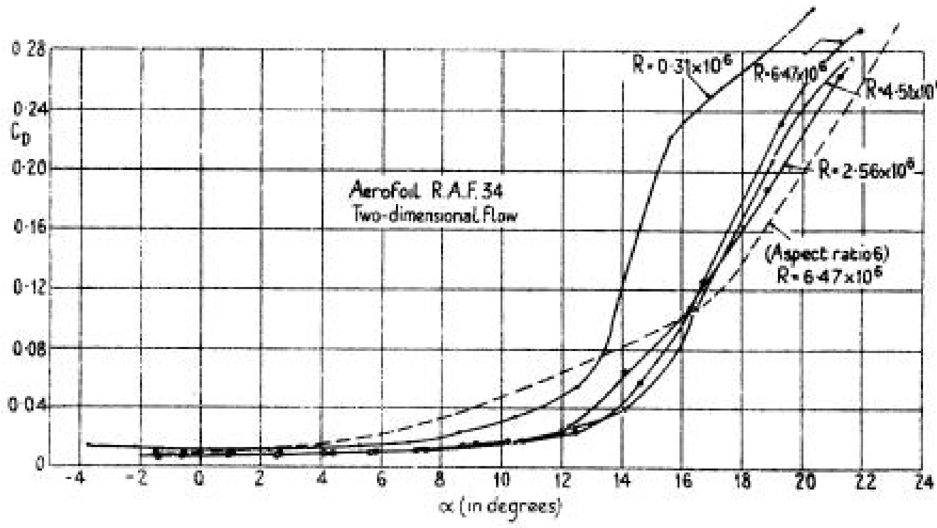



FIG. 171.

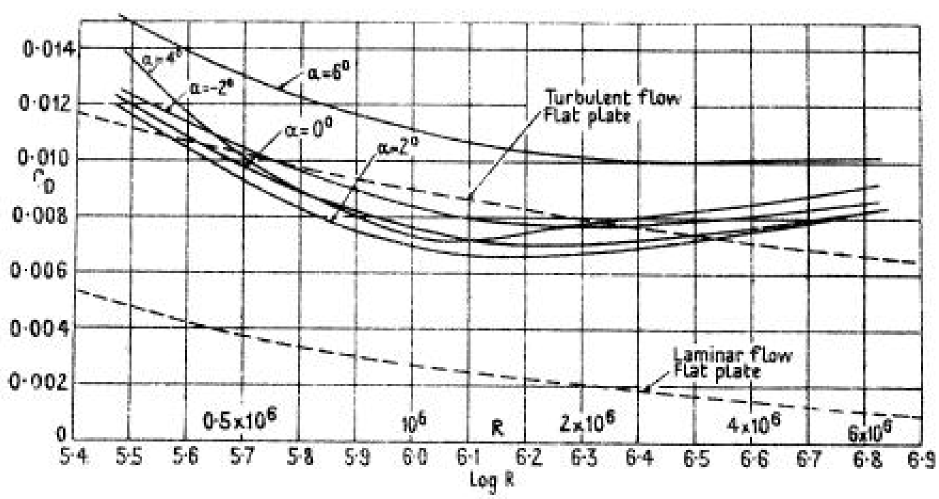



FIG. 172.

Figure 13.3: Drag coefficient $c_D$ as function of angle of attack and Reynolds number.

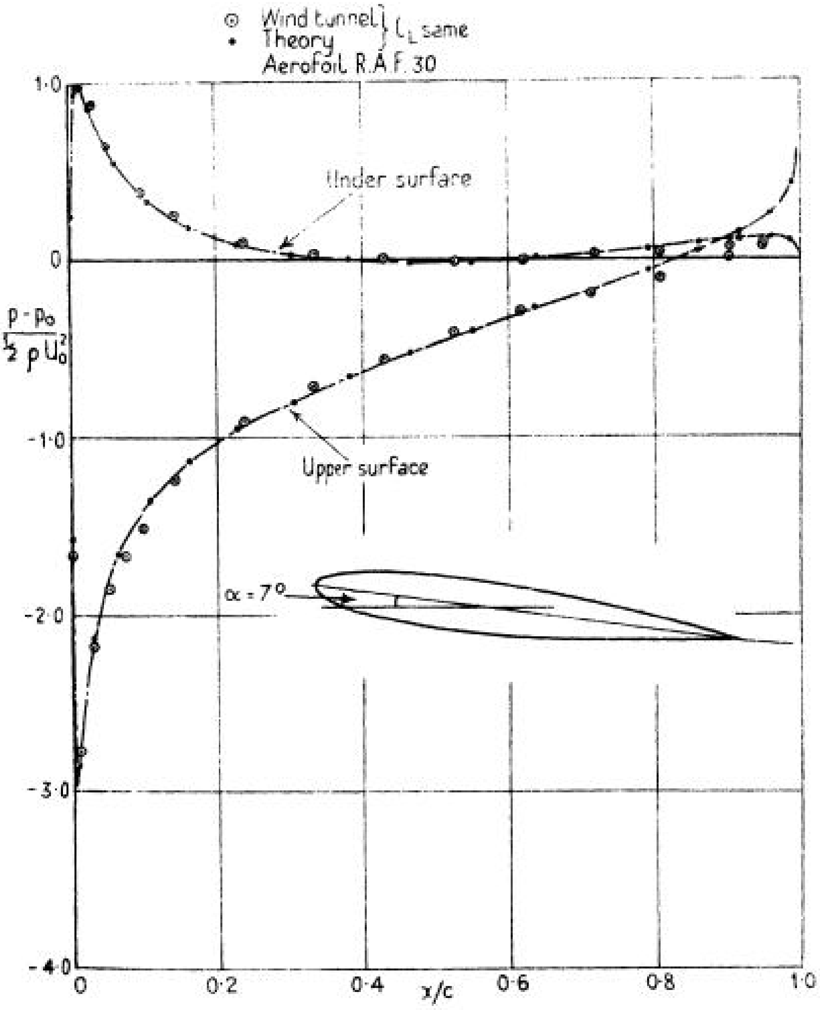



FIG. 179.

Figure 13.4: Comparision of theoretical and observed pressure distribution on upper and lower wing surface for $aoa = 7$. Observe the unphysical theoretical high pressure at the trailing edge.

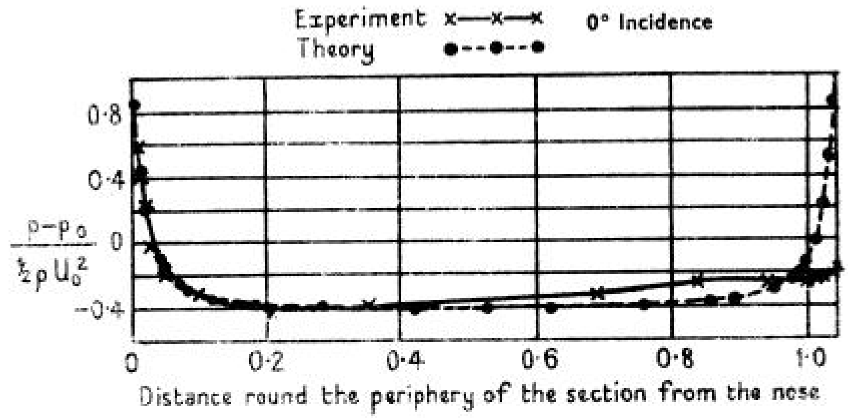



FIG. 191.

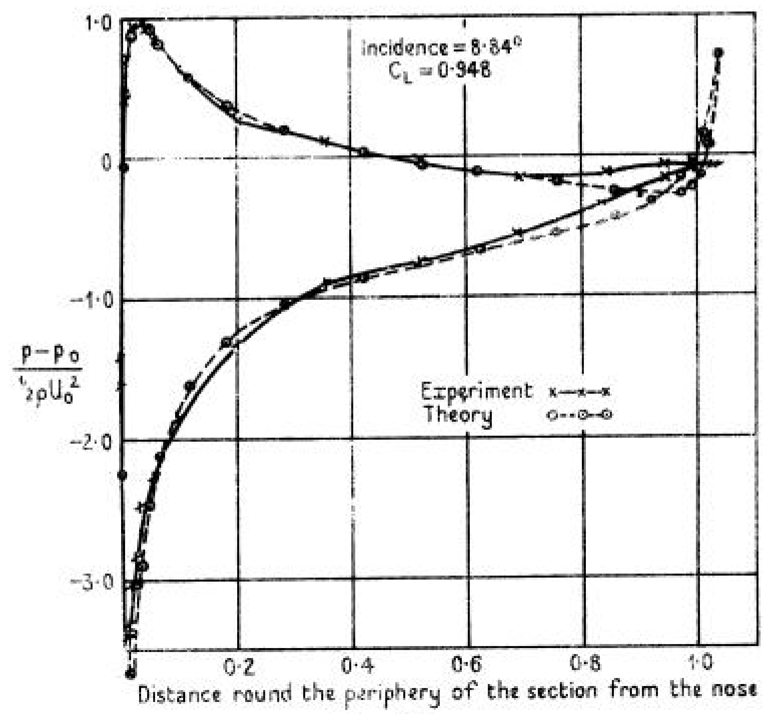



FIG. 192.

Figure 13.5: Comparision of theoretical and observed pressure distribution on upper and lower wing surface for $aoa = 0$ and $8$. Observe the unphysical theoretical high pressure at the trailing edge.

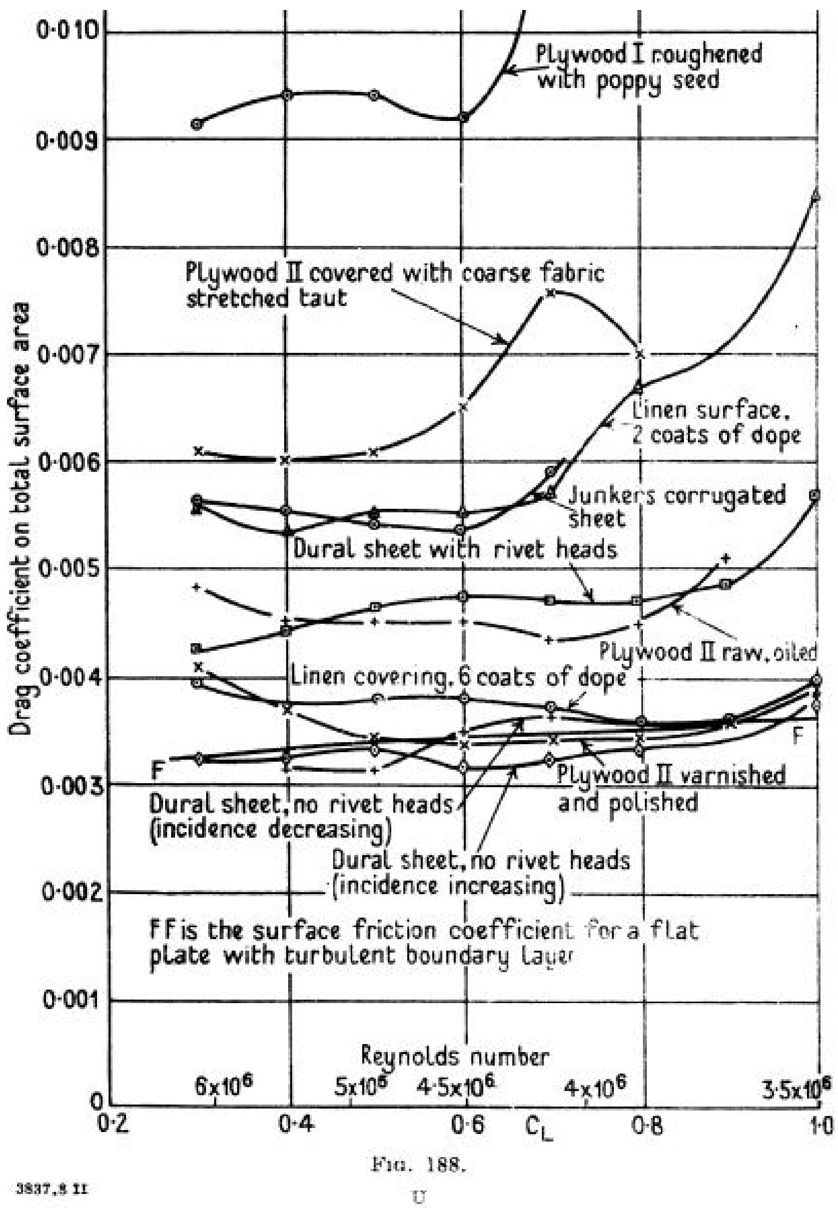



Figure 13.6: Observed drag coefficients for different suraface roughness and Reynolds number. Notice increasing drag for smaller Reynolds numbers.

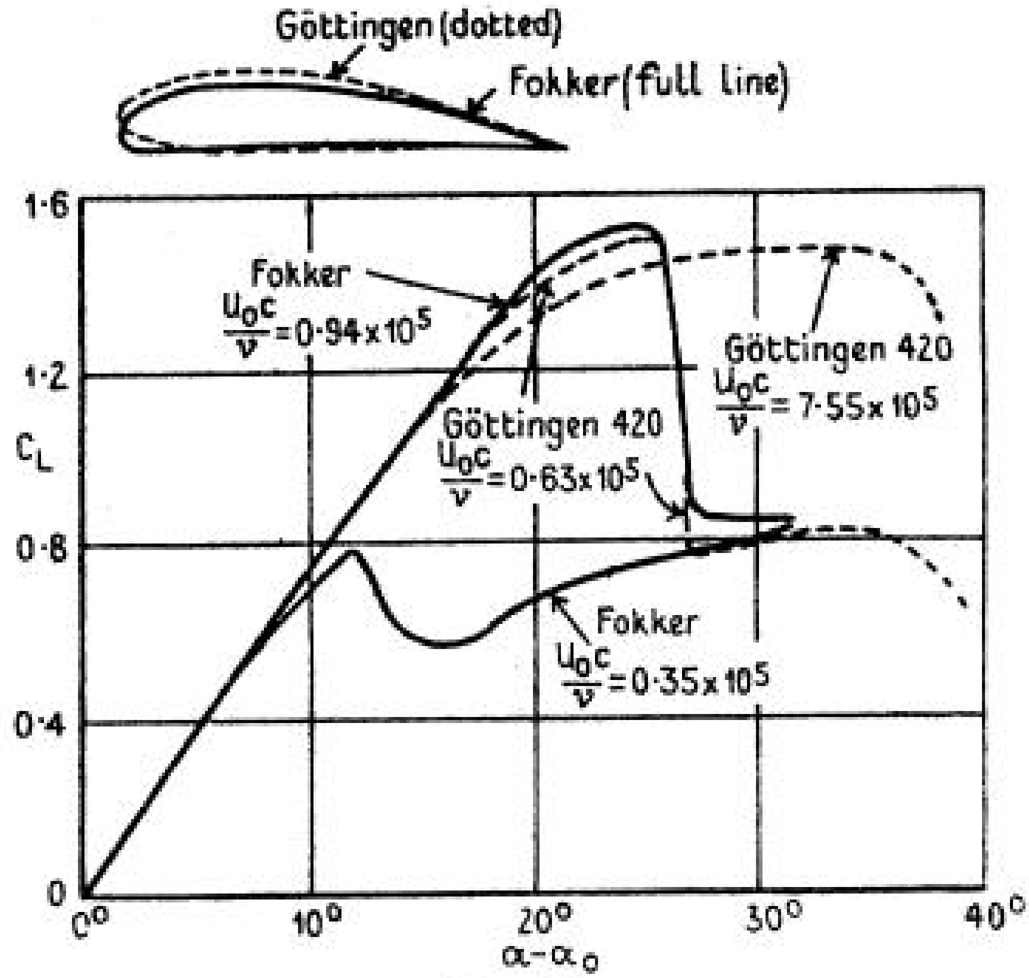



FIG. 184.

Figure 13.7: Observed lift coefficient as function of angle of attack for different airfoils and Reynolds number

.

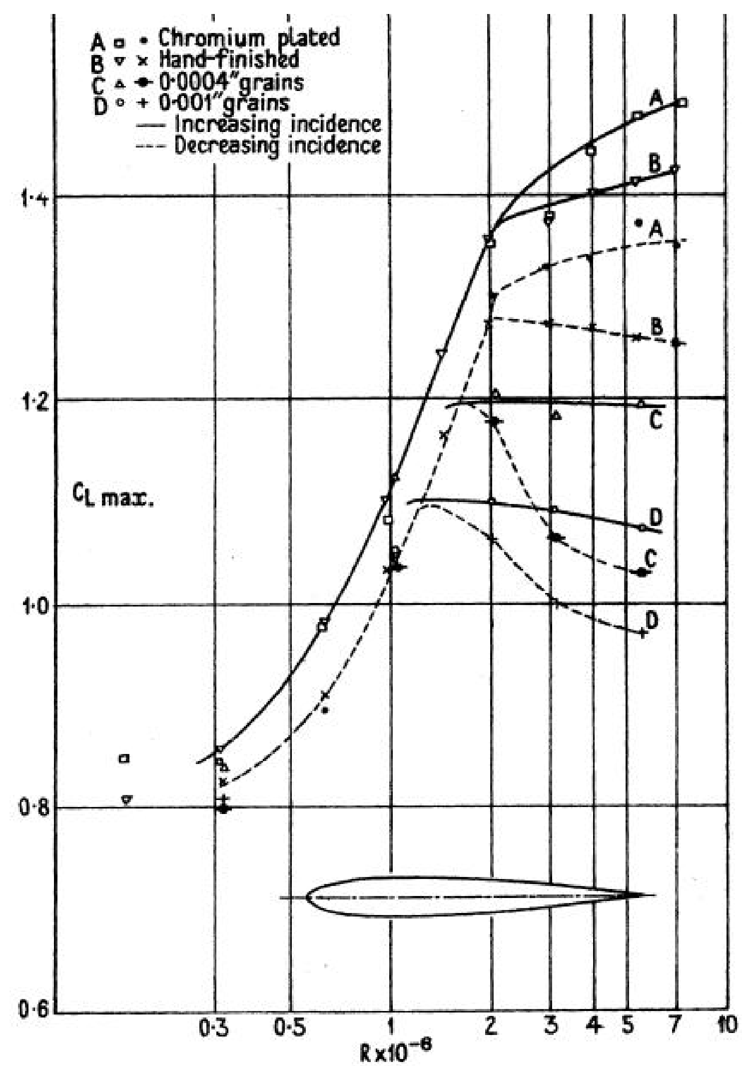



Figure 13.8: Observed maximal lift coefficient as function of Reynolds number for different surface roughness. Observe the branching of the curves as the Reynolds number increases beyond $10^6$ with finished surfaces allowing effective slip to delay separation and increase lift.

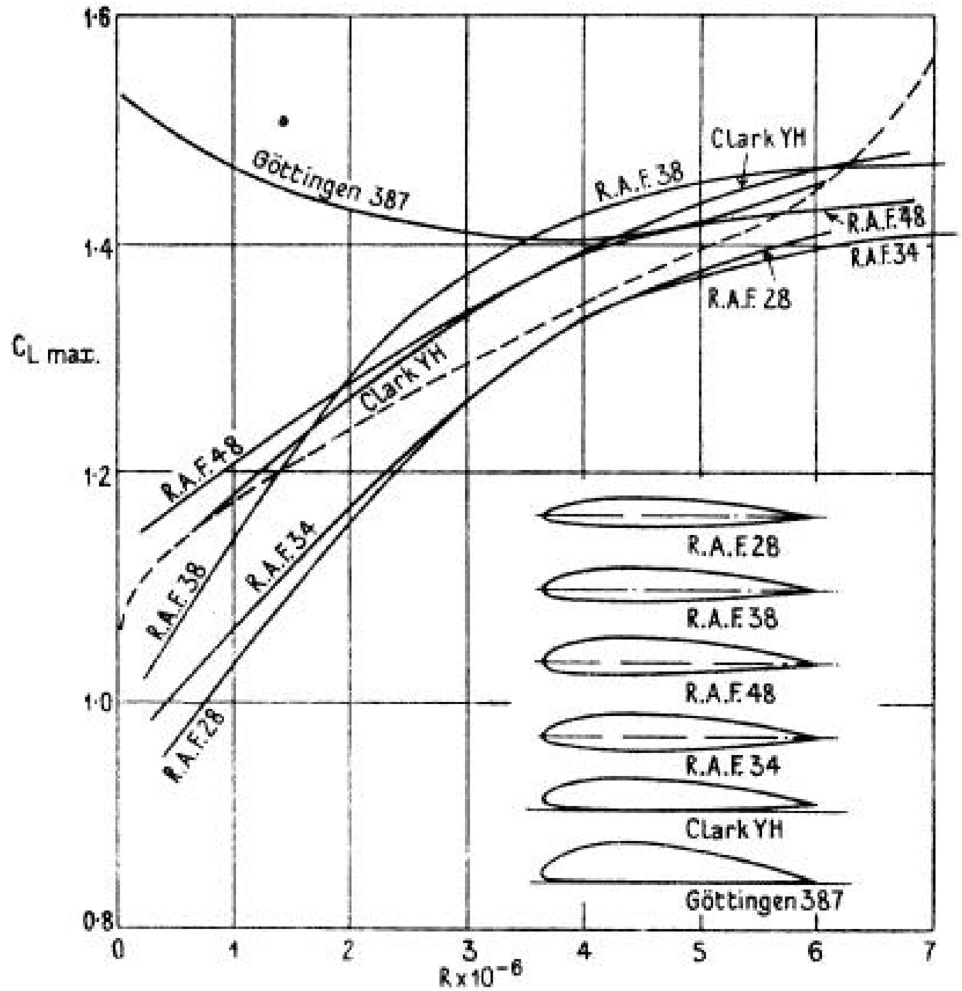



FIG. 167.

Figure 13.9: Observed maximal lift coefficient as function of Reynolds number for different airfoils.

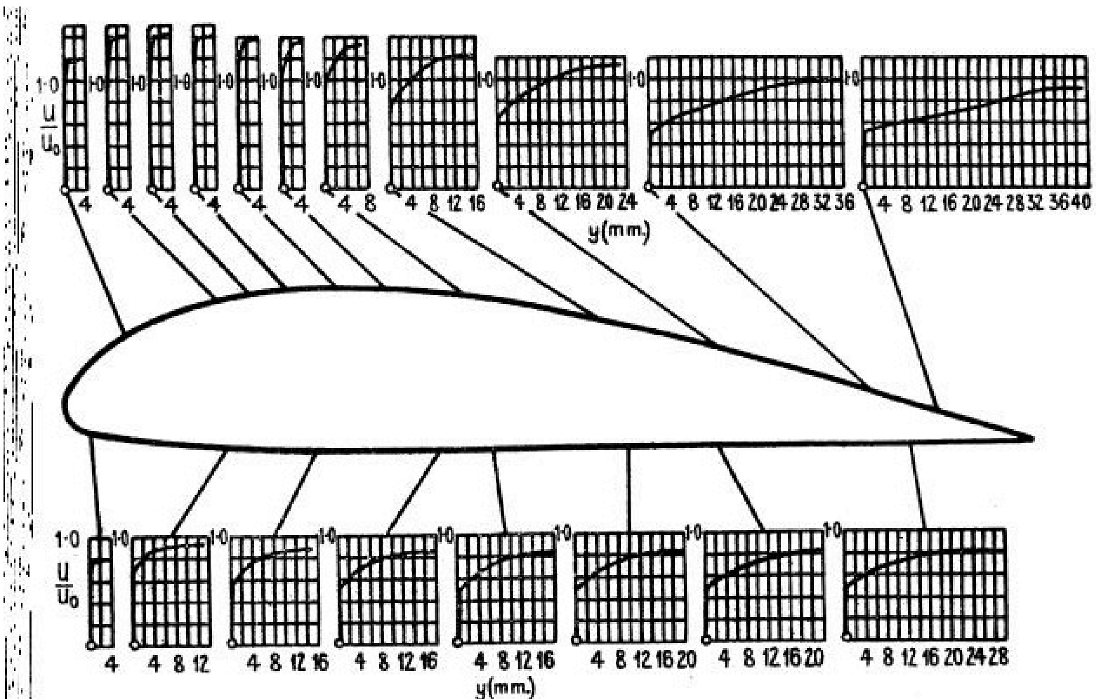



Fig. 185.

Figure 13.10: Observed velocity as function of distance to the wing surface, with $u_0$ the free stream velcocity. Notice the apparent absense of a boundary layer connecting the free stream velocity to a no-slip boundary condition.

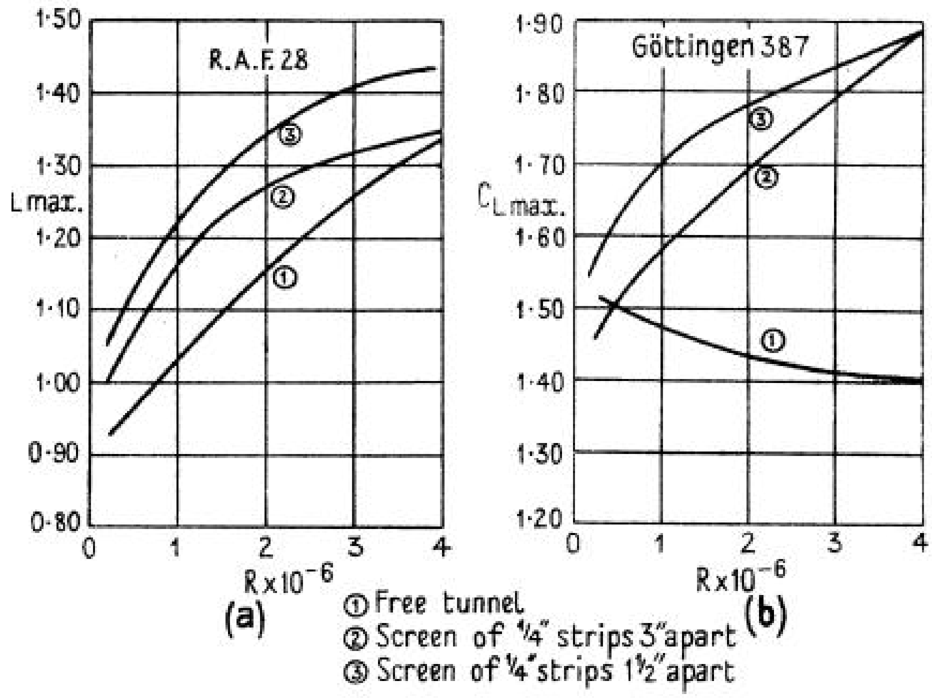



FIG. 169.

Figure 13.11: Observed maximal lift for different levels of inflow turbulence. Observe that the maximal lift increases with increasing level of turbulence indicating delay of separation connecting to an effective slip boundary condition.

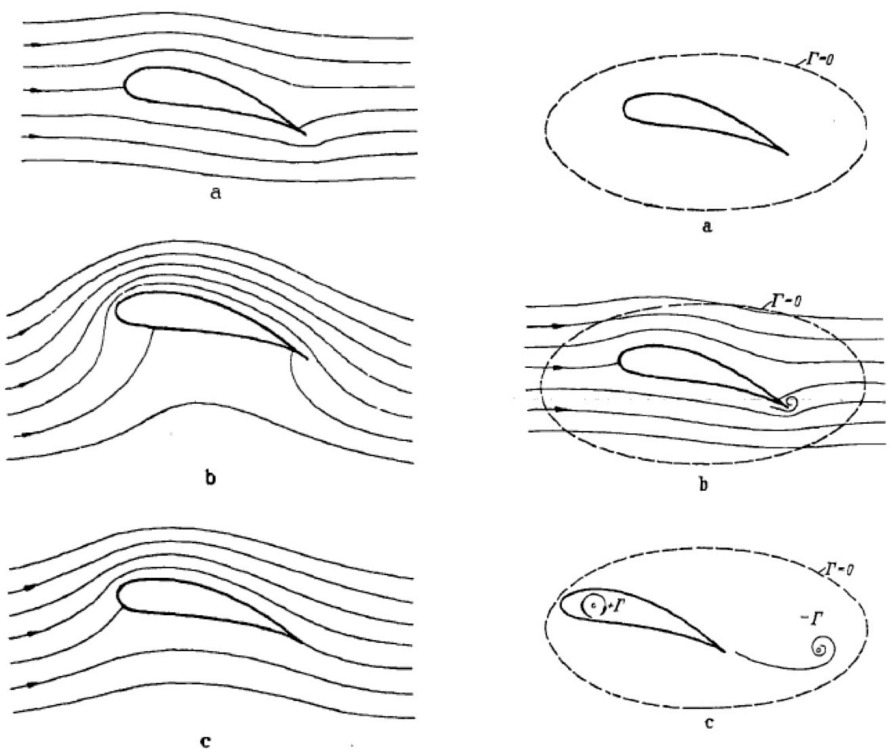



Figure 13.12: The essence of classical circulation theory in pictures, from Schlichting-Truckenbrodt *Aerodynamics of the Airplane*. Top left (a) shows potential flow with zero lift and drag. Bottom left (c) shows potential flow with large scale circulation determined by the Kutta condition of smooth separation at the trailing edge, supposedly generated by the starting vortex shown to the right (b and c). The unphysical nature of circulation theory with a change of the direction of the inflow, is clearly displayed.

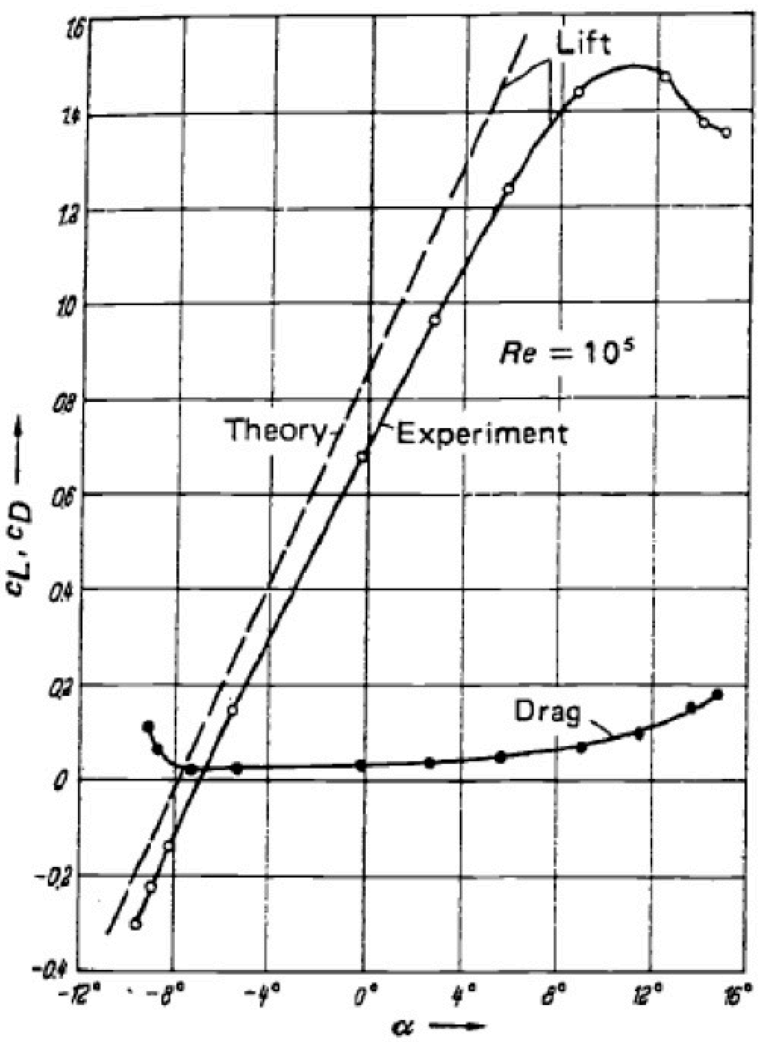



Figure 13.13: Classical circulation theory only gives the slope of the lift curve as one number, but does not include e.g. stall or drag.

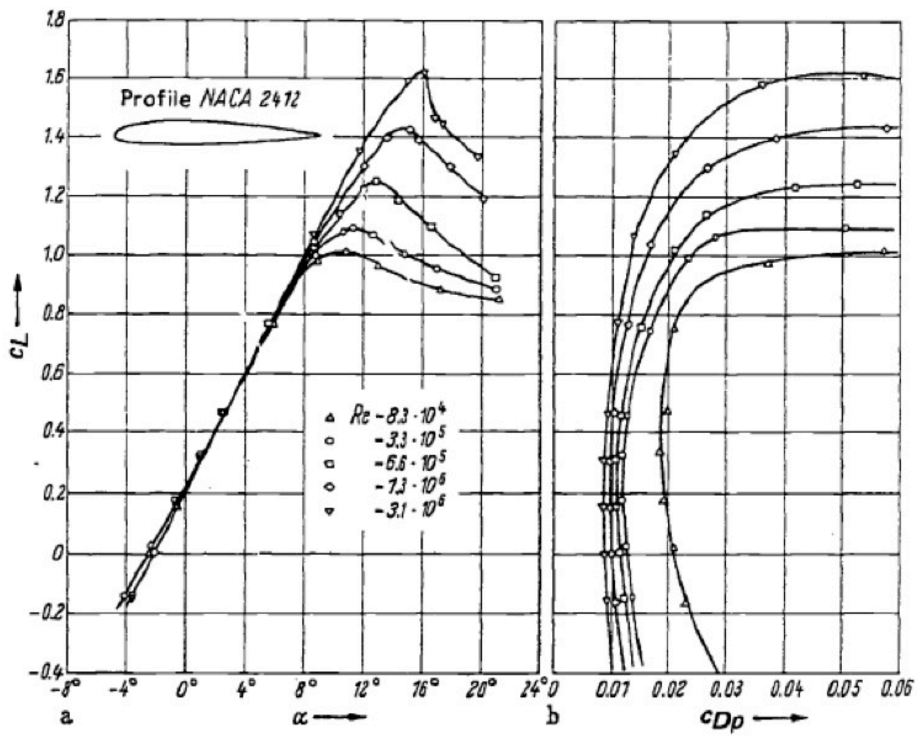



**Figure 2-39** Lift and drag measurements on the NACA 2412 profile at various Reynolds numbers. (*a*) Lift coefficient $c_L$ vs. angle of attack $\alpha$. (*b*) Polar curves $c_L$ vs. $c_{Dp}$.

Figure 13.14: Observation of lift and drag for different angles of attack and Reynolds numbers, from Schlichting-Truckenbrodt *Aerodynamics of the Airplane*.

# Part II

# Observing Navier-Stokes

# Chapter 14

# Shut Up and Calculate

> How can aviation be grounded in such a muddy understanding of the underlying physics? As with many other scientific phenomena, it's not always necessary to understand why something works to make use of it. We engineers are happy if we've got enough practical knowledge to build flying aircraft. The rest we chalk up to magic [87].

## 14.1 Naviers-Stokes vs Schrödinger

The interpretation of the wave functions of the new quantum mechanics of the atomic world as solutions of Schrödinger's wave equations, was intensely debated in the 1920s by Schrödinger, Bohr, Born and Dirac without reaching any agreement. This led a frustated Dirac in an effort to get out of the scientific dead lock to make the appeal ”shut up and calculate”: Simply solve the Schrödinger equation and take what you get as physics. However, Dirac's appeal did not and still does not help much because analytical solution of Schrödinger's equation is possible only for the Hydrogen atom with one electron, and computational solution involves $3N$ space dimensions for $N$ particles, which even today is possible only for small $N$.

The debate thus is shifted to different techniques of solving Schrödinger's equation, considered to harbor the truth, and the debate goes on.

Similarly, solutions of the Navier-Stokes equations may be expected to tell the physics of flight, and so the question is if they are computable or as uncomputable as Schrödinger's equation?

Navier-Stokes solutions at the high Reynold's numbers of flight are turbulent and have thin boundary layers and the standard wisdom expressed by Kim an Moin [43] is that computational resolution requires quadrillions of mesh points beyond the capacity of any forseeable computer.

But there is trick, or mircale, which make the Navier-Stokes equations computable: If we combine the Navier-Stokes equations with a slip boundary condition modeling that the friction force from the air on the wing is small, which it is for slightly viscous flow, then there is no Prandtl boundary layer to resolve and then solutions of the Naviers-Stokes equations can be computed with $10^6$ mesh points, and the lift and drag of these solutions agree very well with experiments. Thus Dirac's appeal works out for flight described by Navier-Stokes equations.

Chosing a slip (or small friction) boundary condition, which we can do because it is a good model of actual physics, we gain in two essential aspects: Solutions become computable and computed solutions tell the truth.

The truth we find this way is that neither lift nor drag originate from a thin boundary layer, and thus that the Kutta-Zhukovsky-Prandtl theory is unphysical and incorrect. State-of-the-art today thus presents a theory of flight which is both unphysical and uncomputable.

## 14.2 Compute - Analyze - Understand

Computed solutions show that a wing creates lift as a reaction force *downwash*, with less than 1/3 coming from the lower wing surface pushing air down and the major remaining part from the upper surface sucking air down, with a resulting *lift/drag quotient* $\frac{L}{D} > 10$.

You could stop here following the device of the physicist Dirac of "shut up and calculate" but as a scientist and rational human beings you would certainly like to "understand" the solutions, that is describe the "mechanism" making a wing generate large lift with small drag.

> For flight with a mean value of 3 degrees wing inclination, the mechanical effort for a human being would be $0.3\,hp$. This amount of energy is within the possibilities of human effort especially after some training. Given a very favourable shape for the flying apparatus, with 15 to 20 square meter area, and not more than 22 pounds weight, rapid horizontal flight in still air becomes feasible. But, at any rate, with such an apparatus and without beating the wings, we can execute a prolonged downward glide, which would present plenty of interest and instruction. (*Birdflight as the Basis of Aviation* )

# Chapter 15

# Observing Pressure, Lift and Drag

> I have been able to solve a few problems of mathematical physics on which the greatest mathematicians since Euler have struggles in vain... But the pride I could have felt over the final results... was considerably diminished by the fact that I knew well how the solutions had almost always come to me: by gradual generalizations of favourable examples, through a succession of felicitous ideas after many false trails. I should compare myself to a mountain climber who, without knowing the way, hikes up slowly and laboriously, often must return because he cannot go further, then, by reflection or by chance, discovers new trails that take him a little further, and who, when he finally reaches his aim, to his shame discovers a royal road on which he could have trodden up if he had been clever enough to find the right beginning. Naturally, in my publications I have not told the reader about the false trails and I have only described the smooth road by which he can now reach the summit without any effort. (Hermann von Helmholtz 1891)

## 15.1 Potential Flow with 3d Rotational Separation

We will now uncover the secret of flight in more detail by inspecting computational solutions of Navier-Stokes equations with a slip boundary condition for a long NACA0012 wing. The distribution of the pressure over the wing surface determines both lift and drag. The pressure acts in a direction normal to the wing surface and its components perpendicular and parallel and perpendicular to the motion of the wing give the distributions of lift and drag over the wing surface which give the total lift $L$ and drag $D$ with by integration. We discover that $\frac{L}{D} = 30 - 50$ for $\alpha < 14$ as recorded in Fig. 1.16, which captures the miracle of flight. For a common wing of finite length, $\frac{L}{D} = 15 - 20$, because of wing-tip effects as discussed below.

We will inspect the computed velocity and vorticity and rationalize what we see in terms of basic fluid mechanics. We will find that the flow can be described as a potential flow modified by *3d rotational slip separation with point stagnation*.

The flow before separation thus will seen to be close to potential flow and there conform with *Bernoulli's Law* stating that the sum of kinetic energy $\frac{1}{2}|u|^2$ with $u$ velocity and pressure $p$ remains constant over the region of potential flow:

$$\frac{1}{2}|u|^2 + p = \text{constant}, \tag{15.1}$$

expressing that the pressure is low where the velocity is high and vive versa.

We recall the principal description in Fig. 10.1 with full potential flow to the left with high pressure $H$ on top of the wing and low pressure $L$ below at the trailing edge destroying lift, and to the right the real flow modified by a certain perturbation at the trailing edge switching $H$ and $L$ to generate lift.

## 15.2 Pressure

Fig. 15.3 shows the pressure distribution for $aoa = 4, 12, 17$. We observe

- low pressure on top/front of the leading edge and high pressure below as expected from potentiual flow,
- no high pressure on top of the trailing edge in contrast to potential flow,
- the pressure distribution intensifies with increasing angle of attack,
- max negative pressure (lift) 5 times bigger than max pressure (drag) on leading edge for $\alpha = 10$,

## 15.3 Lift and Drag Distribution

Fig. 15.2 shows the distributions of lift and drag over the surface of the wing section for $\alpha = 0, 2, 4, 10, 18$. We observe

- lift increases and peaks at the leading edge as $\alpha$ increases towards stall,
- both lift and drag are small att the trailing edge,

- the negative lift on the lower surface for small $\alpha$ shifts to positive lift for larger $\alpha$,
- the lift from the upper surface is several times bigger than from the lower surface
- leading edge suction: negative/positive drag on the upper/lower leading edge balance to give small net drag,
- main drag from leading edge for smaller $\alpha$.

Altogether, we see that the miracle of $\frac{L}{D} > 10$ results from the facts that

- main lift and drag come from leading edge,
- minimal pressure several times bigger than the maximal pressure on the leading edge (factor 5 say)
- leading edge suction reduces drag (another factor 2 say).

## 15.4 Pressure Distribution

Fig.15.4 shows the computed presssure distribution vs experiments for $aoa = 4, 10$.

## 15.5 Total Lift and Drag

Figs. 15.5-15.5 show lift and drag coefficients as functions of time and total number of mesh points. We see that the coefficients are determined up to a tolerance of less than $.5\%$ for lift and $10\%$ for drag.

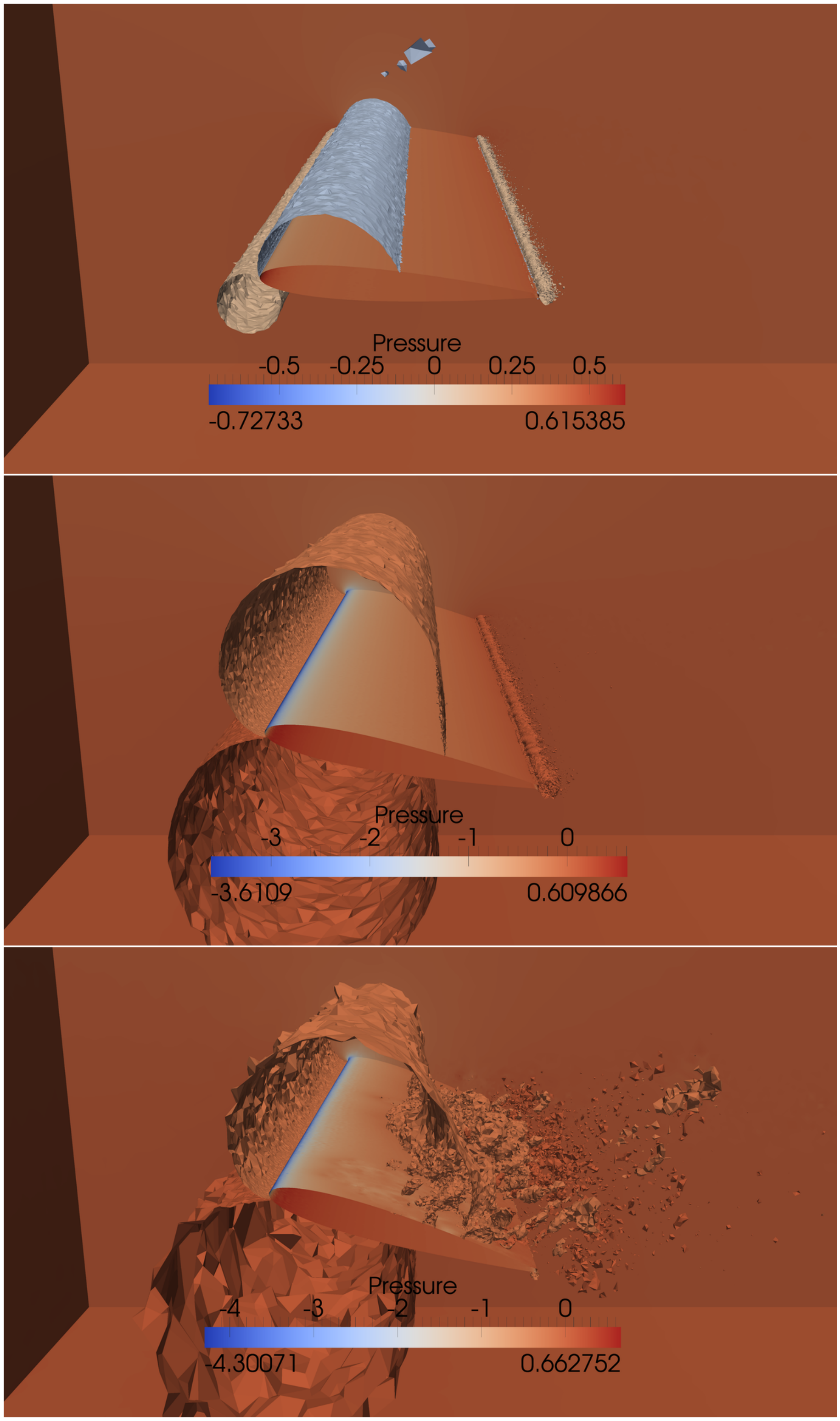



Figure 15.1: Pressure for $aoa = 4, 12, 17$.

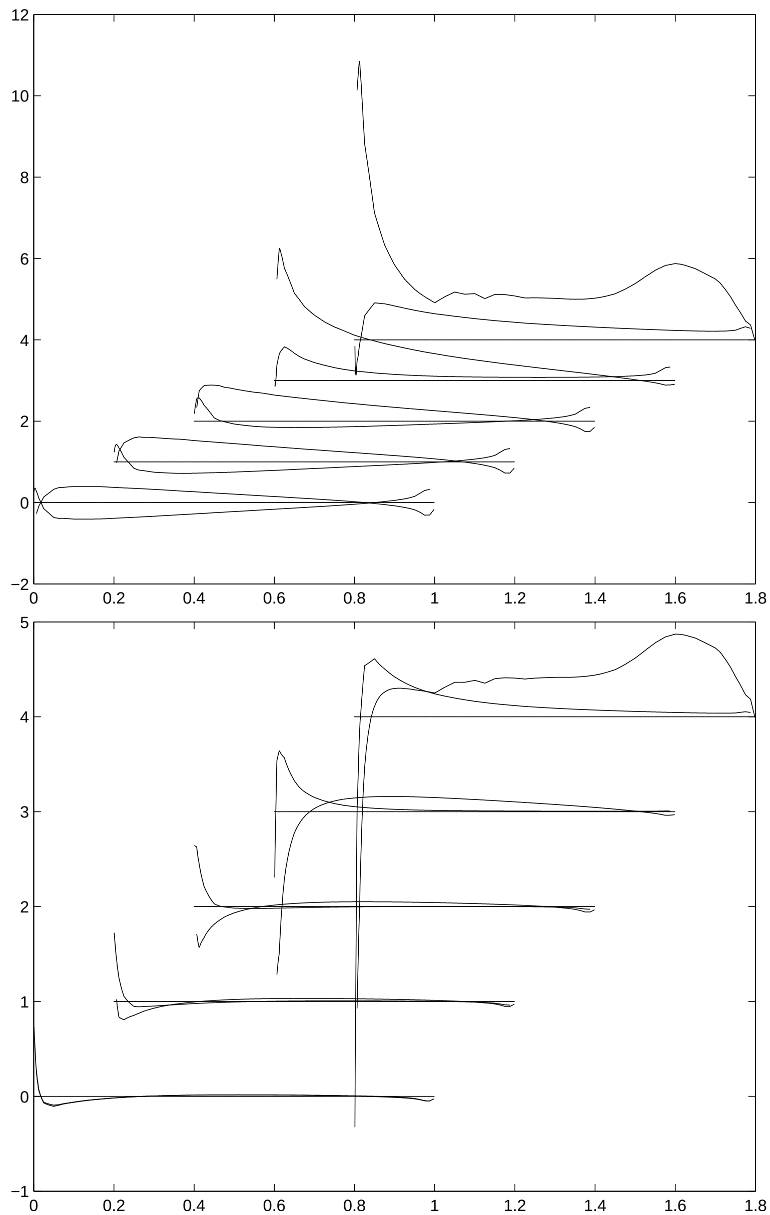



Figure 15.2: G2 computation of normalized local lift force (upper) and drag force (lower) contributions acting along the lower and upper parts of the wing, for angles of attack 0, 2 ,4 ,10 and 18°, each curve translated 0.2 to the right and 1.0 up, with the zero force level indicated for each curve.

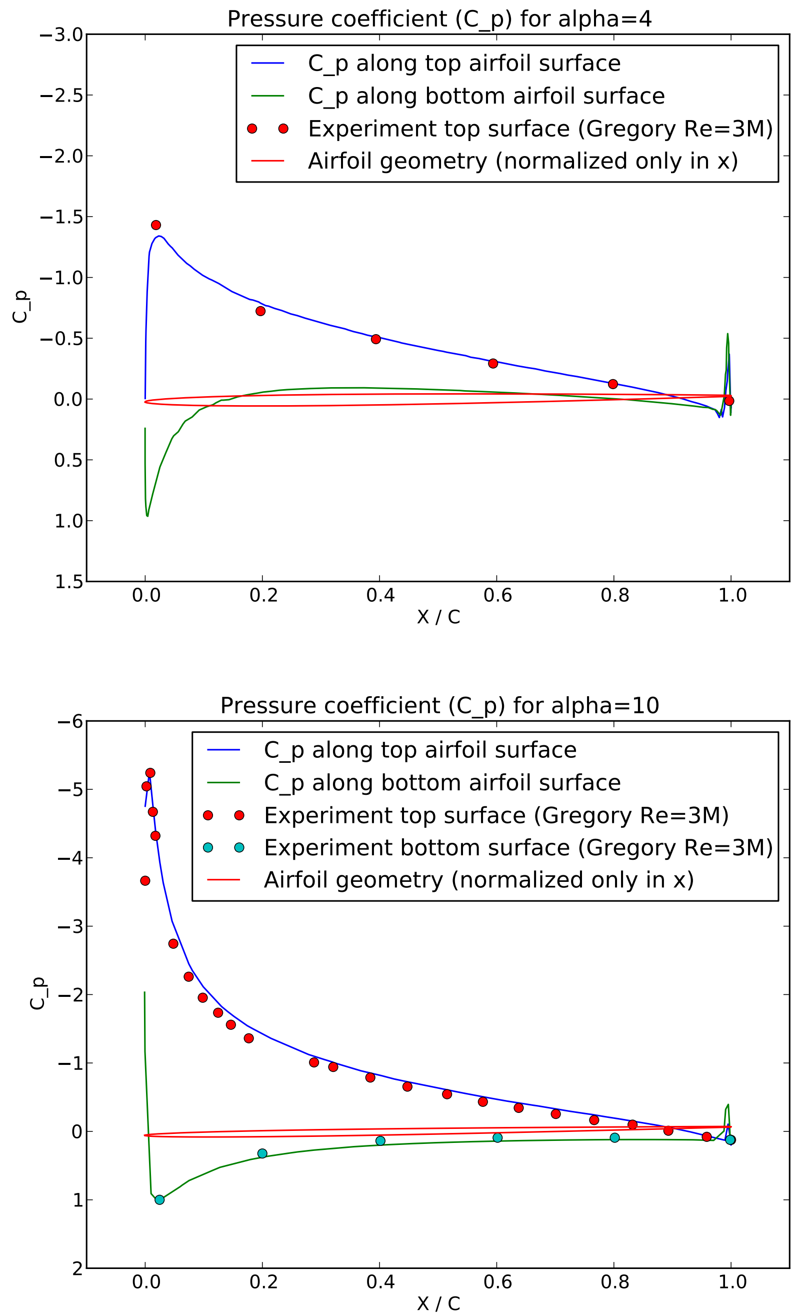
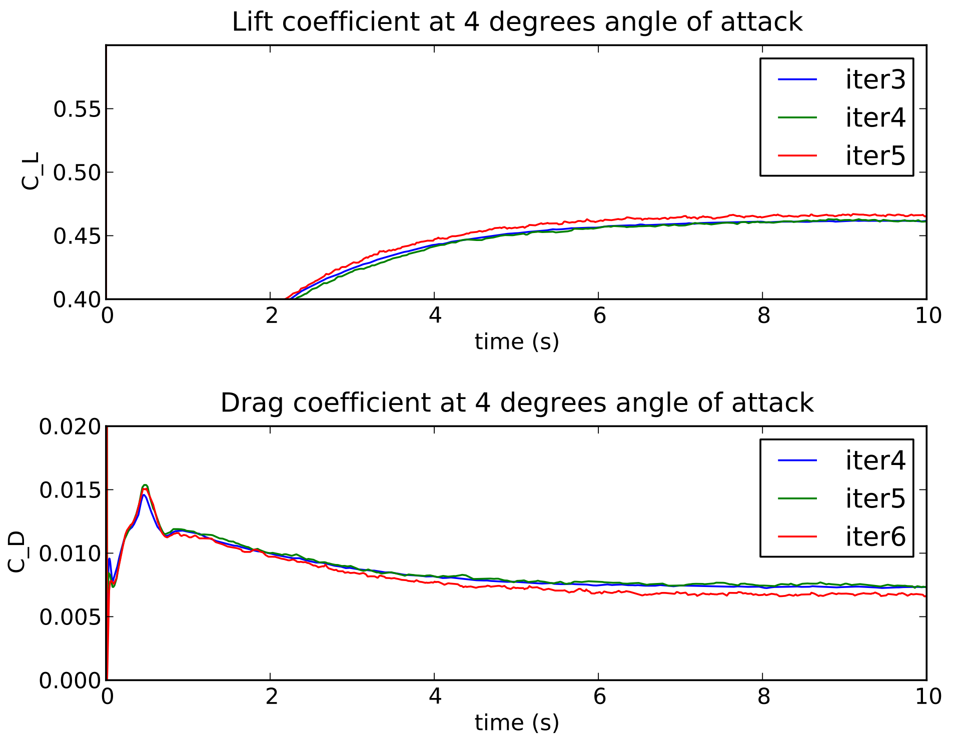
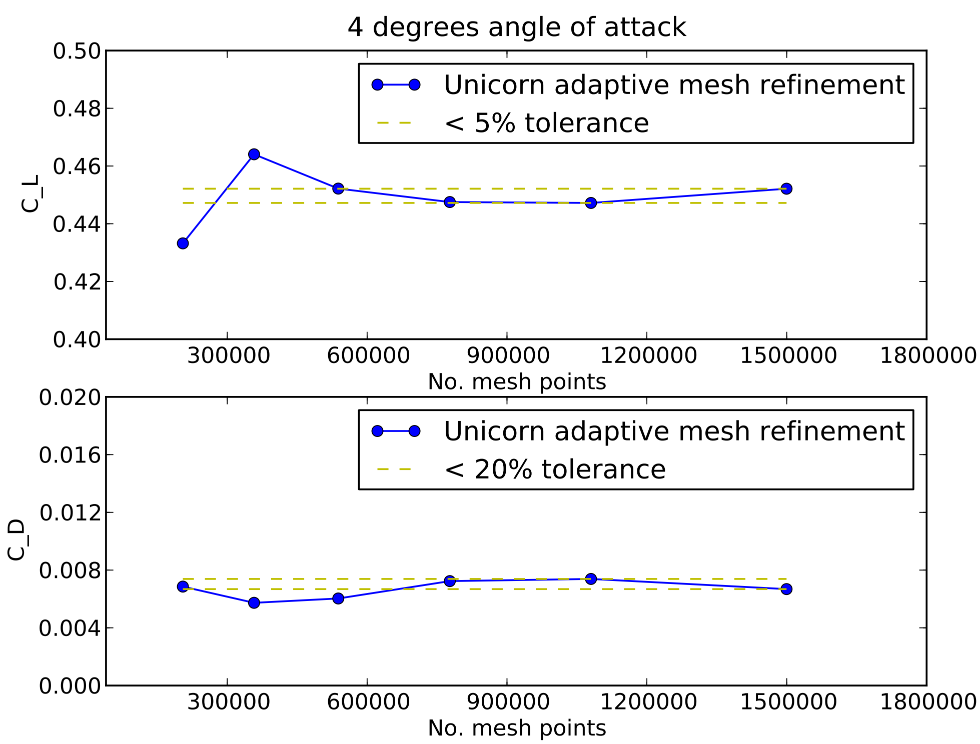
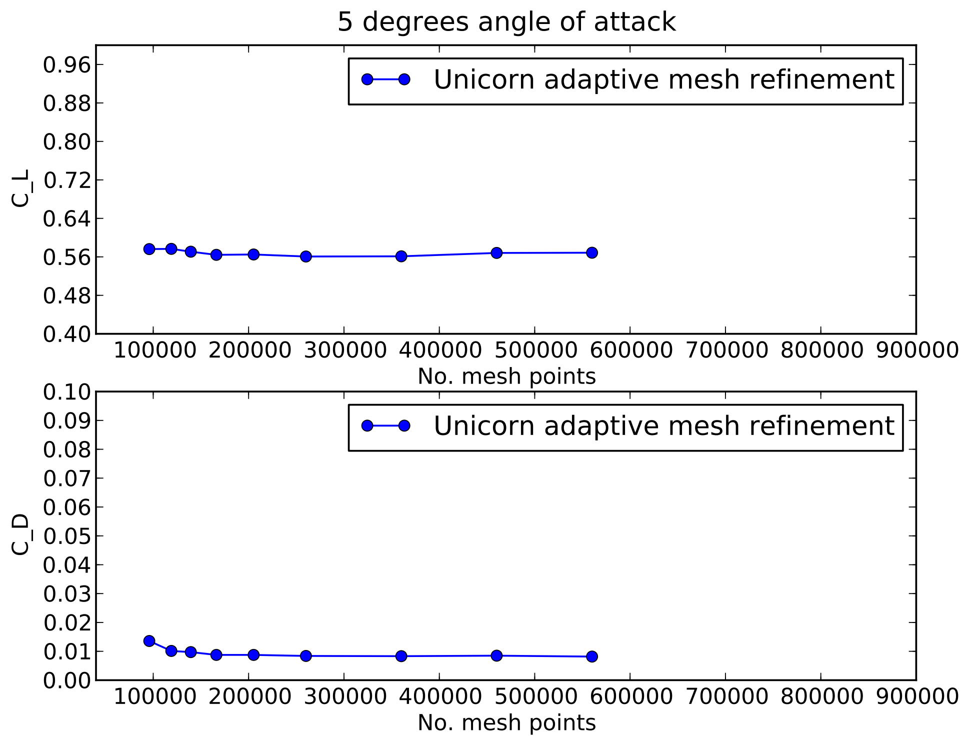
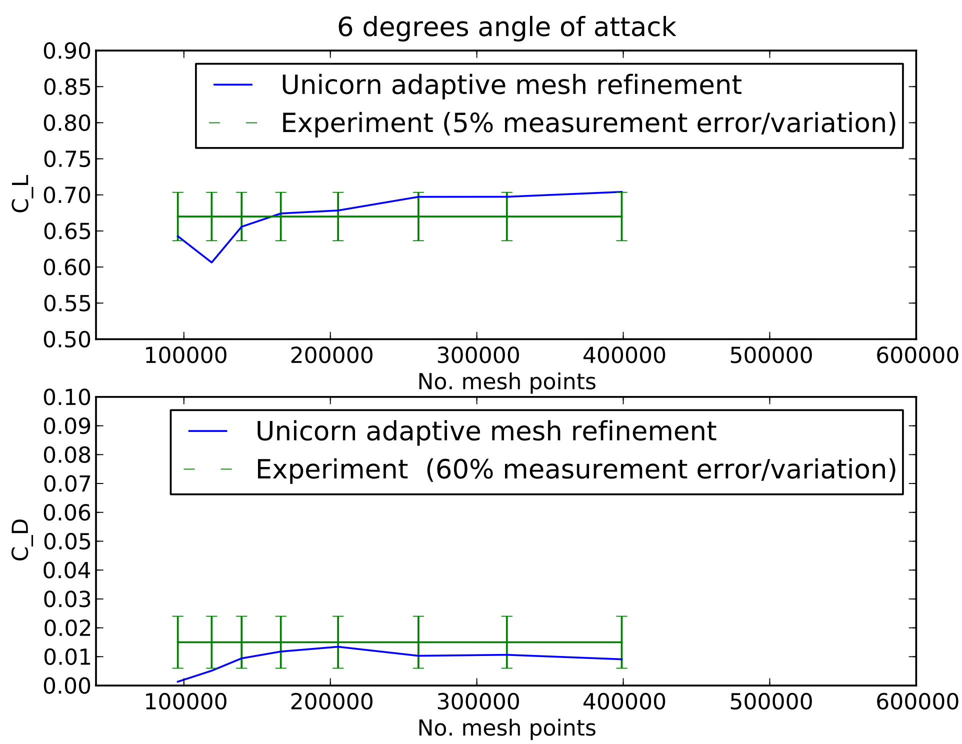


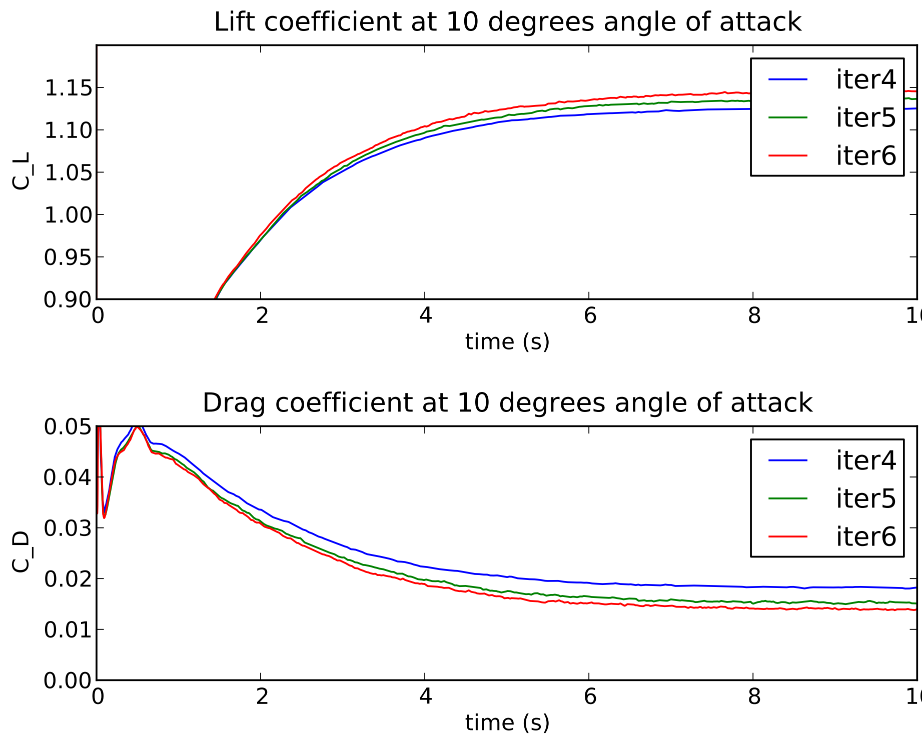

Figure 15.3: Computed pressure distribution vs experiments for $aoa = 4$ and $10$. Notice the pressure distribution at the trailing edge in accordance with observation, without the unphysical high pressure of circulation theory.

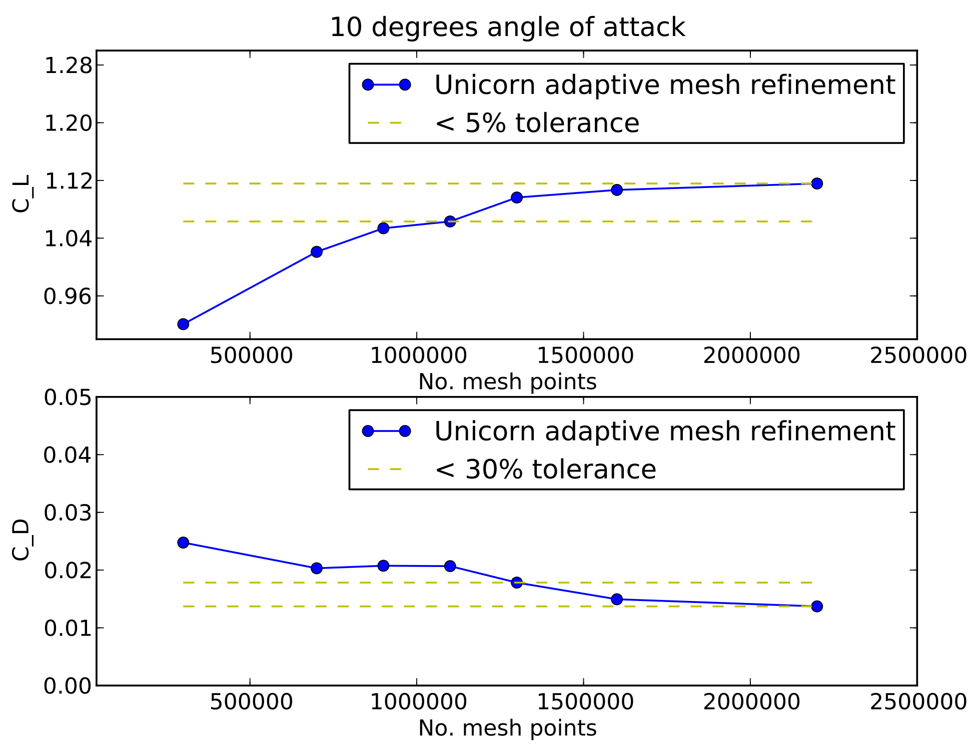

Lift coefficient at 4 degrees angle of attack
C_L
0.55
0.50
0.45
0.40
0
2
4
6
8
10
time (s)
iter3
iter4
iter5
Drag coefficient at 4 degrees angle of attack
C_D
0.020
0.015
0.010
0.005
0.000
0
2
4
6
8
10
time (s)
iter4
iter5
iter6

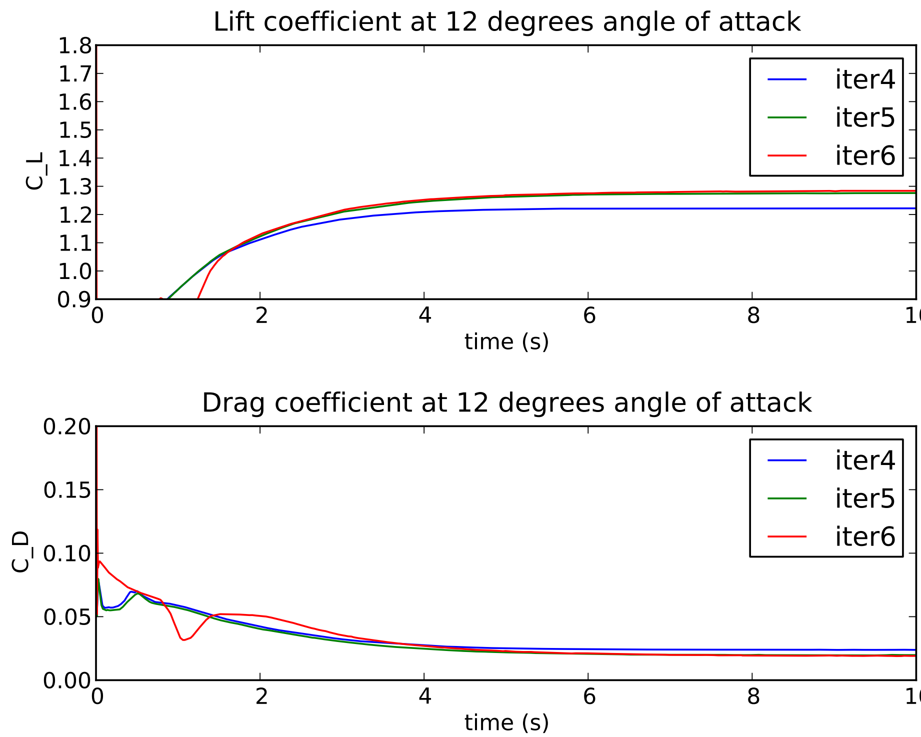

4 degrees angle of attack
C_L
0.50
0.48
0.46
0.44
0.42
0.40
300000
600000
900000
1200000
1500000
1800000
No. mesh points
Unicorn adaptive mesh refinement
< 5% tolerance
C_D
0.020
0.016
0.012
0.008
0.004
0.000
300000
600000
900000
1200000
1500000
1800000
No. mesh points
Unicorn adaptive mesh refinement
< 20% tolerance

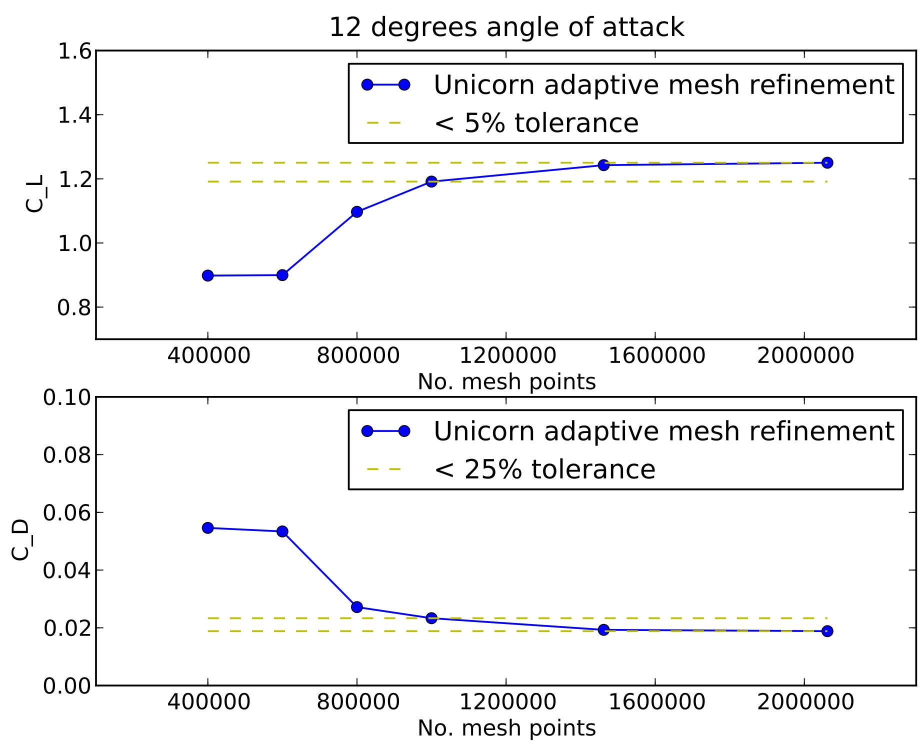

5 degrees angle of attack
Unicorn adaptive mesh refinement
C_L
No. mesh points
Unicorn adaptive mesh refinement
C_D
No. mesh points


6 degrees angle of attack
Unicorn adaptive mesh refinement
Experiment (5% measurement error/variation)
C_L
No. mesh points
Unicorn adaptive mesh refinement
Experiment (60% measurement error/variation)
C_D
No. mesh points


Lift coefficient at 10 degrees angle of attack
C_L
time (s)
iter4
iter5
iter6
Drag coefficient at 10 degrees angle of attack
C_D
time (s)
iter4
iter5
iter6


10 degrees angle of attack
C_L
No. mesh points
Unicorn adaptive mesh refinement
< 5% tolerance
C_D
No. mesh points
Unicorn adaptive mesh refinement
< 30% tolerance


Lift coefficient at 12 degrees angle of attack
C_L
time (s)
iter4
iter5
iter6
Drag coefficient at 12 degrees angle of attack
C_D
time (s)
iter4
iter5
iter6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C_L
No. mesh points
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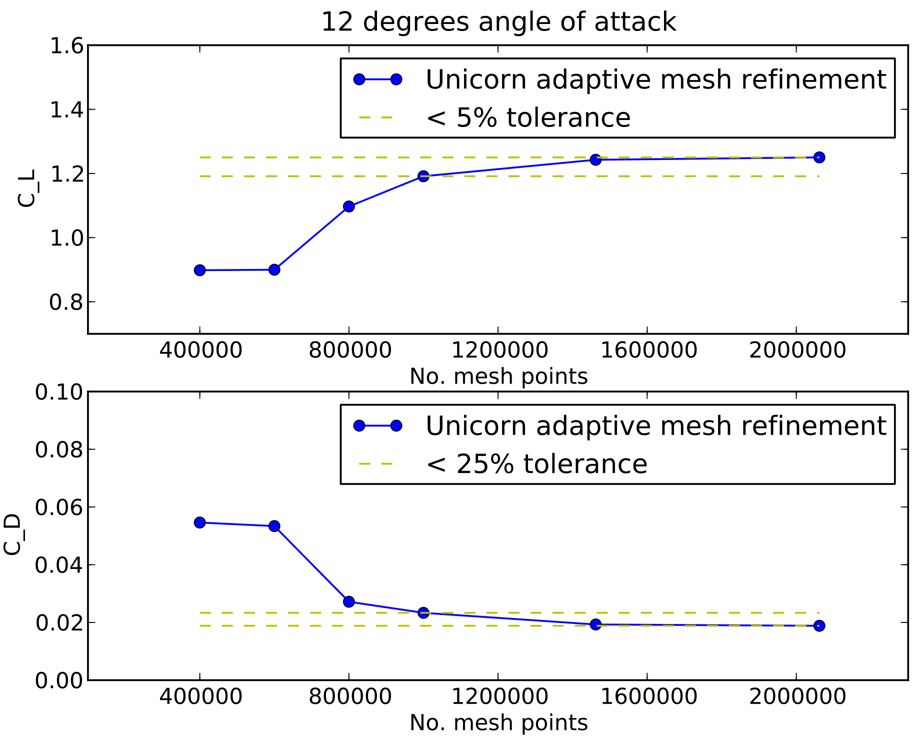
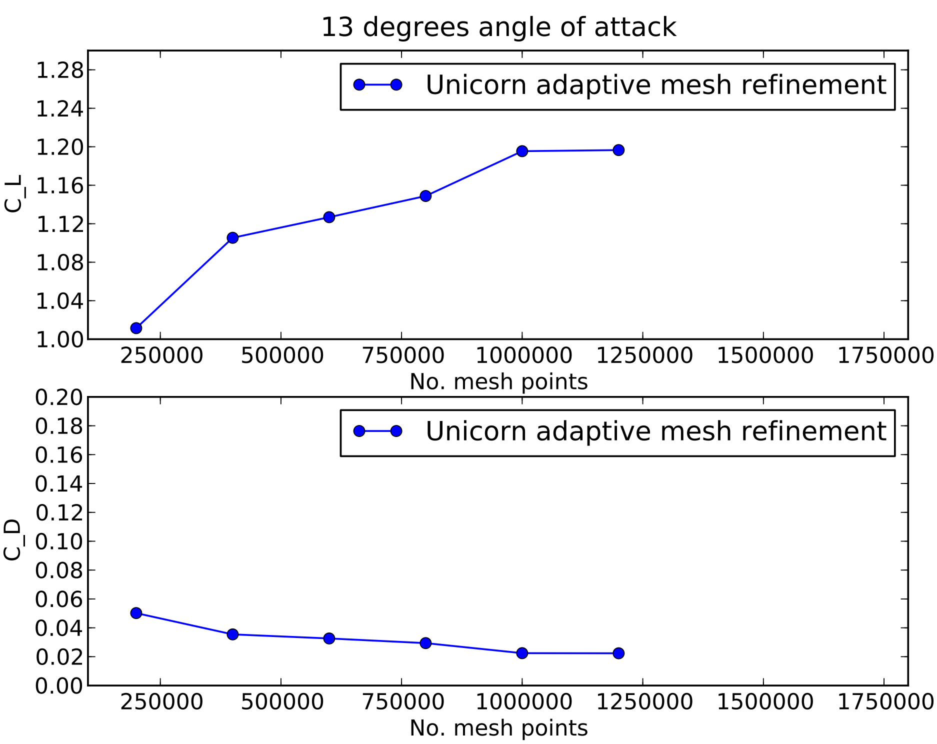
C_D
No. mesh points
Unicorn adaptive mesh refinement
< 25% tolerance


12 degrees angle of attack
Unicorn adaptive mesh refinement
< 5% tolerance
C_L
No. mesh points
Unicorn adaptive mesh refinement
< 25% tolerance
C_D
No. mesh points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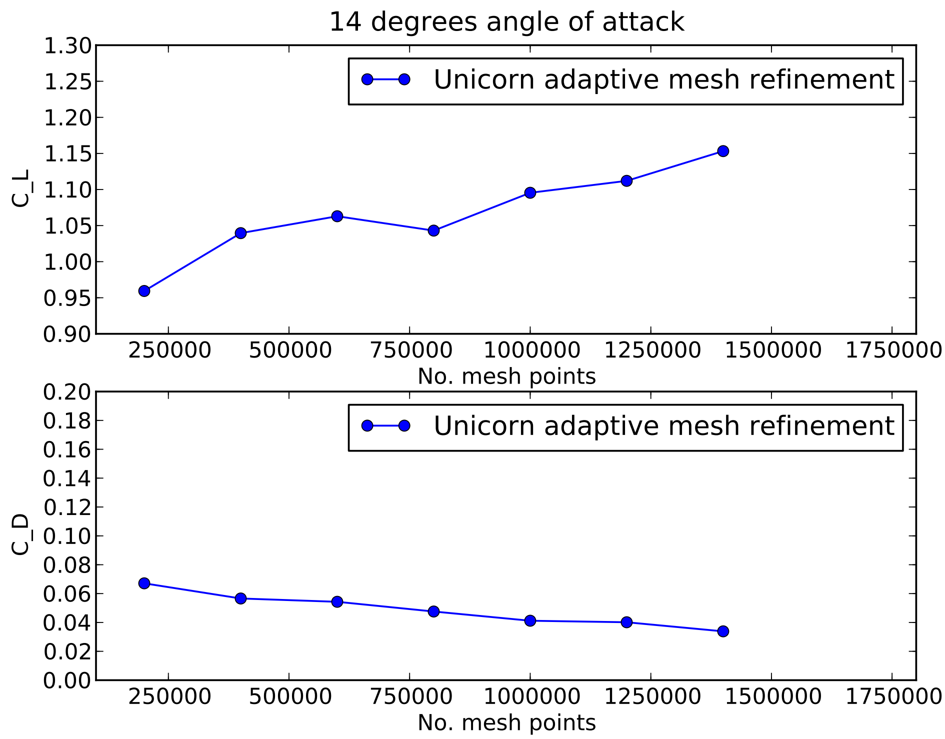
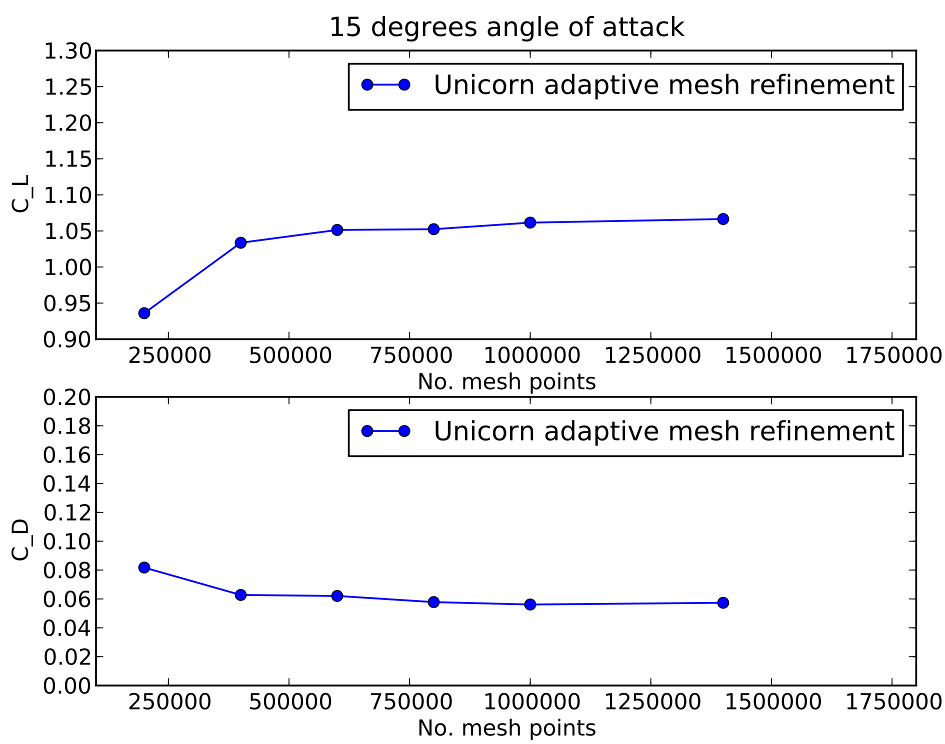
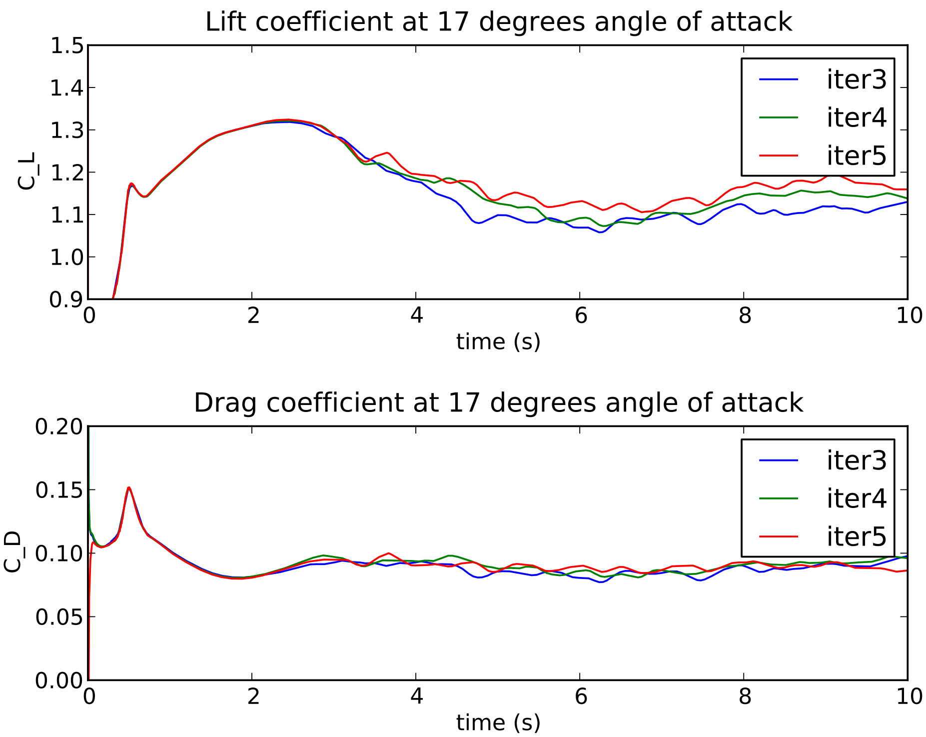
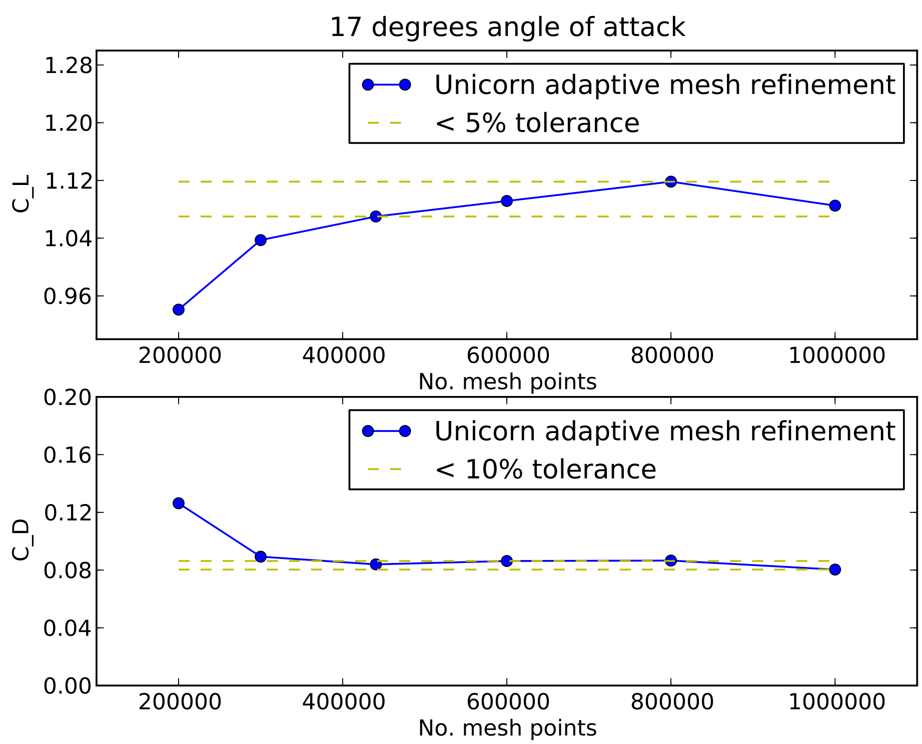
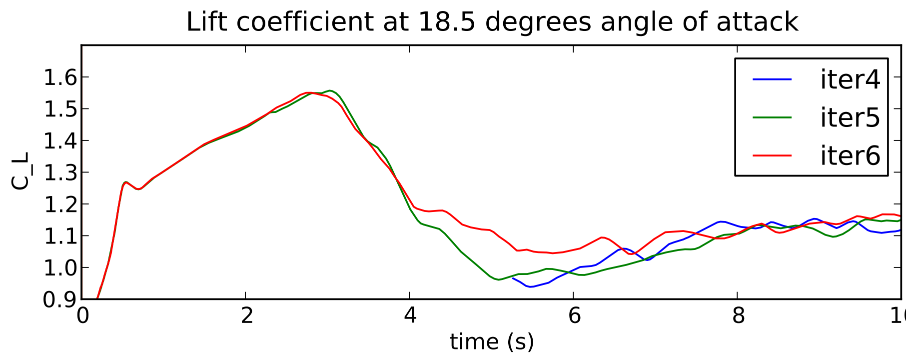
Unicorn adaptive mesh refinement
C_L
No. mesh points
Unicorn adaptive mesh refinement
C_D
No. mesh points

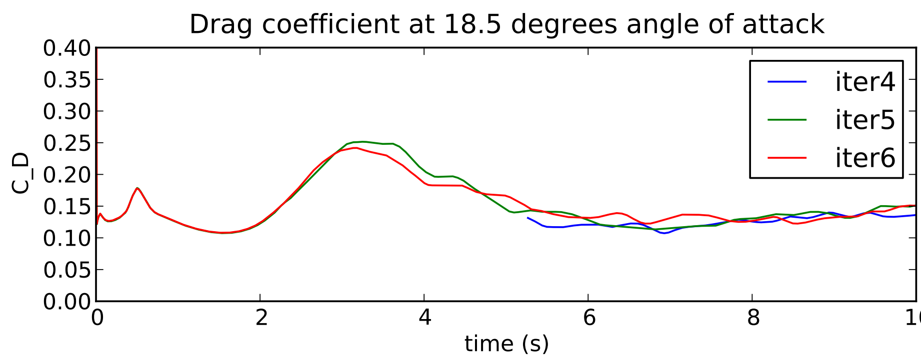

14 degrees angle of attack
Unicorn adaptive mesh refinement
C_L
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
250000
500000
750000
1000000
1250000
1500000
1750000
No. mesh points
Unicorn adaptive mesh refinement
C_D
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
250000
500000
750000
1000000
1250000
1500000
1750000
No. mesh points

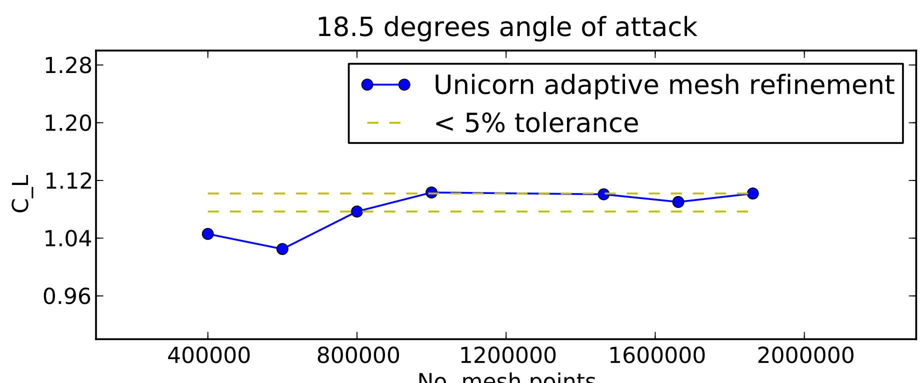

15 degrees angle of attack
Unicorn adaptive mesh refinement
C_L
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
250000
500000
750000
1000000
1250000
1500000
1750000
No. mesh points
Unicorn adaptive mesh refinement
C_D
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
250000
500000
750000
1000000
1250000
1500000
1750000
No. mesh points

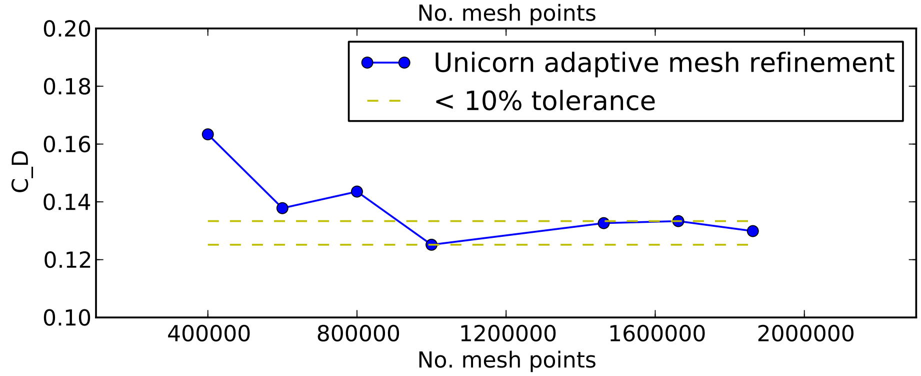

Lift coefficient at 17 degrees angle of attack
C_L
time (s)
iter3
iter4
iter5
Drag coefficient at 17 degrees angle of attack
C_D
time (s)
iter3
iter4
iter5


17 degrees angle of attack
C_L
No. mesh points
Unicorn adaptive mesh refinement
< 5% tolerance
C_D
No. mesh points
Unicorn adaptive mesh refinement
< 10% tolerance


Lift coefficient at 18.5 degrees angle of attack
C_L
time (s)
iter4
iter5
iter6


Drag coefficient at 18.5 degrees angle of attack
C_D
time (s)
iter4
iter5
iter6


18.5 degrees angle of attack
C_L
No. mesh points
Unicorn adaptive mesh refinement
< 5% tolerance


C_D
No. mesh points
Unicorn adaptive mesh refinement
< 10% tolerance

# Chapter 16

# Observing Velocity

> The contribution of skin-frcition to the drag of a cylinder is small...for $Re = 2.2 \times 10^5$ about $2.0\%$. (Gold stein in *Modern Developments in Fluid Dynamics*)

Fig. 16 and 16 show the velocity for $\alpha = 10, 14, 17$. We observe

- high velocity on top/front of the leading edge and low velocity below in conformity with Bernoulli's Principle,
- a wake with low velocity develops as stall is approached,
- well before stall the flow separates smoothly at the trailing edge and not as potential flow at stagnation before the trailing edge,
- the separation moves forward from the trailing edge on the upper surface as stall is approached.

We observe

- 3d rotational slip separation with point stagnation in a zig-zag pattern
- generating rolls of streamwise vorticity with low pressure.

Fig. 16.5-16.9 also show the zig-zag velocity separation pattern with point stagnation.

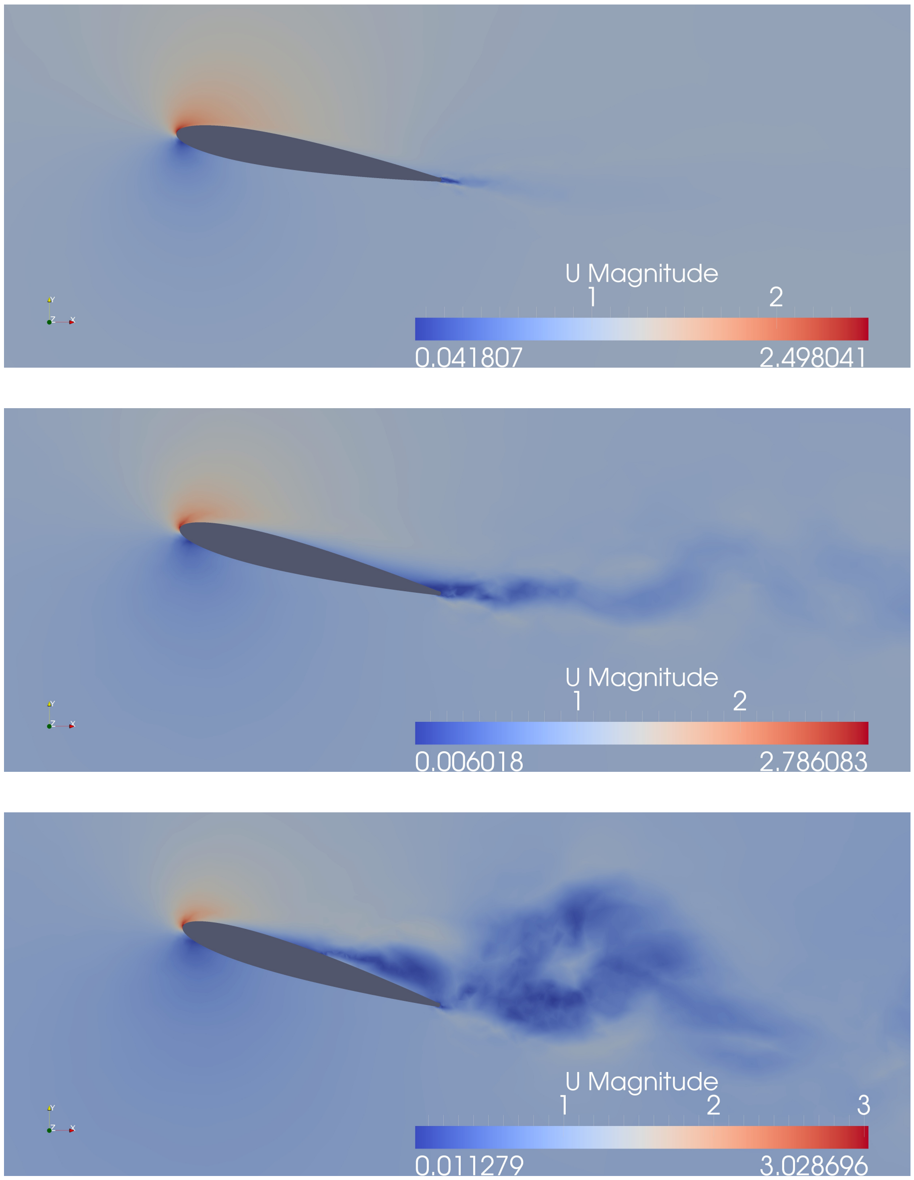



Figure 16.1: Velocity magnitude around the airfoil for $\alpha = 10$ (top), 14 (center) and 17 (bottom).

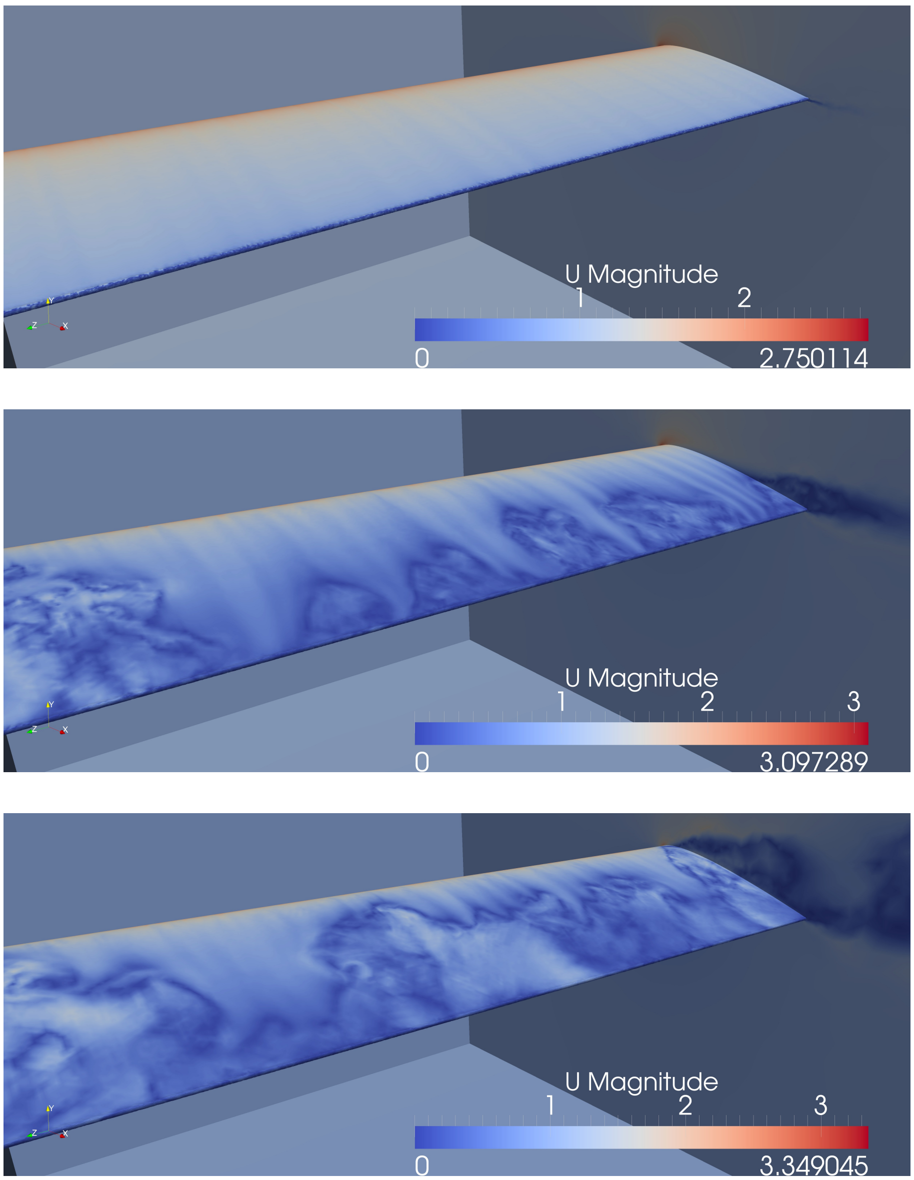



Figure 16.2: Velocity magnitude on the airfoil surface for $\alpha = 10$ (top), 14 (center) and 17 (bottom) showing that separation pattern moves up the airfoil with increasing $\alpha$ towards stall.

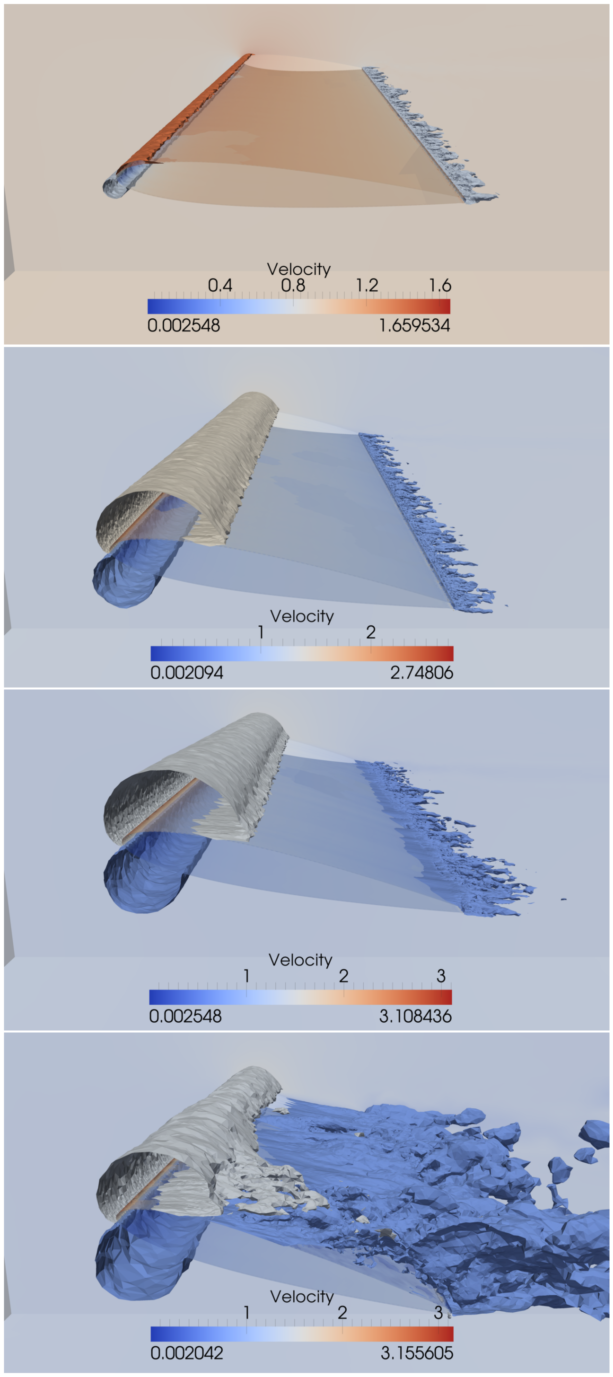



Figure 16.3: Velocity for $aoa = 4, 10, 12, 17$.

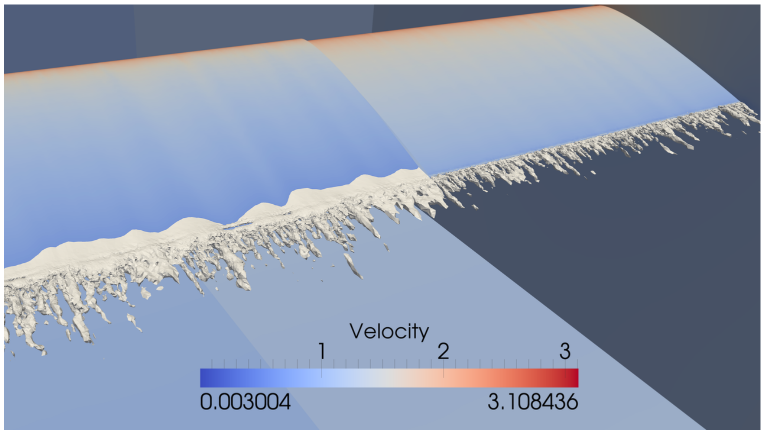



Figure 16.4: Velocity $aoa = 10$.

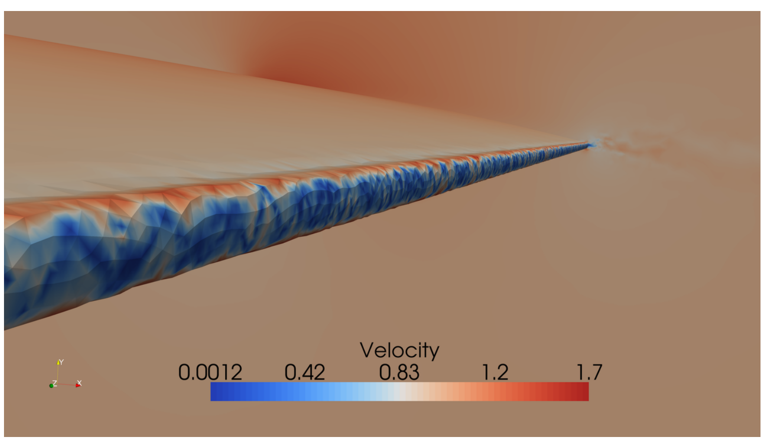



Figure 16.5: Trailing edge zig-zag velocity pattern $aoa = 04$.

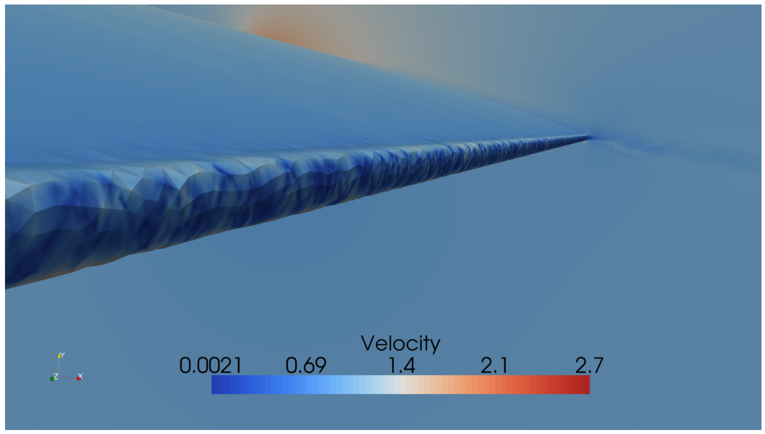



Figure 16.6: Trailing edge zig-zag velocity $aoa = 10$.

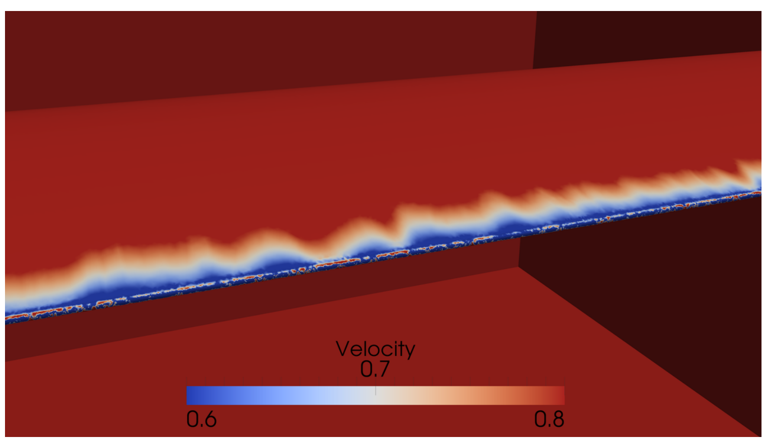



Figure 16.7: Zigzag velocity $aoa = 12$.

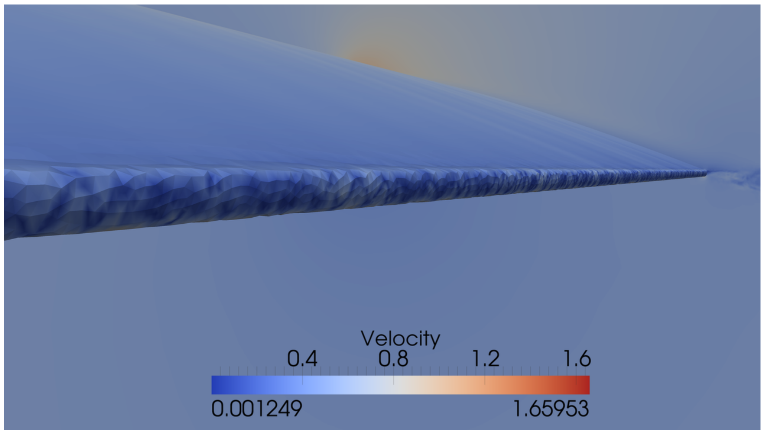



Figure 16.8: Trailing edge zig-zag velocity $aoa = 12$.

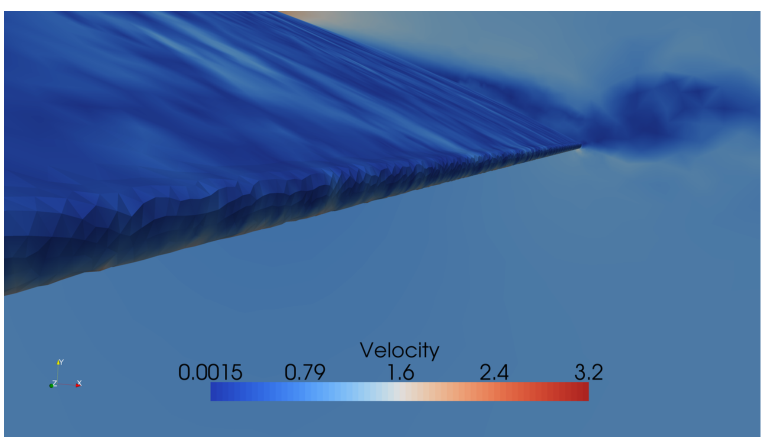



Figure 16.9: Trailing edge zig-zag velocity $aoa = 17$.

# Chapter 17

# Observing Vorticity

> The lift and drag coefficients of an airfoil are usually obtained from wind-tunnel tests on models of finite aspect ratio (usually 6 to 1) and rectangular plan form: coefficients for this type of flow can be predicted by the vortex theory of airfoils from those measured on an airfoil of finite span...The experimental values of surface pressure lie close to the theoretical Kutta-Joukowski formula except for a short part at the tail. (Goldstein in *Modern Developments in Fluid Dynamics*)

Fig. 17.1-17.3 shows side view of pressure, velocity and top view of vorticity for $\alpha = 2, 4, 8, 10, 14, 18$. We observe

- development of rolls of streamwise vorticity at separation at the trailing edge as illustrated in Fig. 10.1,
- pressure is small inside rolls of streamwise vorticity generating som drag,
- separation moves from the trailing edge to upper surface as $\alpha$ approches stall.

Below we will in more detail study the dynamics of the generation of streamwise vorticity as the generic structure of rotational slip separation with point stagnation.

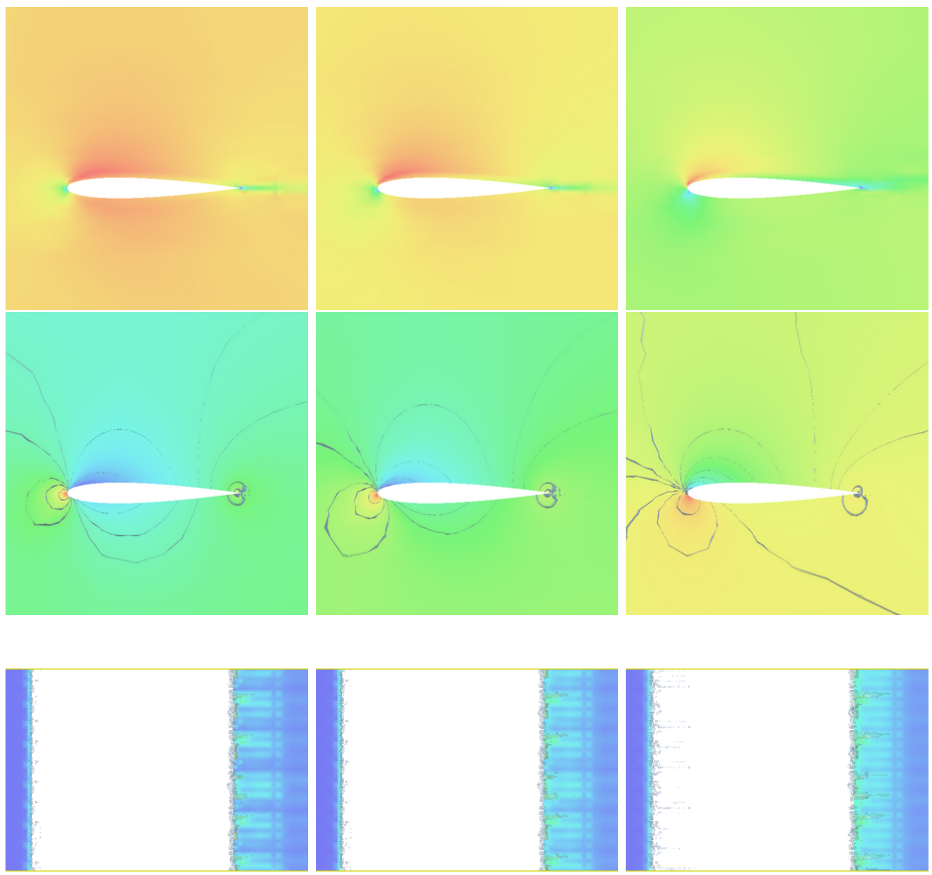


Figure 17.1: G2 computation of velocity magnitude (upper), pressure (middle), and non-transversal vorticity (lower), for angles of attack 2, 4, and 8° (from left to right). Notice in particular the rolls of streamwise vorticity at separation.

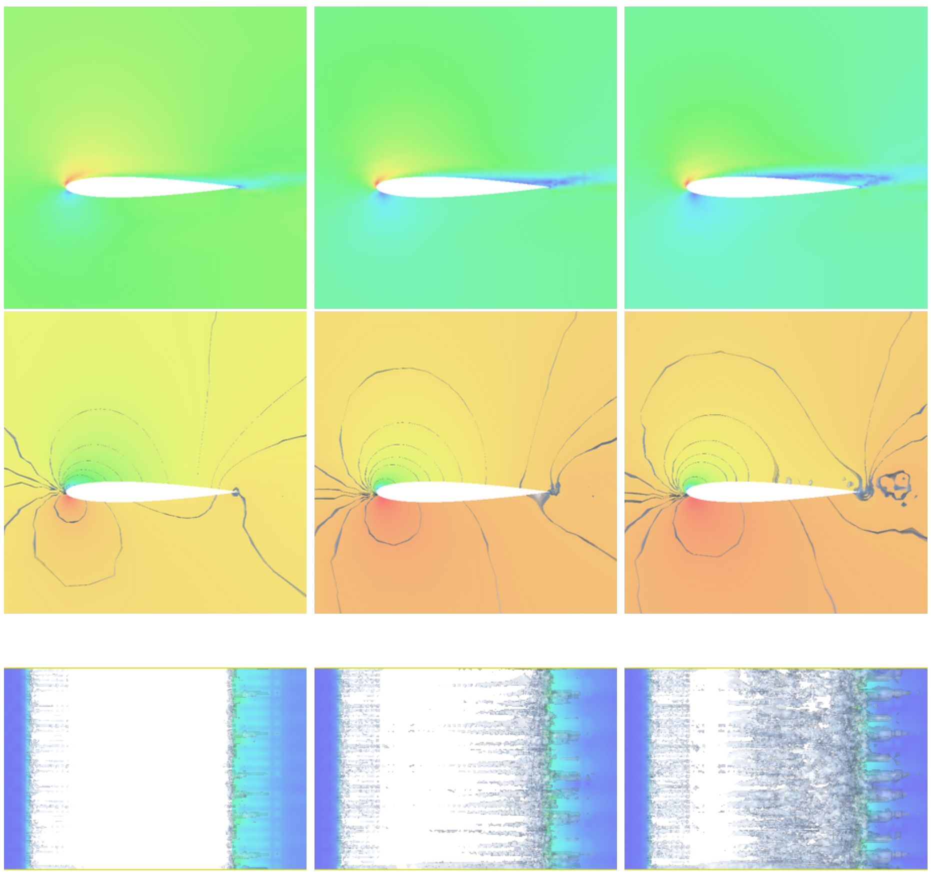


Figure 17.2: G2 computation of velocity magnitude (upper), pressure (middle), and topview of non-transversal vorticity (lower), for angles of attack 10, 14, and 18° (from left to right). Notice in particular the rolls of streamwise vorticity at separation.

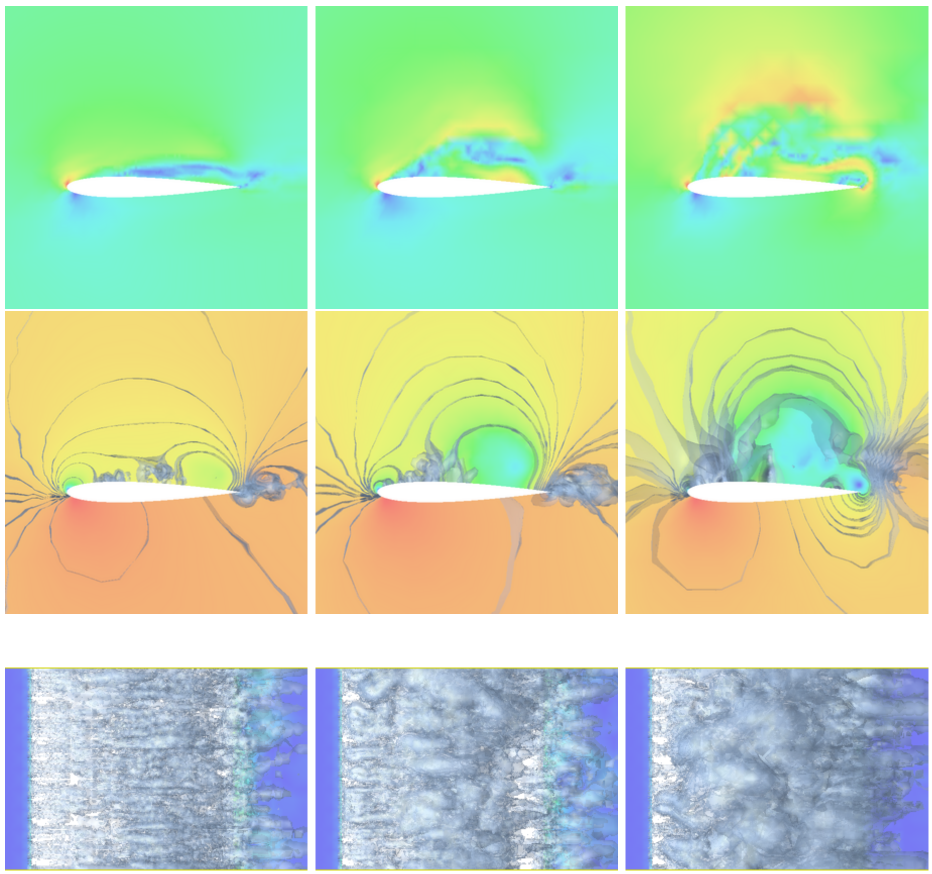


Figure 17.3: G2 computation of velocity magnitude (upper), pressure (middle), and non-transversal vorticity (lower), for angles of attack 20, 22, and 24° (from left to right).

# Chapter 18

# Summary of Observation

> On all airfoils the first sign of an approaching stall is the formation of a thick region of low pressure over the upper surface near the trailing edge, but without definite separation of the stream from the surface. (Goldstein in *Modern Developments in Fluid Dynamics*)

## Phase 1: $0 \leq \alpha \leq 8$

At zero angle of attack with zero lift there is high pressure at the leading edge and equal low pressures on the upper and lower crests of the wing because the flow is essentially potential and thus satisfies Bernouilli's law of high/low pressure where velocity is low/high. The drag is about 0.01 and results from rolls of low-pressure streamwise vorticity attaching to the trailing edge. As $\alpha$ increases the low pressure below gets depleted as the incoming flow becomes parallel to the lower surface at the trailing edge for $\alpha = 6$, while the low pressure above intenisfies and moves towards the leading edge. The streamwise vortices at the trailing edge essentially stay constant in strength but gradually shift attachement towards the upper surface. The high pressure at the leading edge moves somewhat down, but contributes little to lift. Drag increases only slowly because of negative drag at the leading edge.

## Phase 2: $8 \leq \alpha \leq 12$

The low pressure on top of the leading edge intensifies to create a normal gradient preventing separation, and thus creates lift by suction peaking on top of the leading edge. The slip boundary condition prevents separation and downwash is created with the help of the low-pressure wake of streamwise vorticity at rear separation. The high pressure at the leading edge moves further down and the pressure below increases slowly, contributing to the main lift coming from suction above. The net drag from the upper surface is close to zero because of the negative drag at the leading edge, known as *leading edge suction*, while the drag from the lower surface increases (linearly) with the angle of the incoming flow, with somewhat increased but still small drag slope. This explains why the line to a flying kite can be almost vertical even in strong wind, and that a thick wing can have less drag than a thin.

### Phase 3: $12 \leq \alpha \leq 14$

Beginning stall with lift non-increasing while drag is increasing super-linearly.

### Phase 4: $14 \leq \alpha \leq 16$

Stall with rapidly increasing drag.

## 18.1 Summary Lift and Drag

The lift generation in Phase 1 and 3 can rather easily be envisioned, while both the lift and drag in Phase 2 results from a (fortunate) intricate interplay of stability and instability of potential flow: The main lift comes from upper surface suction arising from a turbulent boundary layer with small skin friction combined with rear separation instability generating low-pressure streamwise vorticity, while the drag is kept small by negative drag from the leading edge.

We can thus summarize as follows:

- Substantial lift from suction on upper surface.
- Small drag from suction on leading edge.

# Chapter 19

# Computation vs Experiments

> The drag of a stream-line body is mainly, though not entirely, due to skin-friction...The drag of a bluff obstacle, on the other hand, is mainly the resultant of normal pressure. (Goldstein in *Modern Developments in Fluid Dynamics*)

> When we consider fluids of small viscosity, we shall have to explain first why inviscid fluid theory ever gives approximately correct results, since it involves slip over the surface, how circulation and vorticity make their appearance; how to calculate skin-friction; how to calculate the circulation and the lift for a stream-line body without a salient edge; and how to calculate the pressure distribution, the drag, and the lift for a bluff body. We may state at once that we can make very little progress indeed with the last calculation beyond what has already been mentioned in connection with the theory of the vortex-street...(Goldstein in *Modern Developments in Fluid Dynamics*)

The computations shown are obtained by the finite element solver $G2$ with automatic mesh adaption from a posteriori error estimation of lift and drag by based on sensitivity information obtained by solving a linearized dual Navier-Stokes problem.

## 19.1 Data for Experiments

**Ladson 1**

$Re = 8.95e6, M = 0.15$ Grit level 60W (wraparound). grit acts to trip the boundary layer.

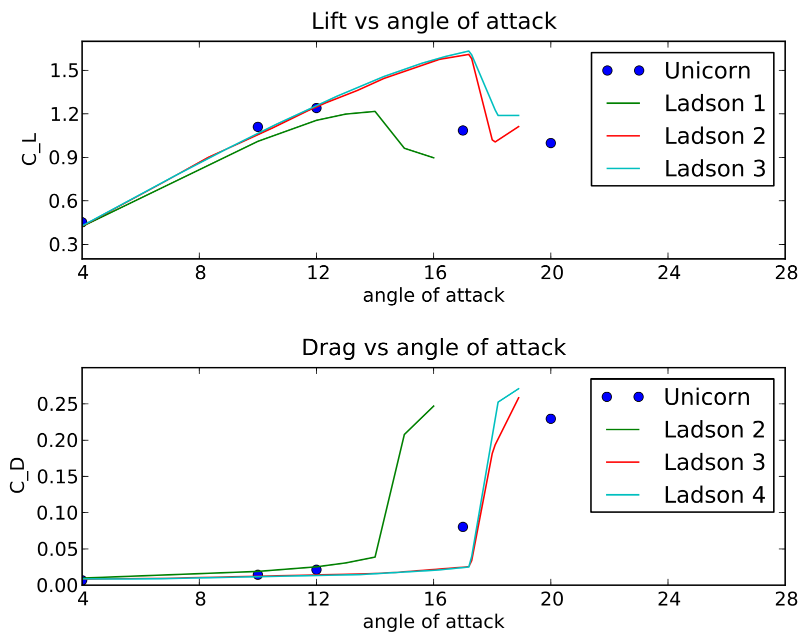



Figure 19.1: Comparison between computed lift and drag coefficient with experimentsfor $aoa = 10$.

**Ladson 2**
: $Re = 6.00e6$, $M = 0.15$ Grit level 60W (wraparound).

**Ladson 3**
: $Re = 8.95e6$, $M = 0.30$ Grit level 120W (wraparound).

**Gregory/O'Reilly**
: $Re = 2.88e6$, $M = 0.16$

where the wraparound grit in the Ladson experiments acts to trip the boundary layer.

## 19.2 Comparing Computation with Experiment

Fig. 19.1 shows that G2 computations lie within variations in experiments [55, 58] as a central message of this book.

In fact, computations may well be more precise than experiment, because G2 with slip models a very large Reynolds number representative of e.g. a jumbojet, which cannot be attained in wind tunnel where scale model are used. Upscaling of test results is cumbersome because boundary layers do not scale. This means that computations may be closer to reality than wind tunnel experiments.

Of particular importance is the maximal lift coefficient, which cannot be predicted by Kutta-Zhukovsky nor in model experiments.

# Chapter 20

# Preparing Understanding

> The only natural phenomena that we can pre- calculate and understand in all their observable details, are those for which small errors in the input of the calculation bring only small errors in the final result. As soon as unstable equilibrium interferes, this condition is no longer met. Hence chance still exists in our horizon; but in reality chance only is a way of expressing the defective character of our knowledge and the roughness of our combining power. (Helmholtz 1876)

## 20.1 New Resolution of D'Alembert's Paradox

We have said that the new flight theory comes out of a *new resolution* of D'Alembert's Paradox [103, 104, 102]. The new resolution is based on a stability analysis showing that zero-lift/drag potential flow is *unstable* and in both computation and reality is replaced by turbulent flow with both lift and drag.

The new resolution is fundamentally different from the classical official resolution attributed to Prandtl [120, 124, 137], which disqualifies potential flow because it satisfies a *slip* boundary condition allowing fluid particles to glide along the boundary without friction force, and does not satisfy a *no-slip* boundary condition requiring the fluid particles to stick to the boundary with zero relative velocity and connect to the free-stream flow through a thin *boundary layer*, as demanded by Prandtl.

## 20.2 Slip/Small Friction Boundary Conditions

In contrast to Prandtl, we complement in the new theory Navier-Stokes equations with a *friction force boundary condition* for *tangential forces* on the boundary with a *small friction coefficient* as a model of the *small skin friction* resulting from a *turbulent boundary layer* of *slightly viscous flow*. In the limit of zero boundary friction this becomes a slip boundary condition, which means that potential flow can be seen as a solution of the Navier-Stokes equations subject to a small perturbation from small viscous stresses. In the new theory we then disqualify potential flow because it is unstable, that is on physical grounds, and not as Prandtl on formal grounds because it does not satisfy no-slip boundary conditions.

The Navier-Stokes equations can be complemented by no-slip or slip/friction boundary conditions, just like Poisson's equation can be complemented by Dirichlet or Neumann boundary conditions. The choice of boundary conditions depends on which data is available. In the aerodynamics of larger birds and airplanes, the skin friction is small and can be approximated by zero friction or a slip boundary condition.

For small insects viscous effects become important, which changes the physics of flight and makes gliding flight impossible. An albatross is a very good glider, while a fruit fly cannot glide at all in the syrup-like air it meets and can only move by using some form of paddling.

## 20.3 Computable + Correct = Secret

We solve the Navier-Stokes equations (NS) with slip/friction boundary condition using an *adaptive stabilized finite element method* with *duality-based a posteriori error control* referred to as *General Galerkin* or *G2* presented in detail in [103] and available in executable open source form from [132]. The stabilization in G2 acts as an *automatic computational turbulence model*, and the only input is the geometry of the wing. We thus find by computation that lift is not connected to circulation in contradiction to Kutta-Zhukovsky's theory and that the curse of Prandtl's laminar boundary layer theory (also questioned in [92, 93, 72]) can be circumvented. Altogether, we show in this book that *ab initio* computational fluid mechanics opens new possibilities of flight simulation ready to be explored and utilized.

- NS with no-slip: uncomputable: hides the secret,

- NS with slip: computable: reveals the secret.

## 20.4 No Lift without Drag

The zero-drag of potential flow has been (and still is) leading aerodynamicis to search for wings with lift but without drag [3], which we have seen is not a feature of Navier-Stokes solutions, and thus is unrealistic.

# Part III

# Solving Navier-Stokes

# Chapter 21

# Navier-Stokes Equations

> Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations. (Clay Mathematics Institute Millennium Problem [106])

> I have always inclined to the belief that the motion of a perfect incompressible liquid, primitively at rest, about a solid which continually progressed, was unstable.(Stokes 1858).

## 21.1 Conservation of Mass, Momentum and Energy

The basic mathematical model of fluid mechanics takes the form of the *Navier-Stokes equations* expressing conservation of mass, momentum and energy of a viscous fluid in the conservation variables of density, momentum and energy with the viscosity as a given coefficient. For an incompressible fluid the equations can be formulated in terms velocity and pressure, referred to as the *incompressible Navier-Stokes equations*, with a decoupled energy equation.

The fluid mechanics of subsonic flight is modeled by the incompressible Navier-Stokes equations for a slightly viscous fluid, which is the focus of this book. The *Reynolds number* $Re = \frac{UL}{\nu}$ where $U$ is a characteristic fluid velocity, $L$ a characteristic length and $\nu$ the fluid viscosity, is used to identify the high Reynolds number flow occuring in aerodynamics with $Re$ of size $10^8$ for large airplanes.

The incompressible Navier-Stokes equations are complemented by initial values for velocity, and boundary conditions specifying either velocities or forces on the boundary. A *no-slip* boundary condition sets the fluid velocity to zero on the boundary, while a *slip* boundary condition sets the velocity normal to the boundary to zero together with the tangential (friction) force. The slip condition is a limit case of a combined normal velocity-tangential stress boundary condition with the tangential stress set to zero as a model of zero skin friction.

The Navier-Stokes equations for an incompressible fluid of unit density with *small viscosity* $\nu > 0$ and *small skin friction* $\beta \geq 0$ filling a volume $\Omega$ in $\mathbb{R}^3$ surrounding a solid body with boundary $\Gamma$ over a time interval $I = [0, T]$, read as follows: Find the velocity $u = (u_1, u_2, u_3)$ and pressure $p$ depending on $(x, t) \in \Omega \cup \Gamma \times I$, such that

$$\begin{array}{rcll} \dot{u} + (u \cdot \nabla)u + \nabla p - \nabla \cdot \sigma & = & f & \text{in } \Omega \times I, \\ \nabla \cdot u & = & 0 & \text{in } \Omega \times I, \\ u_n & = & g & \text{on } \Gamma \times I, \\ \sigma_s & = & \beta u_s & \text{on } \Gamma \times I, \\ u(\cdot, 0) & = & u^0 & \text{in } \Omega, \end{array} \tag{21.1}$$

where $u_n$ is the fluid velocity normal to $\Gamma$, $u_s$ is the tangential velocity, $\sigma = 2\nu\epsilon(u)$ is the viscous (shear) stress with $\epsilon(u)$ the usual velocity strain, $\sigma_s$ is the tangential stress, $f$ is a given volume force, $g$ is a given inflow/outflow velocity with $g = 0$ on a non-penetrable boundary, and $u^0$ is a given initial condition.

We notice the skin friction boundary condition coupling the tangential stress $\sigma_s$ to the tangential velocity $u_s$ with the friction coefficient $\beta$ with $\beta = 0$ for slip, and $\beta >> 1$ for no-slip. We note that $\beta$ is related to the standard *skin friction coefficient* $c_f = \frac{2\tau}{U^2}$ with $\tau$ the tangential stress per unit area, by the relation $\beta = \frac{U}{2}c_f$. In particular, $\beta$ tends to zero with $c_f$ (if $U$ stays bounded).

Prandtl insisted on using a no-slip velocity boundary condition with $u_s = 0$ on $\Gamma$, because his resolution of D'Alembert's Paradox hinged on discriminating potential flow by this condition. On the oher hand, with the new resolution of D'Alembert's Paradox, relying instead on instability of potential flow, we are free to choose instead a friction force boundary condition, if data is available. Now, experiments show [124, 28] that the skin friction coefficient decreases with increasing Reynolds number $Re$ as $c_f \sim Re^{-0.2}$, so that $c_f \approx 0.0005$ for $Re = 10^{10}$ and $c_f \approx 0.007$ for $Re = 10^5$. Accordingly we model a turbulent boundary layer by a friction boundary condition with a friction parameter $\beta \approx 0.03URe^{-0.2}$. For very large Reynolds numbers, we can effectively use $\beta = 0$ in G2 computation corresponding to slip boundary conditions.

## 21.2 Wellposedness and Clay Millennium Problem

The mathematician J. Hadamard identified in 1902 [95] *wellposedness* as a necessary requirement of a solution of a mathematical model, such as the Navier-Stokes equations, in order to have physical relevance: Only *wellposed* solutions which are suitably stable in the sense that small perturbations have small effects when properly measured, have physical significance as observable pheonomena.

Leray's requirement of wellposedness is absolutely fundamental, but the question whether solutions of the Navier-Stokes equations are wellposed, has not been studied because of lack mathematical techniques for quantitative analysis. This is evidenced in the formulation of the Clay Millennium Prize Problem on the Navier-Stokes equations excluding wellposedness [106, 102].

The mathematical Garret Birkhoff became heavily criticized for posing this question in [92], which stopped him from further studies. The first step towards resolution of D'Alembert's Paradox and the mathematical secret of flight is thus to pose the question if potential flow is wellposed, and then to realize that it is not. It took 256 years to take these steps.

## 21.3 Laminar vs Turbulent Boundary Layer

As developed in more detail in [134], we make a distinction between laminar (boundary layer) separation modeled by no-slip and turbulent (boundary layer) separation modeled by slip/small friction. Note that laminar separation cannot be modeled by slip, since a laminar boundary layer needs to be resolved with no-slip to get correct (early) separation. On the other hand, as will be seen below, in turbulent (but not in laminar) flow the interior turbulence dominates the skin friction turbulence indicating that the effect of a turbulent boundary layer can be modeled by slip/small friction, which can be justified by an posteriori sensitivity analysis as shown in [134].

We thus assume that the boundary layer is turbulent and is modeled by slip/small friction, which effectively includes the case of laminar separation followed by reattachment into a turbulent boundary layer.

# Chapter 22

# G2 Computational Solution

> If the velocity is sufficiently great, the motion of the fluid at small distances from its surface all round will always be very nearly the same as if the fluid were inviscid, and the difference will be smaller near the front part than near the rear of the globe. (Lord Kelvin 1887)

## 22.1 G2: Stabilized Finite Element Method

We show in [103, 102, 104] that the Navier-Stokes equations (21.1) can be solved by a weighted least squares residual stabilized finite element referred to as *General Galerkin* or *G2*.

Writing the Navier-Stokes equations in symbolic form as $R(u, p) = 0$, the G2 method determines a piecewise linear computational solution $(U, P)$ on a given finite element mesh such that the residual $R(U, P)$ is small in a mean-value sense (Galerkin property) and with a certain weighted control of $R(U, P)$ in a least-square sense (residual stabilization).

The least squares stabilization of G2 acts as an automatic turbulence model, reflecting that the Euler residual cannot be made pointwise small in turbulent regions. G2 has a posteriori error control based on duality and shows output uniqueness in mean-values such as lift and drag [103, 99, 100]

We find that G2 with slip is capable of modeling slightly viscous turbulent flow with $Re > 10^6$ of relevance in many applications in aero/hydro dynamics, including flying, sailing, boating and car racing, with hundred thousands of mesh points in simple geometry and millions in complex geometry, while according to state-of-the-art quadrillions is required [59]. This is because a friction-force/slip boundary condition can model a turbulent boundary layer, and interior turbulence does not have to be resolved to physical scales to capture mean-value outputs [103].

The idea of circumventing boundary layer resolution by relaxing no-slip boundary conditions introduced in [99, 103], was used in [115, 26] in the form of weak satisfaction of no-slip, which however misses the main point of using a force condition instead of a velocity condition in a model of a turbulent boundary layer.

G2 produces turbulent solutions characterized by substantial turbulent dissipation from the least squares residual stabilization acting as an automatic turbulence model, reflecting that $R(U,P)$ cannot be made small pointwise in turbulent regions.

G2 is equipped with automatic a posteriori error control guaranteeing correct lift and drag coefficients [103, 34, 33, 38, 99, 100] up to an error tolerance of a few percent on meshes with a few hundred thousand or million mesh points number of mesh points depending on geometry complexity.

G2 with slip works because slip is good model of the turbulent boundary layer od slightly viscous flow, and interior turbulence does not have to be resolved to physical scales to capture mean-value outputs [103].

G2 with slip thus offers a wealth of infomation at affordable cost, while Prandtl's requirement of boundary layer resolution cannot be met by any forseeable computer.

A G2 solution $(U,P)$ on a mesh with local mesh size $h(x,t)$ according to [103], satisfies the following energy estimate (with $f=0$, $g=0$ and $\beta=0$):

$$K(U(t)) + D_h(U;t) = K(u^0), \tag{22.1}$$

where

$$K(U(t)) = \frac{1}{2}\int_\Omega |U(t)|^2\,dx\,, \quad D_h(U;t) = \int_0^t\int_\Omega h|R(U,P)|^2\,dxdt, \tag{22.2}$$

with $K(U(t))$ the kinetic energy at time and $D_h(U;t)$ the turbulent disspation up to time $t$ and $R_h(U,P)$ the Euler residual of $(U,P)$. We see that the G2 turbulent viscosity $D_h(U;t)$ arises from penalization of a non-zero Euler residual $R(U,P)$ with the penalty directly connecting to the violation (according the theory of criminology). A turbulent solution is characterized by substantial dissipation $D_h(U;t)$ with $\|R(U,P)\|_0 \sim h^{-1/2}$, and

$$\|R_h(U,P)\|_{-1} \leq \sqrt{h} \tag{22.3}$$

with $\|R(U,P)\|_0$ the mean square norm of $R(U,P)$ and $\|R(U,P)\|_{-1}$ a weaker norm measuring the mean square norm of a smoothed version of $R(U,P)$, see [103] for details.

## 22.2 Wellposedness of Mean-Value Outputs

G2 is equipped with automatic a posteriori error estimation in chosen output quantities such as lift and drag, which expresses the sensitivity of lift and drag with respect to the residual of a computed Navier-Stokes solution in terms of a weight function obtained by solving a dual linearized Navier-Stokes equation with the size of derivatives of the dual velocity and pressure representing the weight. G2 automatically adapts the computational mesh according to the weight so as to optimize computational resources.

Let $M(v) = \int_Q v\psi\, dxdt$ be a *mean-value output* of a velocity $v$ defined by a smooth weight-function $\psi(x,t)$, and let $(u,p)$ and $(U,P)$ be two G2-solutions on two meshes with maximal mesh size $h$. Let $(\varphi,\theta)$ be the solution to the *dual linearized problem*

$$\begin{aligned} -\dot{\varphi} - (u\cdot\nabla)\varphi + \nabla U^\top \varphi + \nabla\theta &= \psi \qquad \text{in } \Omega\times I,\\ \nabla\cdot\varphi &= 0 \qquad \text{in } \Omega\times I,\\ \varphi\cdot n &= g \qquad \text{on } \Gamma\times I,\\ \varphi(\cdot,T) &= 0 \qquad \text{in } \Omega, \end{aligned} \tag{22.4}$$

where $\top$ denotes transpose. Multiplying the first equation by $u-U$ and integrating by parts, we obtain the following output error representation [103]:

$$M(u) - M(U) = \int_Q (R_h(u,p) - R_h(U,P))\cdot\varphi\, dxdt \tag{22.5}$$

where for simplicity the dissipative terms are here omitted, from which follows the a posteriori error estimate:

$$|M(u) - M(U)| \leq S(\|R_h(u,p)\|_{-1} + \|R_h(U,P)\|_{-1}), \tag{22.6}$$

where the stability factor

$$S = S(u,U,M) = S(u,U) = \|\varphi\|_{H^1(Q)}. \tag{22.7}$$

In [103] we present a variety of evidence, obtained by computational solution of the dual problem, that for global mean-value outputs such as drag and lift,

$S << 1/\sqrt{h}$, while $\|R_h\|_{-1} \sim \sqrt{h}$, allowing computation of of drag/lift with a posteriori error control of the output within a tolerance of a few percent. In short, mean-value outputs such as lift and drag are wellposed and thus physically meaningful.

We explain in [103] the crucial fact that $S << 1/\sqrt{h}$, heuristically as an effect of *cancellation* of rapidly oscillating reaction coefficients of turbulent solutions combined with smooth data in the dual problem for mean-value outputs. In smooth potential flow there is no cancellation, which explains why zero lift/drag cannot be observed in physical flows.

## 22.3 Computed Dual Velocity-Pressure

In Fig. 22.1-22.5 we show dual pressure and velocity for lift/drag indicating mesh refinement where derivatives are large. Notice the large pressure on top of the trailing edge for $aoa = 04$.

## 22.4 Computational Meshes

In Figs. 22.6-22.7 we show a sequence of adaptively refined 3d meshes for $aoa = 10, 14$. with up to 1 million mesh points.

In Figs. 22.8-**??** we show computational meshes for $aoa = 4, 10, 12, 17, 20$.

## 22.5 What You Need to Know

To understand flight it is not necessary to get into the details of G2; it is sufficient to understand that G2 solves the Navier-Stokes equations with an automatic control of the the computational error, which guarantees that G2 solutions are proper solutions capable of unraveling the secrets hidden in the Navier-Stokes equations, thereby offering valuable information for both understanding and design.

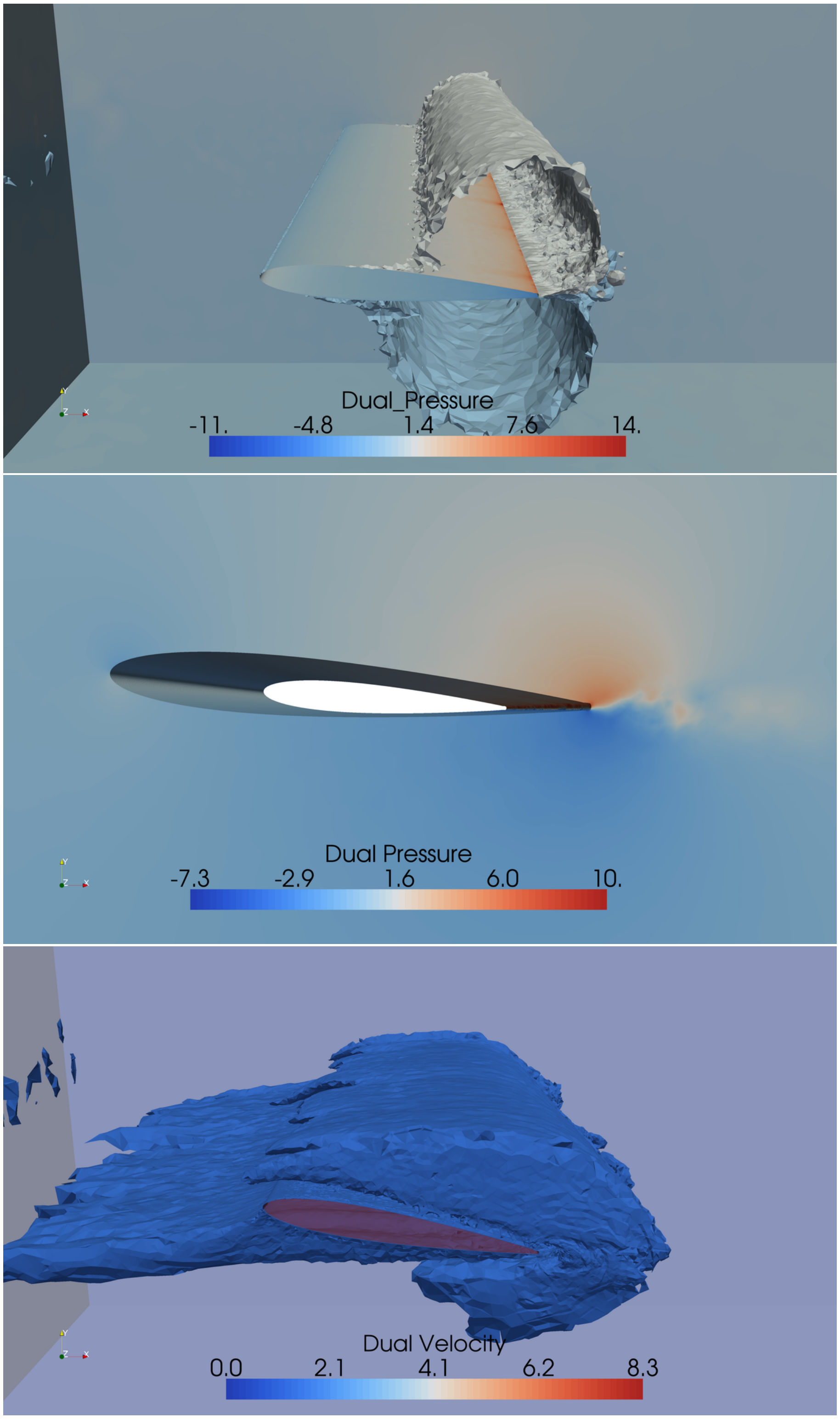



Figure 22.1: Dual pressure and velocity $aoa = 04$.

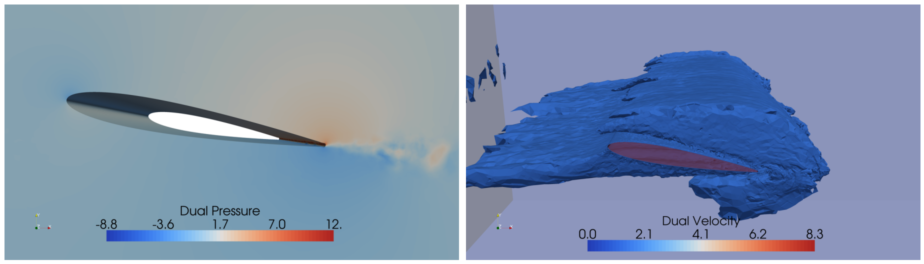



Figure 22.2: Dual pressure and velocity $aoa = 10$.

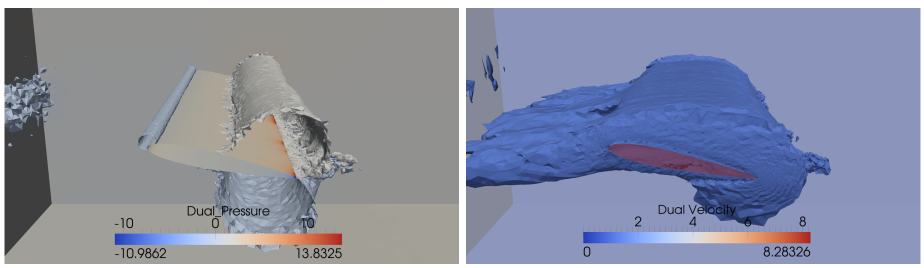



Figure 22.3: Dual pressure and velocity $aoa = 12$.

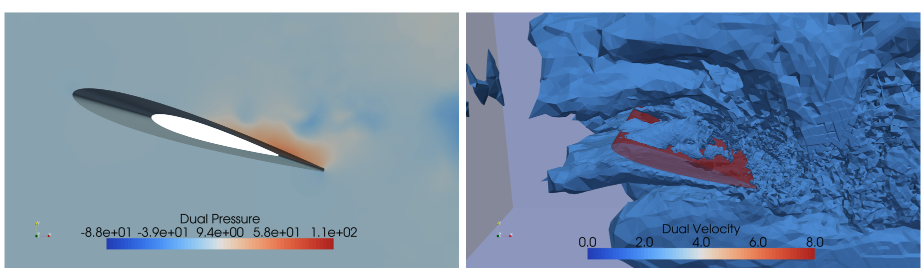



Figure 22.4: Dual pressure and velocity $aoa = 17$.

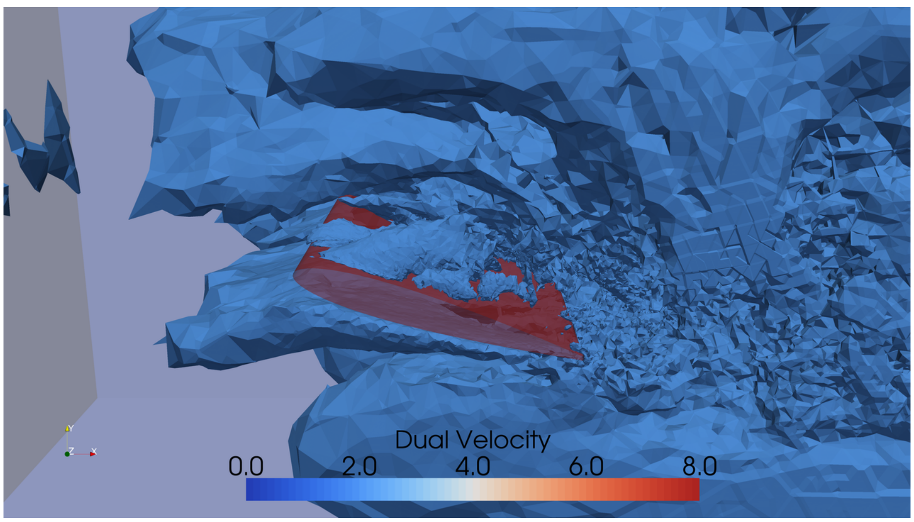



Figure 22.5: Dual velocity for $aoa = 20$.

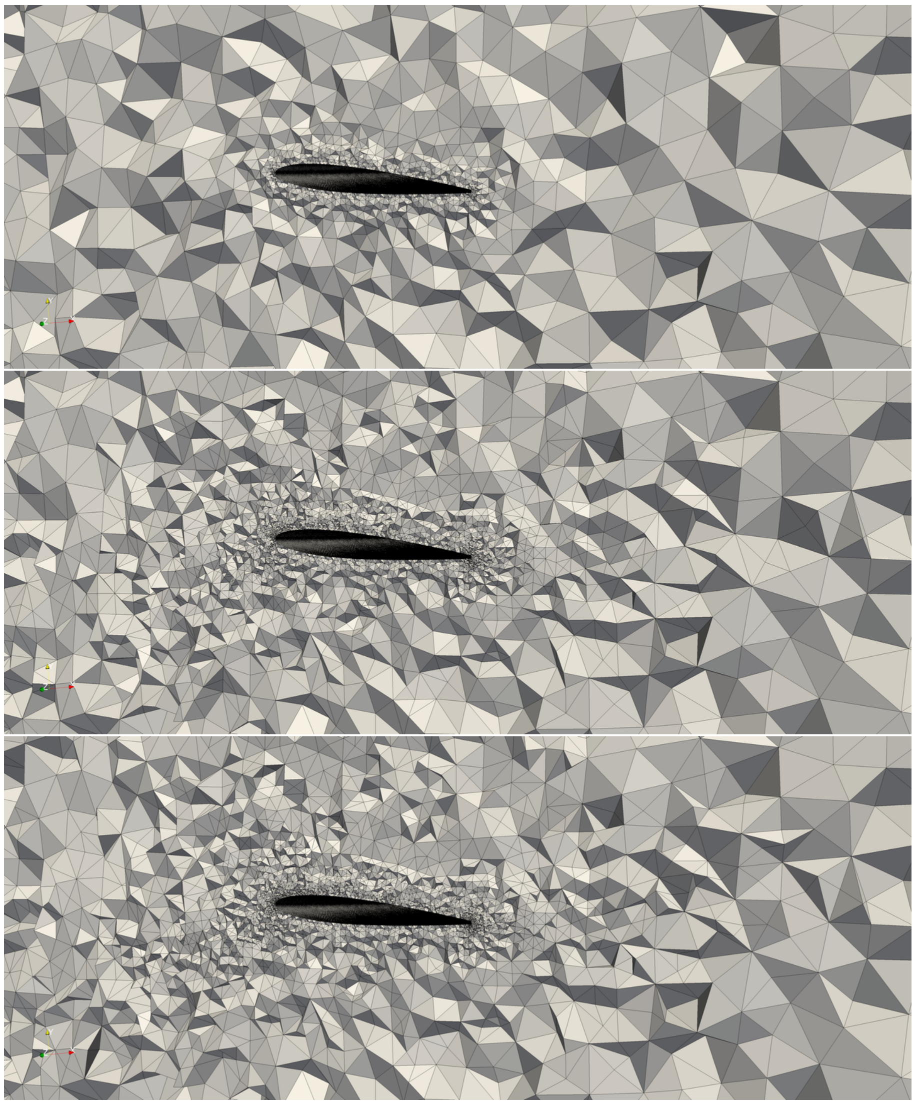


Figure 22.6: Automatically adapted meshes for $aoa = 10$, with inital mesh (top), iteration 4 (center), and iteration 8 (bottom).

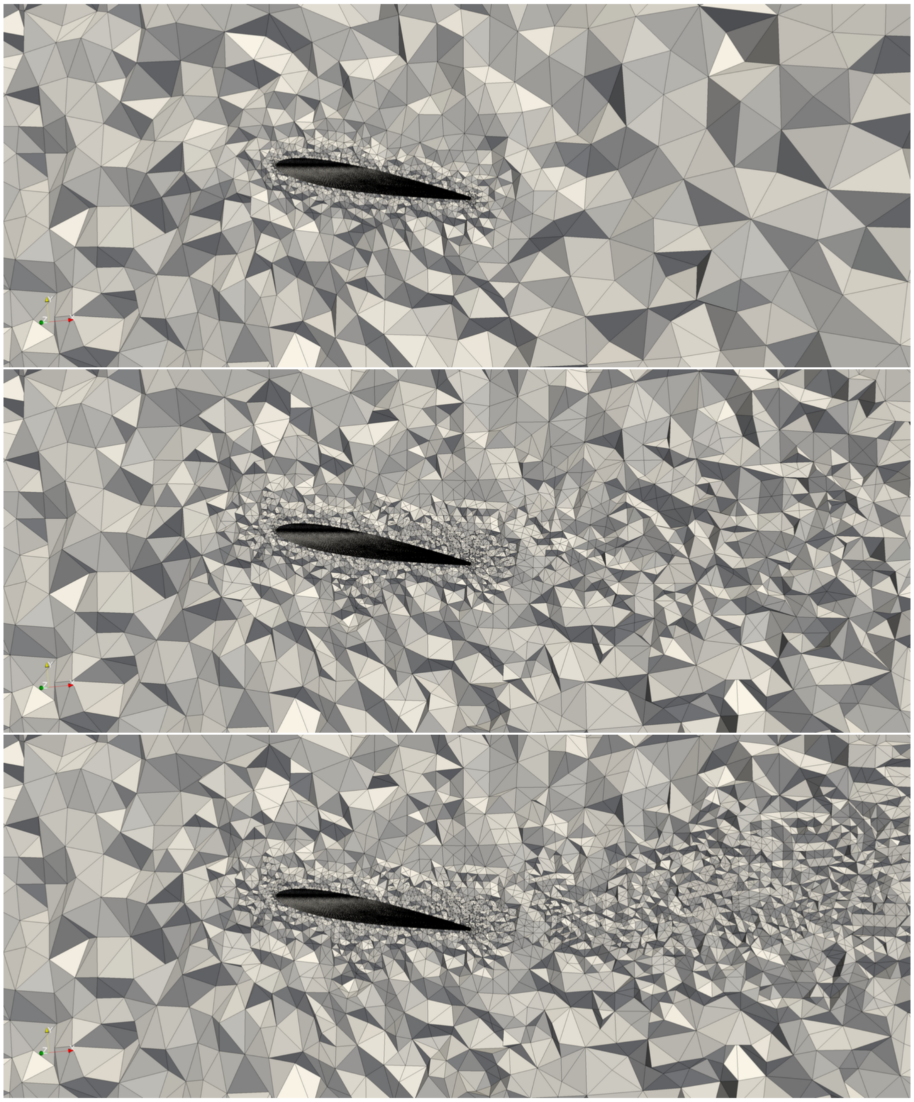


Figure 22.7: Automatically adapted meshes for $aoa = 14$, with inital mesh (top), iteration 4 (center), and iteration 8 (bottom).

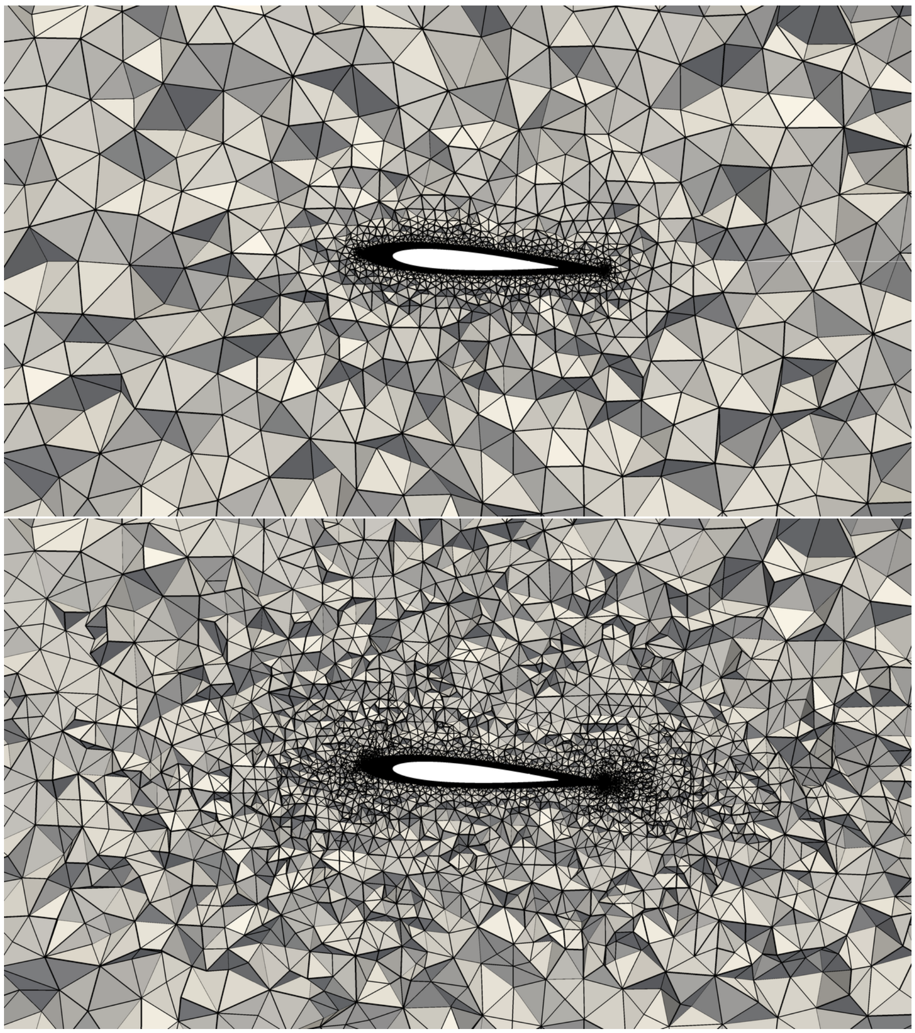


Figure 22.8: Adaptively refined meshes for $aoa = 04$.

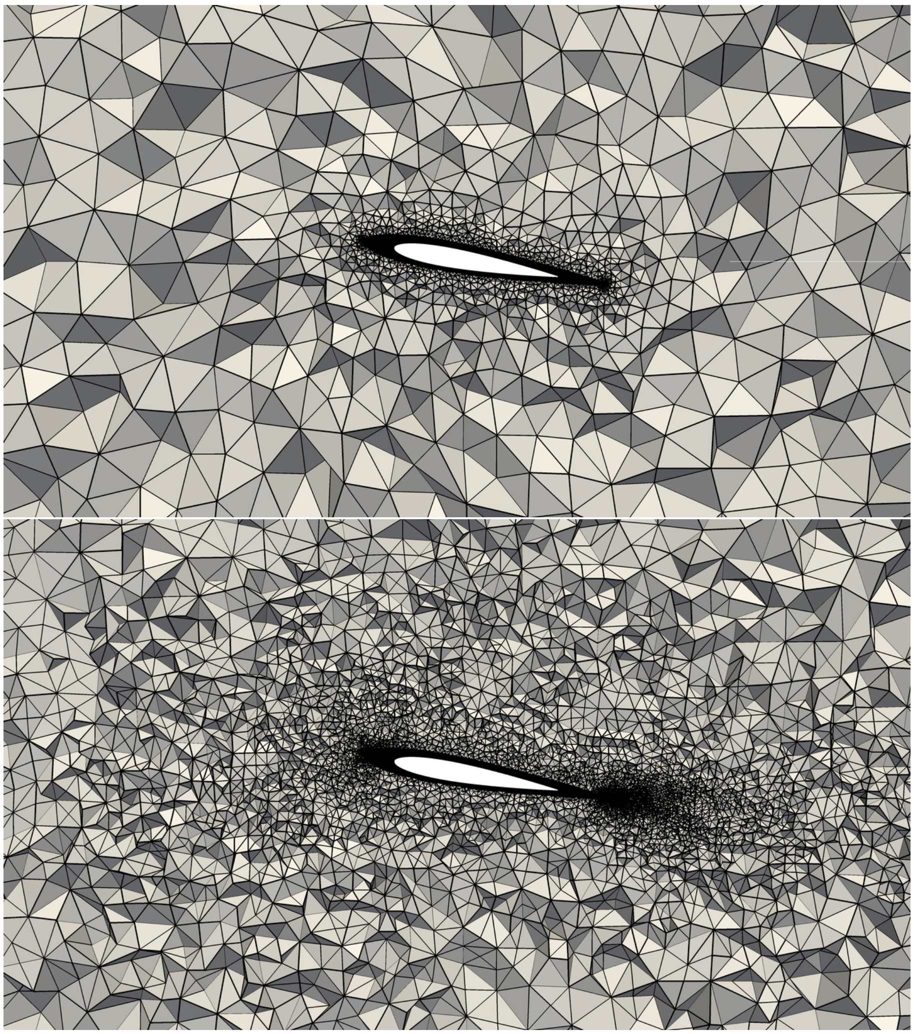


Figure 22.9: Adaptively refined meshes for $aoa = 10$.

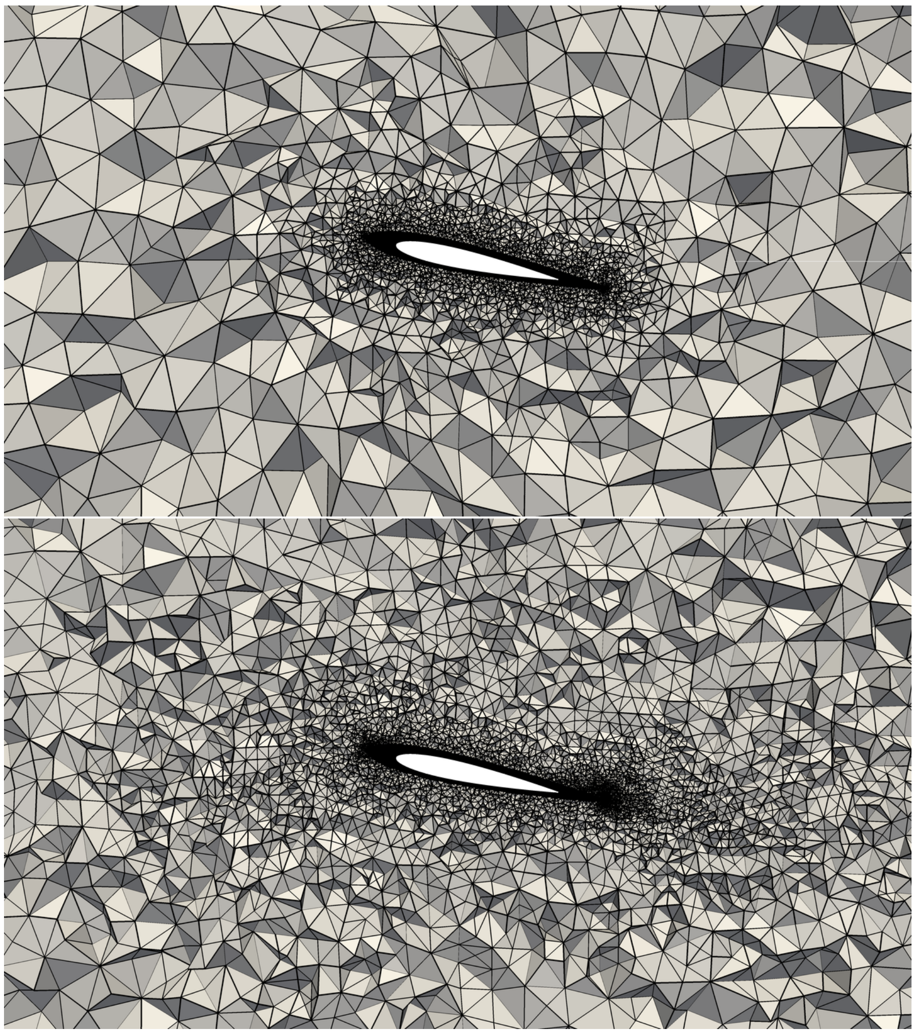


Figure 22.10: Adaptively refined meshes for $aoa = 12$.

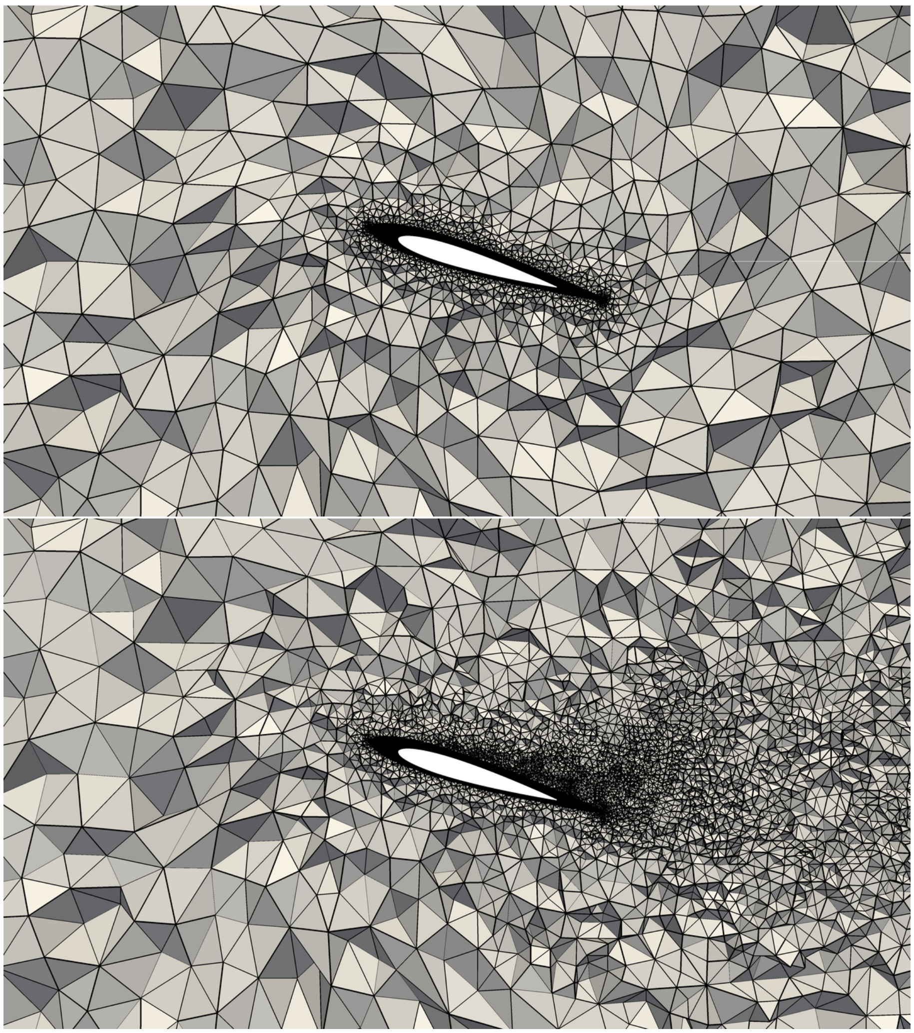


Figure 22.11: Adaptively refined meshes for $aoa = 17$.

# Part IV

# Understanding Navier-Stokes

# Chapter 23

# The Secret

> To those who ask what the infinitely small quantity in mathematics is, we answer that it is actually zero. Hence there are not so many mysteries hidden in this concept as they are usually believed to be. (Leonhard Euler)

> High office, is like a pyramid; only two kinds of animals reach the summit–reptiles and eagles. (d'Alembert)

> Just go on . . . and faith will soon return. (d'Alembert to a friend hesitant with respect to infinitesimals)

The flow of air around a wing is a specific example of slightly viscous flow around a bluff body. If we can understand slightly viscous flow around bluff bodies in general, then we can understand the flow around a wing and thus uncover the secret of flight.

But you say that slightly viscous flow around a bluff body is turbulent and since turbulence is not or cannot be understood, and so how can it be possible to uncover the secret of flight in this way?

But Nature comes to our help; even if turbulence in general hides its secrets, it turns out that slightly viscous around a bluff body can be described and thus understood as

- **potential flow modified by 3d slip separation.**

Since potential flow has been well understood since the time of Euler, it remains to understand 3d slip separation. So we now proceed to uncover the secret of flight by first recalling basic aspects of potential flow and then describe 3d slip separation in mathematical terms.

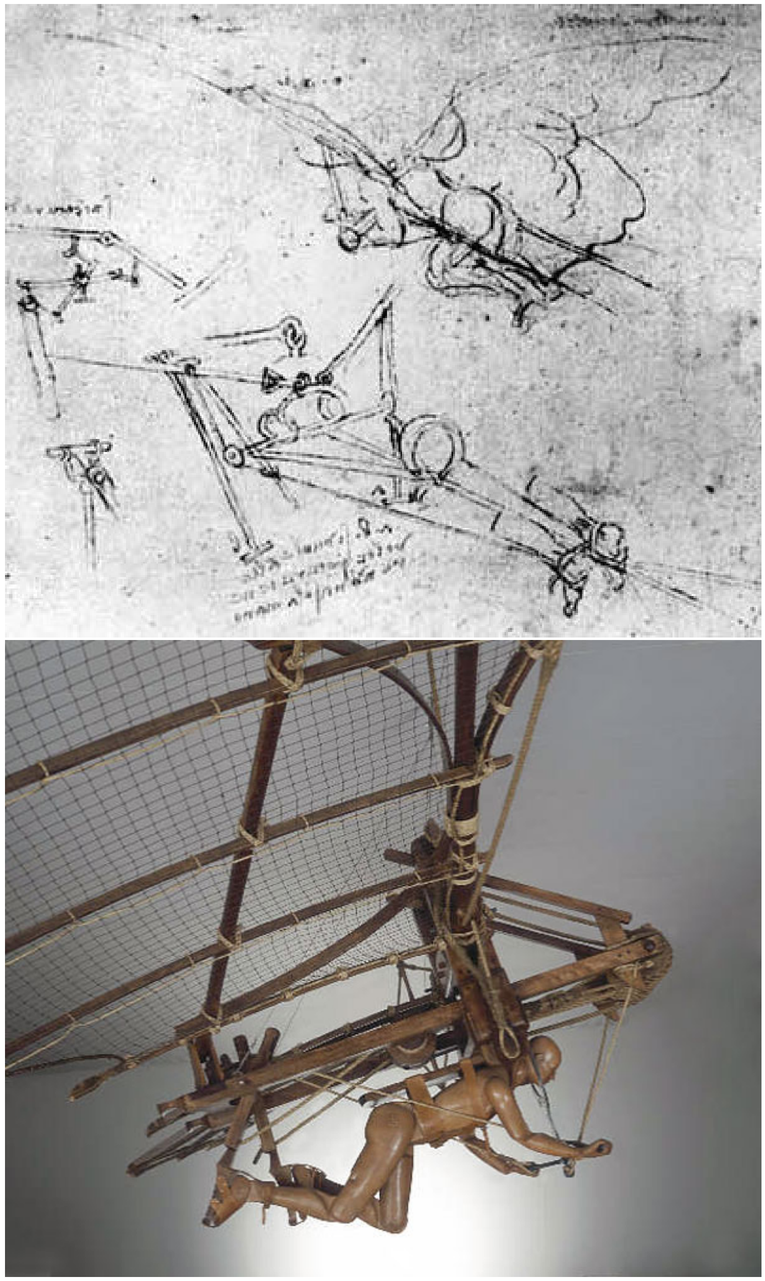


Figure 23.1: Unsuccessful attempt to uncover the secret of flight by da Vinci: sketch and model.

# Chapter 24

# Potential Flow

> Because of D'Alembert's Paradox) fluid mechanics was from start split into the field of hydraulics, observing phenomena which could not be explained, and mathematical or theoretical fluid mechanics explaining phenomena which could not be observed. (Chemistry Nobel Laureate Sir Cyril Hinshelwood [63])

> If one looks at all closely at the middle of our own century, the events that occupy us, our customs, our achievements and even our topics of conversation, it is difficult not to see that a very remarkable change in several respects has come into our ideas; a change which, by its rapidity, seems to us to foreshadow another still greater. Time alone will tell the aim, the nature and limits of this revolution, whose inconveniences and advantages our posterity will recognize better than we can. (d'Alembert on the Enlightment)

## 24.1 The Euler Equations

The basic equations in fluid mechanics expressing *conservation of momentum* or *Newton's 2nd law* connecting force to accelleration combined with *conservation of mass* in the form of incompressibility, were formulated by Euler in 1755 as the *Euler equations* for an *incompressible inviscid* fluid (of unit density) enclosed in a volume $\Omega$ in $\mathbb{R}^3$ with boundary $\Gamma$: Find the velocity $u = (u_1, u_2, u_3)$ and pressure $p$ such that

$$\begin{array}{rcll} \dot{u} + (u \cdot \nabla)u + \nabla p &=& f & \text{in } \Omega \times I, \\ \nabla \cdot u &=& 0 & \text{in } \Omega \times I, \\ u \cdot n &=& g & \text{on } \Gamma \times I, \\ u(\cdot, 0) &=& u^0 & \text{in } \Omega, \end{array} \tag{24.1}$$

which is the Navier-Stokes equations (21.1) with vanishing viscosity and skin friction corresponding to setting $\nu = 0$ $\beta = 0$.

## 24.2 Euler's Optimism vs D'Alembert's Paradox

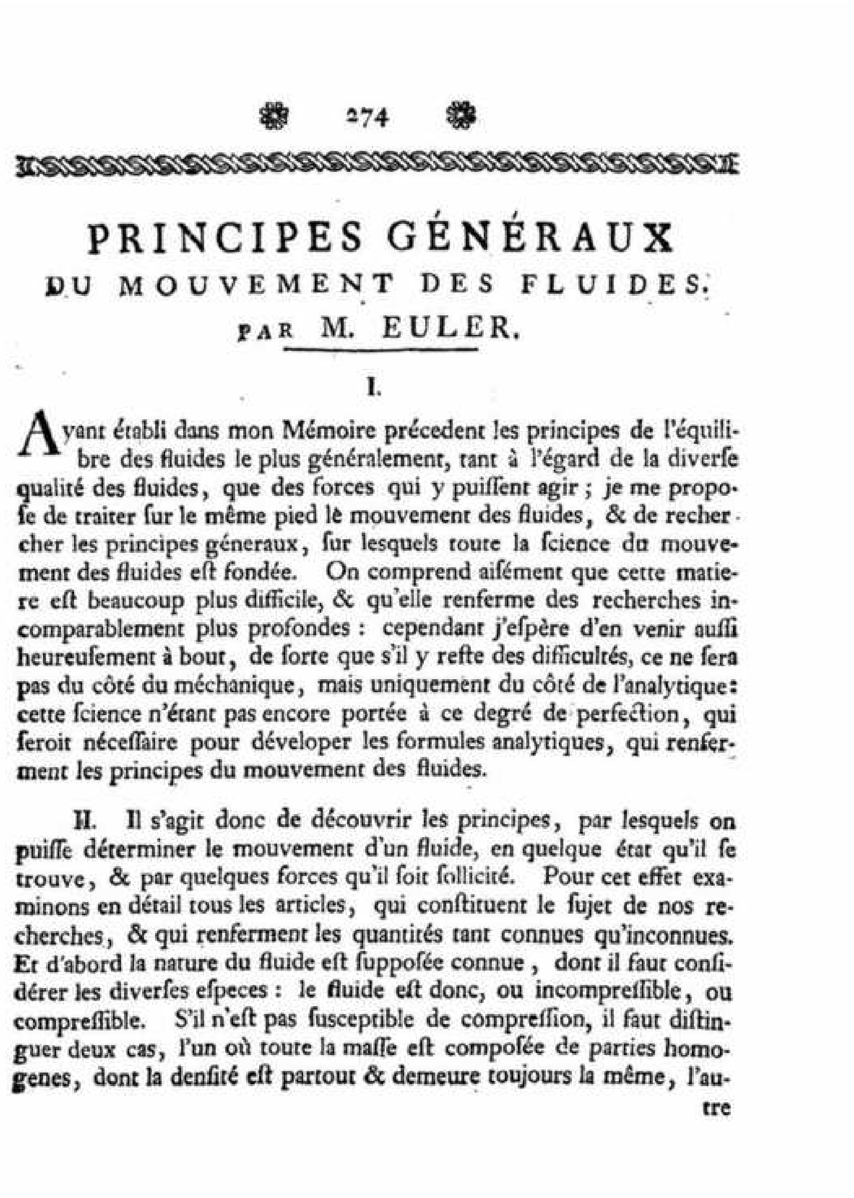
Mathematical fluid mechanics kick-started when Euler in the 1740s discovered certain solutions in two space dimensions of the Euler equations with velocities of the form $u = \nabla\varphi$ with the potential $\varphi$ a *harmonic function* satisfying $\Delta\varphi = 0$ in the fluid domain. These were named *potential solutions* characterized as

- inviscid: vanishingly small viscosity,
- incompressible: $\nabla \cdot u = 0$,
- irrotational: $\nabla \times u = 0$
- stationary: $\dot{u} = 0$.

This promised a fluid mechanics boom for mathematicians as experts of harmonic functions, but the success story quickly collapsed when d'Alembert in 1752 showed that both lift and drag of potential solutions are zero, which showed that the wonderful potential solutions were unphysical and thus were doomed as useless. But potential solution were essentially the only solutions of the Euler equations which could be constructed analytically, which led to a long-lasting split of fluid mechanics into according to the description of Hinshelwood:

- practical fluid mechanics or hydraulics: observing flow with non-zero lift and drag, which cannot be explained as potential flow with zero lift and drag,
- theoretical fluid mechanics: explaining flow as potential flow with zero lift and drag which cannot be observed.

We shall see that it took 254 years to resolve the paradox, but once it was resolved the mystery of flight could be uncovered without split between theory and reality.



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# PRINCIPES GÉNÉRAUX
## DU MOUVEMENT DES FLUIDES.
PAR M. EULER.

I.

Ayant établi dans mon Mémoire précedent les principes de l'équilibre des fluides le plus généralement, tant à l'égard de la diverſe qualité des fluides, que des forces qui y puiſſent agir ; je me propoſe de traiter ſur le même pied lè mouvement des fluides, & de rechercher les principes géneraux, ſur lesquels toute la ſcience du mouvement des fluides eſt fondée. On comprend aiſément que cette matiere eſt beaucoup plus difficile, & qu'elle renferme des recherches incomparablement plus profondes : cependant j'eſpère d'en venir auſſi heureuſement à bout, de ſorte que s'il y reſte des difficultés, ce ne ſera pas du côté du méchanique, mais uniquement du côté de l'analytique: cette ſcience n'étant pas encore portée à ce degré de perfection, qui ſeroit néceſſaire pour développer les formules analytiques, qui renferment les principes du mouvement des fluides.

II. Il s'agit donc de découvrir les principes, par lesquels on puiſſe déterminer le mouvement d'un fluide, en quelque état qu'il ſe trouve, & par quelques forces qu'il ſoit ſollicité. Pour cet effet examinons en détail tous les articles, qui conſtituent le ſujet de nos recherches, & qui renferment les quantités tant connues qu'inconnues. Et d'abord la nature du fluide eſt ſuppoſée connue, dont il faut conſidérer les diverſes eſpeces : le fluide eſt donc, ou incompreſſible, ou compreſſible. S'il n'eſt pas ſusceptible de compreſſion, il faut diſtinguer deux cas, l'un où toute la maſſe eſt compoſée de parties homogenes, dont la denſité eſt partout & demeure toujours la même, l'au-

tre

Figure 24.1: First page of Euler's *General Principles concerning the Motion of Fluids* from 1757 [80].

## 24.3 Potential Flow as Near Navier-Stokes Solution

Potential flow $(u, p)$ with velocity $u = \nabla\varphi$, where $\varphi$ is harmonic in $\Omega$ and satisfies a homogeneous Neumann condition on $\Gamma$ and suitable conditions at infinity, can be seen as a solution of the Navier-Stokes equations for slightly viscous flow with slip boundary condition, subject to

- perturbation of the volume force $f = 0$ in the form of $\sigma = \nabla \cdot (2\nu\epsilon(u))$,
- perturbation of zero friction in the form of $\sigma_s = 2\nu\epsilon(u)_s$,

with both perturbations being small because $\nu$ is small and a potential flow velocity $u$ is smooth. Potential flow can thus be seen as a solution of the Navier-Stokes equations with small force perturbations tending to zero with the viscosity. We can thus express D'Alembert's Paradox as the zero lift/drag of a Navier-Stokes solution in the form of a potential solution, and resolve the paradox by realizing that potential flow is unstable and thus cannot be observed as a physical flow.

Potential flow is like an inverted pendulum, which cannot be observed in reality because it is unstable and under infinitesimal perturbations turns into a swinging motion. A stationary inverted pendulum is a fictious mathematical solution without physical correspondence because it is unstable. You can only observe phenomena which in some sense are stable, and an inverted pendelum or potential flow is not stable in any sense.

## 24.4 2d Potential Flow Separates only at Stagnation

2d potential flow has the following crucial property which partly will be inherited by real turbulent flow, and which explains why a flow over a wing subject to small skin friction can avoid separating at the crest and thus generate downwash, unlike viscous flow with no-slip, which separates at the crest without downwash. We will conclude that gliding flight is possible only in slightly viscous incompressible flow.

**Theorem.** Let $\varphi$ be harmonic in the domain $\Omega$ in the plane and satisfy a homogeneous Neumann condition on the smooth boundary $\Gamma$ of $\Omega$. Then the streamlines of the corresponding velocity $u = \nabla\varphi$ can only separate from $\Gamma$ at a point of stagnation with $u = \nabla\varphi = 0$.

**Proof.** Let $\psi$ be a harmonic conjugate to $\varphi$ with the pair $(\varphi, \psi)$ satisfying the Cauchy-Riemann equations (locally) in $\Omega$. Then the level lines of $\psi$ are the streamlines of $\varphi$ and vice versa. This means that as long as $\nabla\varphi \neq 0$, the boundary curve $\Gamma$ will be a streamline of $u$ and thus fluid particles cannot separate from $\Gamma$ in bounded time.

## 24.5 Point Stagnation vs Line Stagnation

2d potential flow around a circular cylinder shows unstable separation at a *line of stagnation* in 3d. On the other hand, 3d rotational slip separation with instead *point stagnation* represents a more stable separation pattern.

## 24.6 Bernoulli's Principle

The momentum equation of (24.1) can alternatively be formulated

$$\dot{u} + \nabla(\frac{1}{2}|u|^2 + p) + u \times \omega = f, \tag{24.2}$$

where

$$\omega = \nabla \times u$$

is the *vorticity* of the velocity $u$. This follows from the following calculus identity:

$$\frac{1}{2}\nabla|u|^2 = (u \cdot \nabla)u + u \times (\nabla \times u).$$

For a stationary irrotational velocity $u$ with $\dot{u} = 0$ and $\omega = \nabla \times u = 0$, we find that if $f = 0$, then

$$\frac{1}{2}|u|^2 + p = C \tag{24.3}$$

where $C$ is a constant, which is nothing but *Bernoulli's principle* coupling small velocity to large pressure and vice versa.

We conclude that a potential flow velocity $u = \nabla\varphi$ solves the Euler equations with the pressure $p$ given by Bernoulli's law.

## 24.7 Potential Flow: Unstable in 3d Stable in 2d

Our resolution of d'Alembert's paradox is based on the fact that potential flow is unstable in 3d and thus cannot be observed as physical flow.

On the other hand, 2d potential flow is stable. This follows from the fact that in 2d the vorticity $\omega = \nabla \times u$ with $u$ a velocity satisfying $\dot{u} + u \cdot \nabla u + \nabla p = f$, satisfies the following transport equation without exponential growth:

$$\nabla \times f = \nabla \times (\dot{u} + u \cdot \nabla u + \nabla p) == \dot{\omega} + u \cdot \nabla \omega = \frac{D\omega}{Dt}. \tag{24.4}$$

This means that potential flow (stationary irrotational incompressible inviscid flow) remains potential flow and as solution to Laplace's equation is stable.

On the other hand, in 3d the vorticity equation takes the form

$$\dot{\omega} + u \cdot \nabla \omega = \nabla \times f + (\omega \cdot \nabla) u, \tag{24.5}$$

with exponential growth from the presence of the zero-order term $(\omega \cdot \nabla)u$. This means that potential flow does not remain irrotational and stability as solution to Laplace's equation is lost.

# Chapter 25

# 3D Slip Separation

> At the time of the first human flight, no theory existed that would explain the sustenation obtained by means of a curved surface at zero angle of attack. It seemed that the mathematical theory of fluid motion was unable to explain the fundamental facts revealed by experimental aerodynamics. (von Karman)

> When the surface of an obstacle is made to move sufficiently rapidly in the direction of the current, it has no retarding action on the fluid; consequently there is no separation. Thus an airfoil would not stall if the top surface were part of a rotating band. (Goldstein in *Modern Developments in Fluid Dynamics*)

We now consider in more detail the central problem of flow separation i slightly viscous bluff body flow such as the flow around a wing modeled by the Navier-Stokes equations with slip boundary conditions. We shall consider three basic cases of *3d slip separation* with the main flow velocity $u$ in the $x_1$-direction as illustrated in Fig. 25.1:

1. Rotational separation modeled by $u = (x_1, 0, -x_3)$ for $x_1 \geq 0$,

2. Parallel flow separation modeled by $u = (1, -x_2, x_3)$ for $x_3 \geq 0$,

3. Flat plate separation modeled by $u = (1, -x_2, x_3)$ for $x_3 \geq 0$.

In case 1. the flow separates from the plane $x_1 = 0$ with flow velocity $u_1 = x_1$ and in case 2. and 3. the flow separates from the plane $x_3 = 0$ with flow velocity $x_3$. The main flow velocity $(1, 0, 0)$ is subject to perturbation in case 2. but not in case 3. Notice that to satisfy incompressibility the positive derivative $\frac{\partial u_1}{\partial x_1} = 1$ in case 1. is balanced by $\frac{\partial u_3}{\partial x_3} = -1$ with $u_2 = 0$. Similarly $\frac{\partial u_3}{\partial x_3} = 1$ in case 2. is balanced by $\frac{\partial u_2}{\partial x_2} = -1$ with now $u_1 = 1$ as the main velocity component.

The flow velocity $u_3 = -x_3$ in case 1 and $u_2 = -x_2$ is case 2 and 3 describes opposing flow with an effect of exponential instability. In case 2 we assume that this effect is damped by the presence of the main flow velocity $u_1 = 1$ which makes the opposing flow into a second order effect from being a first order effect in case 1 with $u_1 = 0$. In case 3 we assume that this damping effect is missing allowing rotational separation to develop as a second order effect.

We shall thus discover opposing flow to be the main source of instability, but we shall also find that the instability can be moderated by diverting the flow.

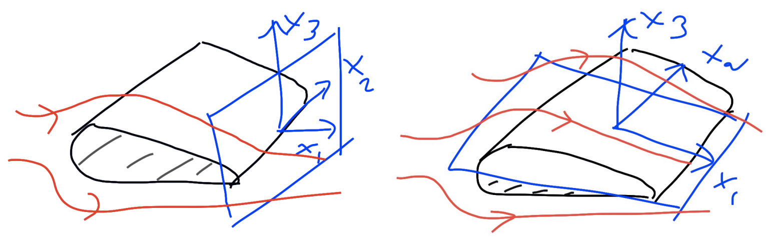


Figure 25.1: Bluff body separation at trailing edge and on upper surface of a wing, modeled as separation in two half-planes, one with normal in the direction of the main flow (rear) and one tangent to the upper surface (top).

We shall start with case 1. covering separation at the trailing edge of a wing before stall, while case 3. models separation on the upper wing surface at and beyond stall using the analysis of case 1. Case 2. connects to separation at e.g. wing tips.

We shall see that in case 1. the separation can be described as *opposing flow rotational slip separation with point stagnation* also covering case 3. as a secondary effect. We shall then refer to case 1. as *rotational separation* and to case 3. as *flat plate slip separation*, with different forms of rotational slip separation with point stagnation. We shall discover separation with point stagnation arising as quasi-stable flow from separation with *line stagnation*, typical of unstable 2d potential flow.

In case 2. the main instability of case 1. is prevented by perturbations in the streamwise direction and thus the base flow can persist.

Figure 25.2: Oil film visualization of rotational separation (left) and parallel separation (right).

For angles of attack well before stall, only case 1. applies, while as stall approaches the trailing edge separation moves upward-forward onto the upper surface and and combines with flat plate separation on top of the wing.

# Chapter 26

# Rotational Separation

> The fear of making permanent commitments can change the mutual love of husband and wife into two loves of self - two loves existing side by side, until they end in separation. (Pope John Paul II)

> Problems of the flow of fluids are of great interest and complexity...the chief incentive to explore such problems in recent years has been provided by the study of the practical problems of flight. (Goldstein in *Modern Developments in Fluid Dynamics*)

We now turn to a detailed mathematical analysis of flow separation, which we will find uncovers the secret of generation of both lift and drag of a body moving through air such as a wing. We know that the flow around the body *attaches* somewhere in the front, typically around a *point of stagnation*, where the flow velocity is zero, and *separates* somewhere somehow in the rear. In many cases attachment is governed by smooth (laminar) potential flow, while separation effectively is a generator of turbulence. We shall thus find that drag can be seen as a "cost of separation", which for a wing also pays for generating lift.

We will present a scenario for separation in slightly viscous turbulent flow, which is fundamentally different from the scenario for viscous laminar flow by Prandtl based on adverse pressure gradients retarding the flow to stagnation at separation. We make a distinction betweeen separation from a laminar boundary layer with no-slip boundary condition and from a turbulent boundary layer with slip. We thus make a distinction between

- **laminar separation with no-slip in (very) viscous flow**

considered by Prandtl of relevance for viscous flow, and

- **turbulent separation with slip in slightly viscous flow**

of relevance in aerodynamics.

We noted above that separation occurs if $\frac{\partial p}{\partial n} < \frac{U^2}{R}$, where $\frac{\partial p}{\partial n}$ is the pressure gradient normal to the boundary into the fluid, $U$ is a flow speed close to the boundary and $R$ the curvature of the boundary, positive for a convex body. We note that in a laminar boundary layer $\frac{\partial p}{\partial n} > 0$ only in contracting flow, which causes separation as soon as the flow expands after the crest of the body. We observe that in a turbulent boundary layer with slip, $\frac{\partial p}{\partial n} > 0$ is possible also in expanding flow which can delay separation. We present a basic mechanism for tangential separation with slip based on instability at rear points of stagnation generating low-pressure rolls of streamwise vorticity reducing $\frac{\partial p}{\partial n}$.

## 26.1 From Unstable to Quasi-Stable Separation

We are concerned with the fundamental problem of fluid mechanics of the motion of a solid body, such as a subsonic airplane, car or boat, through a slightly viscous incompressible fluid such as air at subsonic speeds or water. We focus on incompressible flow at large Reynolds number (of size $10^6$ or larger) around both bluff and streamlined bodies, which is always partly turbulent.

The basic problem is to determine the forces acting on the surface of the body from the motion through the fluid, with the *drag* being the total force in the direction of the flow and the *lift* the total force in a transversal direction to the flow.

As a body moves through a fluid initially at rest, like a car or airplane moving through still air, or equivalently as a fluid flows around a body at rest, approaching fluid particles are deviated by the body in contracting flow, switch to expanding flow at a crest and eventually leave the body. The flow is said to *attach* in the front and *separate* in the back as fluid particles approach and leave a proximity of the body surface.

In high Reynolds number slightly viscous flow the tangential forces on the surface, or *skin friction* forces are small and both drag and lift mainly result from pressure forces and the pressure distribution at turbulent separation is of particular concern.

Separation requires *stagnation* of the flow to zero velocity somewhere in the back of the body as opposing flows are meeting. Stagnation requires *retardation* of the flow, which requires a streamwise increasing pressure, or *adverse pressure gradient*. We show by a linearized stability analysis that retardation from opposing

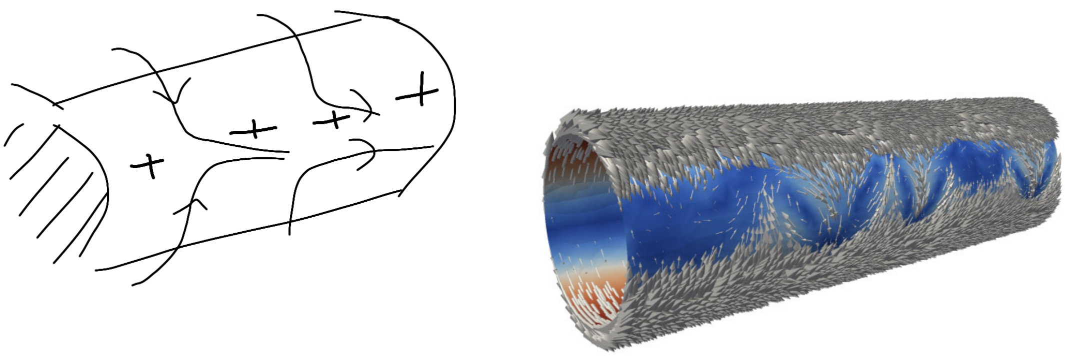

Figure 26.1: Unstable irrotational separation of potential flow around a circular cylinder (left) from line of stagnation surrounded by a high pressure zone indicated by $+$, with corresponding opposing flow instability (right).

flows is exponentially unstable, which in particular shows *potential flow* to be unstable as indictaed in Fig. 32.6. Since unstable flow cannot persist over time, we expect to find a *quasi-stable* separation pattern resulting from the most unstable mode of potential flow, as a flow without streamwise retardation from opposing flows. By quasi-stable we mean a flow which is not exponentially unstable and thus may have a certain permanence over time.

Both experiment and computation show that there is such a quasi-stable separation pattern arising from transversal reorganization of opposing potential flow in the back into a set of counter-rotating vortex tubes of swirling flow (streamwise vorticity) attaching to the body, accompanied by a zig-zag pattern of alternating low and high pressure zones around points of stagnation with low pressure inside the vortex tubes. This pattern is illustrated in Fig. 2 for a cylinder along with computation and experiment, where we see how the flow finds a way to separate with unstable streamwise retardation in opposing flows replaced by quasi-stable transversal accelleration close to the surface before separation and in the swirling flow after separation. We see this phenomenon in the swirling flow in a bathtub drain, which is a stable configuration with transversal accelleration replacing the unstable opposing flow retardation of fully radial flow.

We refer to this quasi-stable pattern as *3d rotational separation*. This is a macroscopic phenomenon with the stagnation points spaced as widely as possibe. From macroscopic point of view the small skin friction of slightly viscous flow can be modeled with a *slip boundary condition* expressing vanishing skin fric-

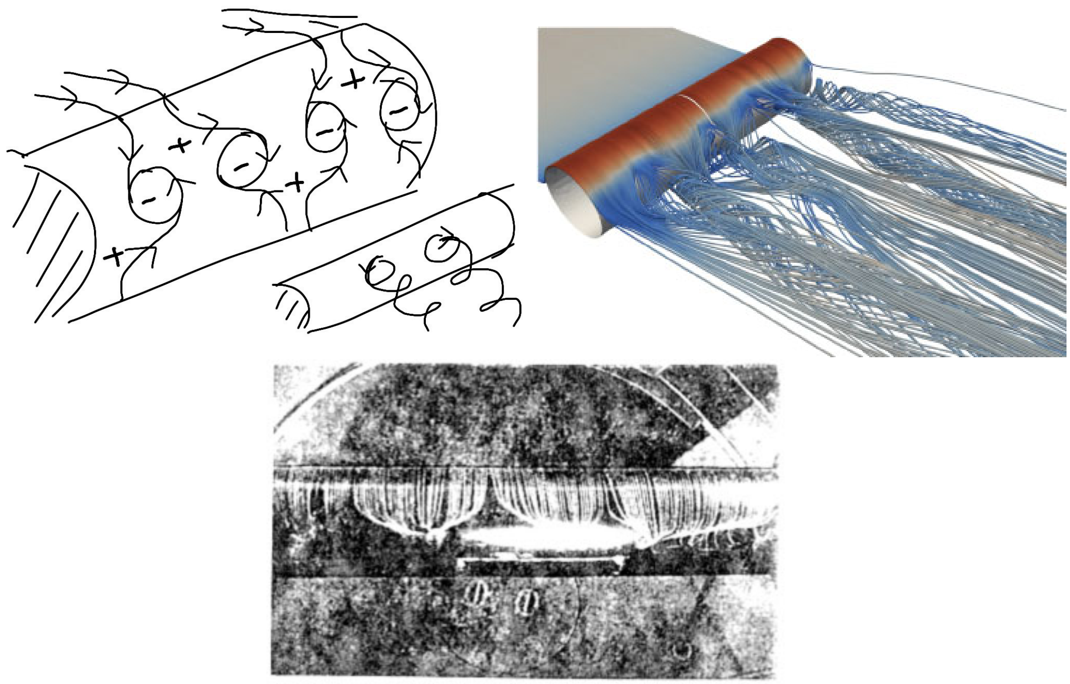

Figure 26.2: Quasi-stable 3d rotational separation from alternating high/low pressure: principle, computation and experiment

tion. We show that computational solution of Navier-Stokes equations with slip is possible at affordable cost, because with slip there are no boundary layers to resolve, which makes it possible to compute both drag and lift of a of a car, boat or airplane arbitrary shape without the quadrillions of mesh points for boundary layer resolution commonly believed to be required [43].

The single high pressure zone stretching along the stagnation line of potential flow around a circular cylinder (creating instability) in Fig. 32.6, is thus broken down into a pattern of high and low pressure zones by the development of low pressure vortical flow in Fig. 2, which allows the fluid to separate without unstable streamwise retardation in opposing flow. The so modified pressure creates drag of a bluff body and lift of a wing from the zero drag and lift of potential flow.

We present evidence in the form of mathematical stability analysis and computation that high Reynolds number incompressible flow around a body moving through a fluid can be described as

- quasi-stable potential flow before separation,
- quasi-stable 3d rotational separation.

This scenario is also supported by observation presented in e.g. [121, 122, **?**] and our evidence thus consists of mathematical theory/computation and observation in strong accord.

We show that both drag and lift critically depend on the pressure distribution of 3d rotational separation. We remark that in the attaching flow in the front the flow is retarded by the body and not by opposing flows as in the back, which allows stable potential flow attachment. We show that drag and lift of a body of arbitrary shape can be accurately computed by solving the Navier-Stokes equations with slip.

The description and analysis of the crucial flow feature of separation presented here is fundamentally different from that of Prandtl, named the father of modern fluid mechanics, based on the idea that both drag and lift originate from a thin viscous boundary layer, where the flow speed relative to the body rapidly changes from the free stream speed to zero at the body surface corresponding to a no-slip boundary condition. Prandtl's scenario for separation, which has dominated 20th century fluid, can be described as *2d boundary layer no-slip separation*, to be compared with our entirely different scenario of *3d no-boundary layer slip separation*.

The unphysical aspect of Prandtl's scenario of separation is illuminated in [**?**]:

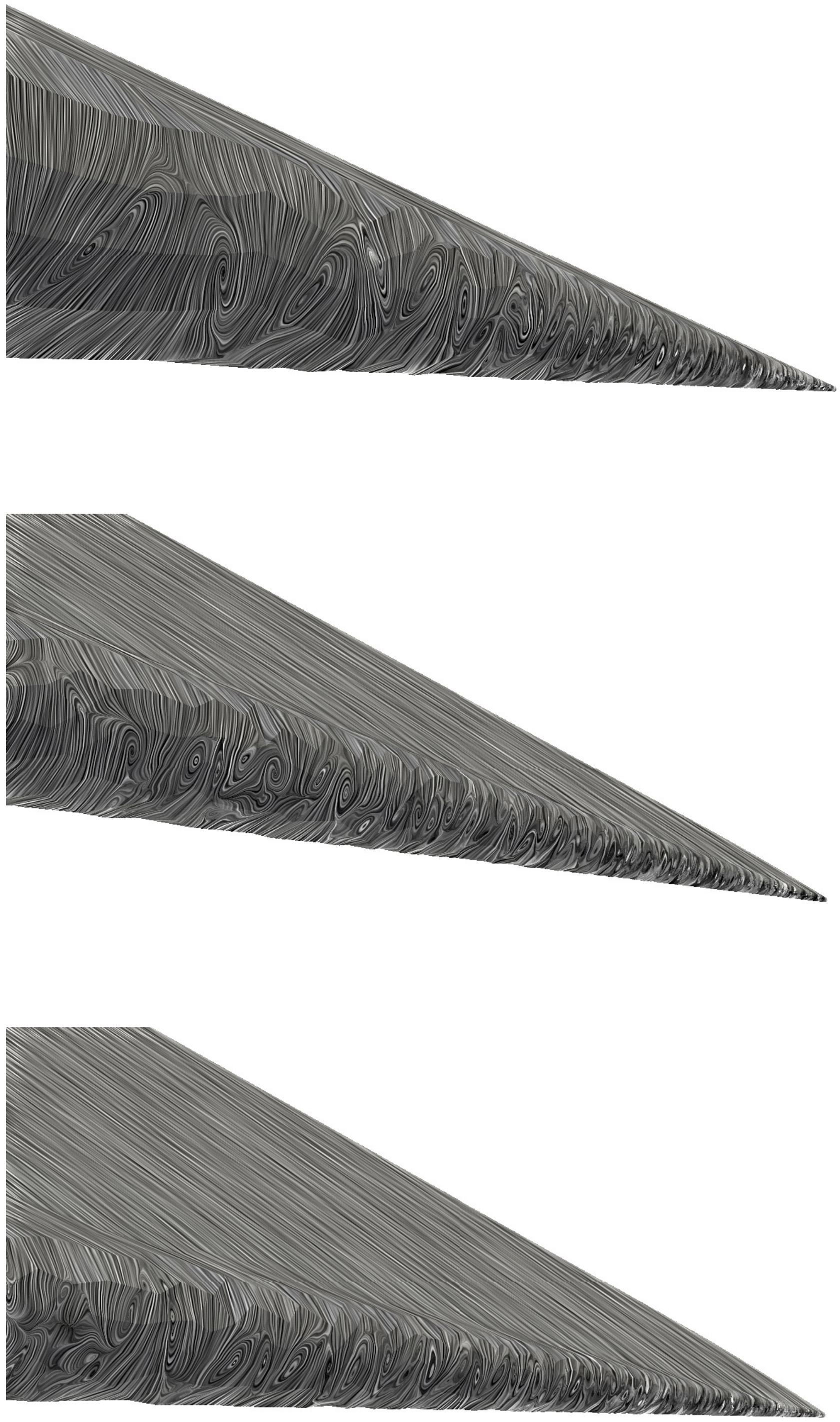

Figure 26.3: Oilfilm plots for $aoa = 4, 10, 12$ showing point stagnation as a characteristic of 3d rotational slip separation Notice that the stagnation points move towards the upper surface as $aoa$ increases.

- *The passage from the familiar 2d to the mysterious 3d requires a complete reconsideration of concepts apparently obvious (separation and reattachment points, separated bubble, recirculation zone) but inappropriate and even dangerous to use in 3d flows.*

## 26.2 Stability Analysis by Linearization

The stability of a Navier-Stokes solution with vansihing viscosity or solution of the Euler equations (24.1) is expressed by the linearized equations

$$\begin{array}{rcll} \dot{v} + (u\cdot\nabla)v + (v\cdot\nabla)\bar{u} + \nabla q &=& f - \bar{f} & \text{in } \Omega\times I, \\ \nabla\cdot v &=& 0 & \text{in } \Omega\times I, \\ v\cdot n &=& g - \bar{g} & \text{on } \Gamma\times I, \\ v(\cdot,0) &=& u^0 - \bar{u}^0 & \text{in } \Omega, \end{array} \tag{26.1}$$

where $(u,p)$ and $(\bar{u},\bar{p})$ are two Euler solutions with slightly different data, and $(v,q)\equiv(u-\bar{u},p-\bar{p})$. Formally, with $u$ and $\bar{u}$ given, this is a linear convection-reaction problem for $(v,q)$ with growth properties governed by the reaction term given by the $3\times 3$ matrix $\nabla\bar{u}$. By the incompressiblity, the trace of $\nabla\bar{u}$ is zero, which shows that in general $\nabla\bar{u}$ has eigenvalues with real values of both signs, of the size of $|\nabla u|$ (with $|\cdot|$ some matrix norm), thus with at least one exponentially unstable eigenvalue, except in the neutrally stable case with purely imaginary eigenvalues, or in the non-normal case of degenerate eigenvalues representing parallel shear flow [103].

The linearized equations in velocity-pressure indicate that, as an effect of the reaction term $(v\cdot\nabla)\bar{u}$:

- streamwise retardation is exponentially unstable in velocity,
- transversal accelleration is neutrally stable,

where transversal signifies a direction orthogonal to the flow direction.

Additional stability information is obtained by applying the curl operator $\nabla\times$ to the momentum equation to give the vorticity equation

$$\dot{\omega} + (u\cdot\nabla)\omega - (\omega\cdot\nabla)u = \nabla\times f \quad \text{in } \Omega, \tag{26.2}$$

which is also a convection-reaction equation in the vorticity $\omega = \nabla\times u$ with coefficients depending on $u$, of the same form as the linearized equation (26.5), with a sign change of the reaction term. The vorticity is thus locally subject to exponential growth with exponent $|\nabla u|$:

- streamwise accelleration is exponentially unstable in streamwise vorticity.

We sum up as follows: The linearized equations (26.5) and (32.1) indicate exponential growth of perturbation of velocity in streamwise retardation and of streamwise vorticity in streamwise accelleration. We shall see in more detail below 3d rotational separation results from exponential instability of potential flow in retardation followed by vortex stretching in accelleration, with the retardation replaced by neutrally stable transversal accelleration.

Note that in classical analysis it is often argued that from the vorticity equation (32.1), it follows that vorticity cannot be generated starting from potential flow with zero vorticity and $f = 0$, which is *Kelvin's theorem*. But this is an incorrect conclusion, since perturbations of $\bar{f}$ of $f$ with $\nabla \times \bar{f} \neq 0$ must be taken into account, even if $f = 0$. What you effectively see in computations is local exponential growth of vorticity on the body surface in rear retardation and by vortex stretching in accelleration, even if $f = 0$, which is a main route of instability to turbulence as well as separation.

## 26.3 Instability of 2d Irrotational Separation

We now analyze the stability of 2d irrotational separation considered by Planck in the following model of the potential flow around a circular cylinder studied in more detail below: $u(x) = (x_1, -x_2, 0)$ in the half-plane $\{x_1 > 0\}$ with stagnation along the line $(0, 0, x_3)$ and

$$\frac{\partial u_1}{\partial x_1} = 1 \quad \text{and} \quad \frac{\partial u_2}{\partial x_2} = -1, \tag{26.3}$$

expressing that the fluid is *squeezed* by *retardation* in the $x_2$-direction and *accelleration* in the $x_1$-direction. We first focus on the retardation with the main stability feature of (26.5) captured in the following simplified version of the $v_2$-equation of (26.5), assuming $x_1$ and $x_2$ are small,

$$\dot{v}_2 - v_2 = f_2,$$

where we assume $f_2 = f_2(x_3)$ to be an oscillating perturbation depending on $x_3$ of a certain wave length $\delta$ and amplitude $h$, for example $f_2(x_3) = h\sin(2\pi x_3/\delta)$, expecting the amplitude to decrease with the wave length. We find, assuming $v_2(0, x) = 0$, that

$$v_2(t, x_3) = (\exp(t) - 1)f_2(x_3).$$

We next turn to the accelleration and then focus on the $\omega_1$-vorticity equation, for $x_2$ small and $x_1 \geq \bar{x}_1 > 0$ with $\bar{x}_1$ small, approximated by

$$\dot{\omega}_1 + x_1 \frac{\partial \omega_1}{\partial x_1} - \omega_1 = 0, \tag{26.4}$$

with the “inflow boundary condition”

$$\omega_1(\bar{x}_1, x_2, x_3) = \frac{\partial v_2}{\partial x_3} = (\exp(t) - 1)\frac{\partial f_2}{\partial x_3}.$$

The equation for $\omega_1$ thus exhibits exponential growth, which is combined with exponential growth of the “inflow condition”. We can see these features in principle and computational simulation in Figs. 26.1 and **??** showing how opposing flows at separation generate a pattern of alternating surface vortices from pushes of fluid up/down, which act as initial conditions for vorticity stretching into the fluid generating counter-rotating low-pressure tubes of streamwise vorticity.

The above model study can be extended to the full linearized equations linearized at $u(x) = (x_1, -x_2, 0)$:

$$\begin{aligned} Dv_1 + v_1 &= -\tfrac{\partial q}{\partial x_1}, \\ Dv_2 - v_2 &= -\tfrac{\partial q}{\partial x_2} + f_2(x_3), \\ Dv_3 &= -\tfrac{\partial q}{\partial x_3}, \\ \nabla \cdot v &= 0 \end{aligned} \tag{26.5}$$

where $Dv = \dot{v} + u \cdot \nabla v$ is the convective derivative with velocity $u$ and $f_2(x_3)$ as before. We here need to show that the force perturbation $f_2(x_3)$ will not get cancelled by the pressure term $-\frac{\partial q}{\partial x_2}$ in which case the exponential growth of $v_2$ would get cancelled. Now $f_2(x_3)$ will induce a variation of $v_2$ in the $x_3$ direction, but this variation does not upset the incompressibility since it involves the variation in $x_2$. Thus, there is no reason for the pressure $q$ to compensate for the force perturbation $f_2$ and thus exponential growth of $v_2$ is secured.

We thus find streamwise vorticity generated by a force perturbation oscillating in the $x_3$ direction, which in the retardation of the flow in the $x_2$-direction creates exponentially increasing vorticity in the $x_1$-direction, which acts as inflow to the $\omega_1$-vorticity equation with exponential growth by vortex stretching. Thus, we find exponential growth at rear separation in both the retardation in the $x_2$-direction and the accelleration in the $x_1$ direction, as a result of the squeezing expressed by (26.3).

Since the combined exponential growth is independent of $\delta$, it follows that large-scale perturbations with large amplitude have largest growth, which is also seen in computations with $\delta$ the distance between streamwise rolls as seen in Fig. 32.1 which does not seem to decrease with decreasing $h$. The perturbed flow with swirling separation is large scale phenomenon, which we show below is more stable than potential flow.

The corresponding pressure perturbation changes the high pressure at separation of potential flow into a zig-zag alternating more stable pattern of high and low pressure with high pressure zones deviating opposing flow into non-opposing streaks which are captured by low pressure to form rolls of streamwise vorticity allowing the flow to spiral away from the body. This is similar to the vortex formed in a bathtub rain.

Notice that in attachment in the front the retardation does not come from opposing flows but from the solid body, and the zone of exponential growth of $\omega_2$ is short, resulting in much smaller perturbation growth than at rear separation.

We shall see that the tubes of low-pressure streamwise vorticity change the normal pressure gradient to allow separation without unstable retardation, but the price is generation of drag by negative pressure inside the vortex tubes as a "cost of separation".

## 26.4 Quasi-Stable Rotational 3d Separation

We discover in computation and experiment that the rotational 3d separtion pattern just detected as the most unstable mode of 2d, represents a quasi-stable flow with unstable retardation in opposing flows replaced by transversal acceleration.

As a model of flow with transversal accelleration we consider the potential velocity $u = (0, x_3, -x_2)$ of a constant rotation in the $x_1$-direction, with corresponding linearized equations linearized problem

$$\dot{v}_1 = 0, \quad \dot{v}_2 + v_3 = 0, \quad \dot{v}_3 - v_2 = 0, \tag{26.6}$$

which model a neutrally stable harmonic oscillator without exponential growth corresponding to imaginary eigenvalues of $\nabla u$.

Further, shear flow may represented by $(x_2, 0, 0)$, which is marginal unstable with linear perturbation growth from degenerate zero eigenvalues of $\nabla u$, as analyzed in detail in [103].

## 26.5 Quasi-Stable Potential Flow Attachment

The above analysis also shows that potential flow attachment, even though it involves streamwise retardation, is quasi-stable. This is because the initial perturbation $f_2$ in the above analysis is forced to be zero by the slip boundary condition requiring the normal velocity to vanish. In short, potential flow attachment is stable because the flow is retarded by the solid body and not by opposing flows as in separation.

This argument further shows that a flow retarded by a high pressure zone is quasi-stable in approach because it is similar to attachment.

## 26.6 Resolution of D'Alembert's Paradox

Potential flow can be viewed as an approximate solution of the Navier-Stokes equations at high Reynolds number with a slip boundary condition, but potential flow is unphysical because both drag and lift are zero, as expressed in D'Alembert's Paradox [104]. Inspection of potential flow shows unstable *irrotational separation* of retarding opposing flow, which is impossible to observe as a physical flow. D'Alembert's Paradox is thus resolved by observing that potential flow with zero drag and lift is unstable [104] and thus unphysical, and not by the official resolution suggested by Prandtl stating that the unphysical feature is the slip boundary condition.

Although 3d rotational separation has a macroscopic features the flow is *turbulent* at separation in the sense that the dissipation in the flow is substantial even though the viscosity is very small, following the definition of turbulent flow in [103].

## 26.7 Magnus Effect by Unsymmetric Separation

The Magnus effect of a rotating ball results from unsymmetric separation with separation delayed where the skin friction is smaller, that is where the relative velocity is smaller. Small skin friction corresponds to a slip boundary condition allowing potential flow which cannot separate without stagnation, and thus has delayed separation as compared to viscous flow with no-slip, which tends to separate on the crest of the flow, see Fig. 29.6.

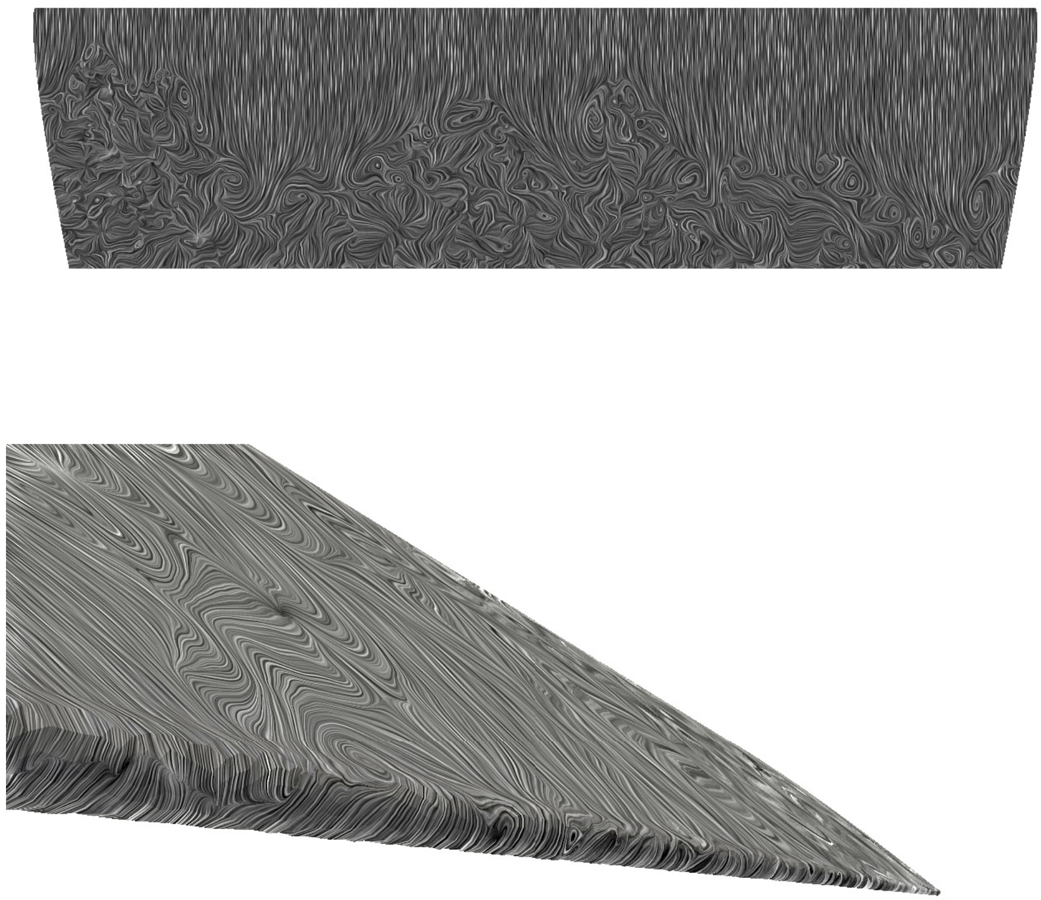

Figure 26.4: Oilfilm plots for $aoa = 17$: upper wing surface (top) (middle) and trailing edge (below).

# Chapter 27

# Parallel Separation

> Oft habe ich Kuttas Leben reich und beneidenswert gefunden wegen seiner Aufgeschlossenheit für so viele Seiten menschlichen Geisteslebens, oft aber fand ich es auch arm und bedauernswert in seiner Einsamkeit und Zurückgezogenheit. (Pfeiffer)

> The actual processes are often in such unsatisfactory agreement with the theoretical conclusions that technology has addopted its own procedure to deal with hydrodynamical propblems, which is usually called hydrualics. This latter speciality, however, lacks so much of a strict method, in its foundations a well as its conclusions, that most of its results do not deserve a higher value tha that of empirical formulae with very limited range of validity. (1900 text book, from *The Dawn of Fluid Dynamics* by M. Eckert)

> Outside Göttingen boundary layer theory was largely ignored for almost two decades after Prandtl's first publication in 1904....Both boundary layer theory and airfoil theory became the subject matter of heated debates in tht 1920s. ( *The Dawn of Fluid Dynamics*)

We now consider the case of parallel flow separation seen on top of the horisontal surface in Fig. 25, modeled by

$$u = (1, -x_2, x_3) \text{ for } x_3 \geq 0, \tag{27.1}$$

as an extension of the analysis in preceding chapter, which with a switch of coordinates concerns the case $u = (0, -x_2, x_3)$. We thus consider the effect of a change of $u_1$ from 0 to 1.

In the above case of $u_1 = 0$, we considered an oscillating perturbation $(0, f_2(x_1), 0)$ in the linearized equation with an effect of exponential growth generating a pattern of counter-rotating rolls of $x_3$ vorticity, thus without perturbation of $u_1 = 0$.

In the case $u_1 = 1$, we also have to allow a perturbation in $u_1$ which may destroy the development of the organized pattern of counter-rotating rolls in the $x_3$ direction and thus allow separation into $x_3 > 0$ according to the unperturbed base flow as a possibly stable flow.

Effectively, without a perturbation of the form $(0, f_2(x_1), 0)$, the base flow $u = (1, -x_2, x_3)$ is a 2d potential flow and as such is stable by the argument in Section 24.7. In this case the main flow $(1, 0, 0)$ acts to stabilize the flow by effectively restricting it to 2d.

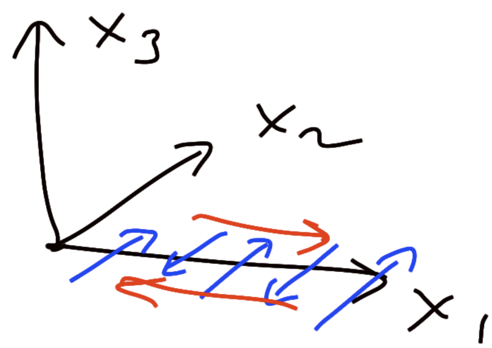


Figure 27.1: Oscillating perturbation of $u_2$ (blue) cancelled by perturbation of $u_1$ (red).

# Chapter 28

# Flat Plate Separation

> We aerodynamicists were always more modest (than physicists) and did not attempt to change basic beliefs of the human mind or to interfere with the business of the good Lord or divine Providence. (von Karman)

We consider now the case 3. of flat plate separation in parallel flow such as the flow on top of a wing approaching stall. In this case the opposing flow required for separation occurs as a secondary transversal effect of counter-rotating rolls of streamwise vorticity emerging from the exponential instability of streamwise vorticity in the accellerating flow before the crest of the wing as evidenced by (26.4). We then assume that the perturbation in the streamwise direction is small allowing the secondary effect to develop.

Notice that streamwise rolls on the top wing surface come along with a tranversal variation of the pressure which can lead to violation of the condition $\frac{\partial p}{\partial n} = \frac{|u|^2}{R}$ where $R$ is the radius of curvature of the surface, and thus trigger local separation.

This local separation can take the form described in the previous chapter: Transversally opposing flow in the valley between each second streamwise roll may trigger generation of counter-rotating rolls of transversal vorticity stretching into the flow from the wing surface which bend into the main flow direction and possibly recombine into hairpin vortices as illustrated in Fig. 28.1 and Fig. 28.2 and evidenced in e.g. [29, 30] and Fig. 28.3.

As a model of rolls of streamwise vorticity in a layer attaching to the $x_3 = 0$ plane, we may consider

$$u = (1, \cos(x_3), -\cos(x_2)), \quad p = -\sin(x_2)\sin(x_3). \tag{28.1}$$

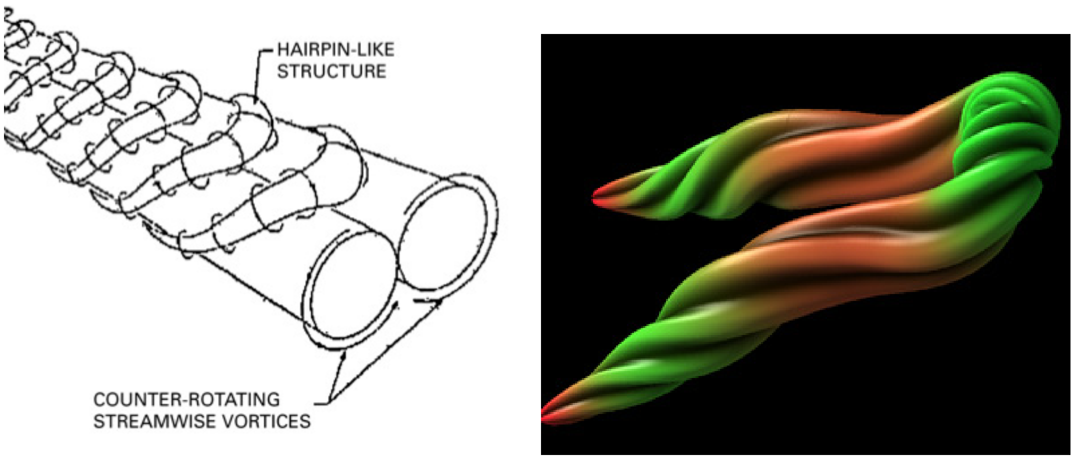


Figure 28.1: Development of hairpin vortices.

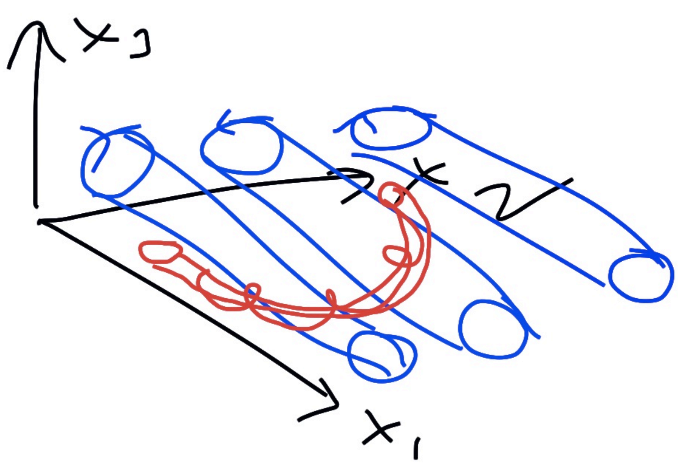


Figure 28.2: Two-step mechanism of flow separation in in parallel flow from non-normal growth of streamwise velocity followed by rotational separation from transversal opposing flow organizing into hairpin vortices.

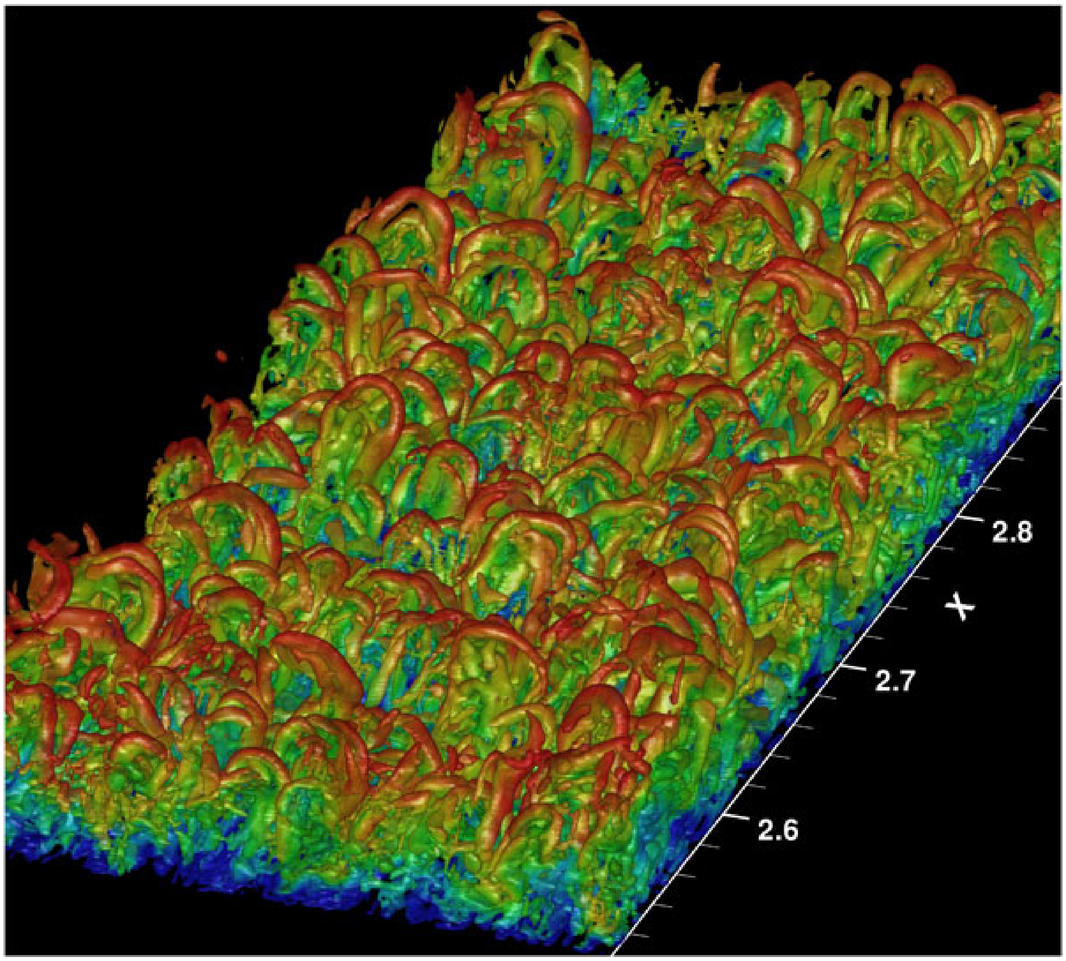


Figure 28.3: Shear flow over flat plate generates $x_1$ vorticity which generates secondary transversal opposing flow which generates rolls of $x_2$-vorticity attaching to the plate and bending into the flow, like a forest of sea tulips attaching to the sea bottom.

# Chapter 29

# Bluff Body Flow

> It seems evident that the mere viscosity of water would be utterly insufficient to account for [the eddies] when they are formed on a large scale, as in a mill pool or whirlpool?. Of course eddies are modified by viscosity, but except on quite a small scale I hold that viscosity is subordinate. Of course, it prevents a finite slip, which it converts into a rapid shear, but viscosity tends to stability, not to instability. (Stokes 1900)

We now give some examples in pictures of our main thesis that slightly viscous bluff body flow can be described as potential flow modified by 3d slip separation:

- landing gear: Fig. 29.1,
- sphere: Fig.29,
- NACA0012 wing: Fig. 29,
- car: Fig. 29.4,
- hill: Fig. 29.5.

> The brothers Wright started with gliding flights, using the results of our experiments on the lifting properties of curved surfaces. Their leading idea was to utilize this lifting effect in calm air by pushing these surfaces forward through the action of motors and propellers, foregoing the assistance of the wind, and thus attaining security and independence from the erratic changes of wind direction. (Addendum to *Birdflight as the Basis of Aviation* 1911).

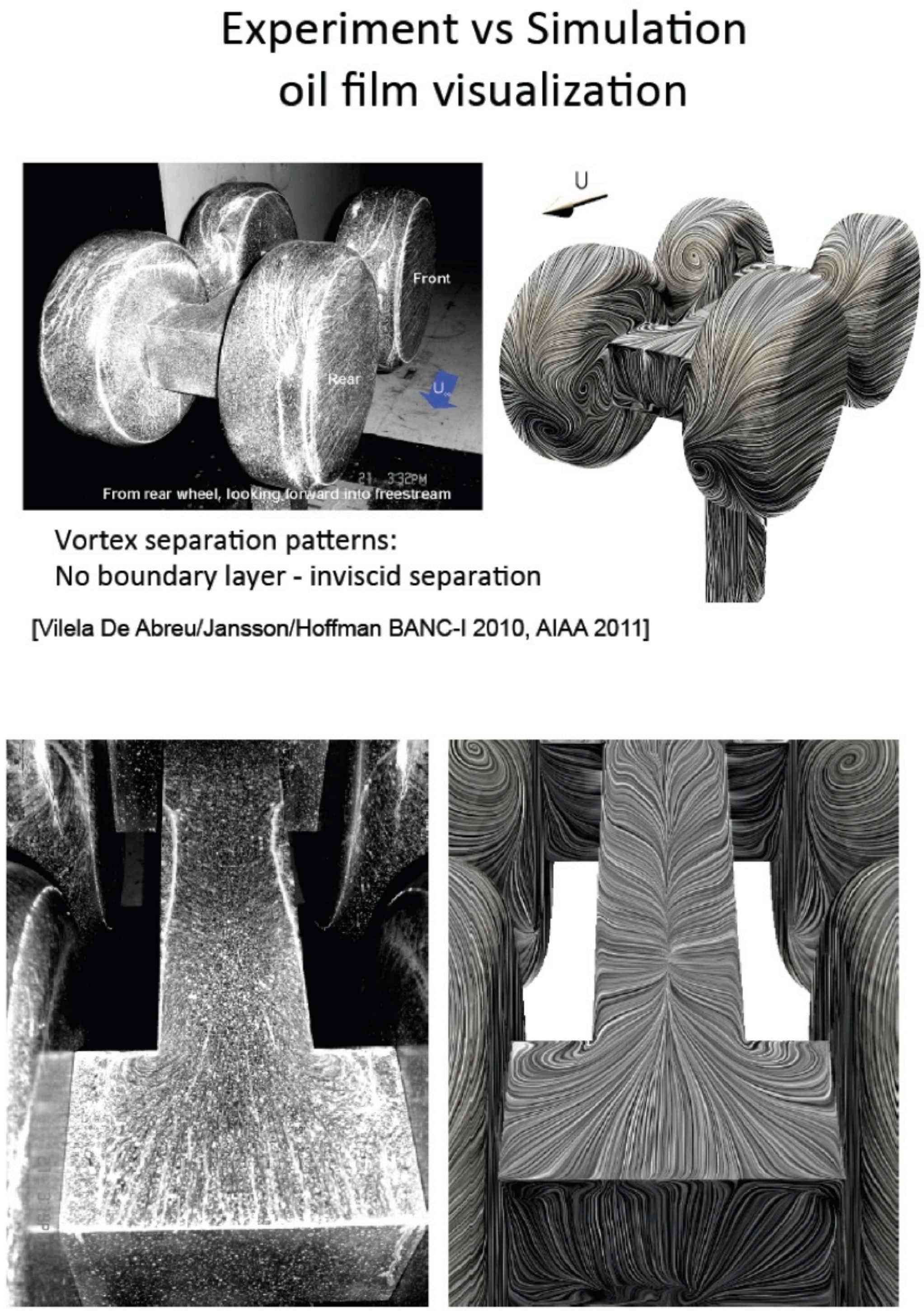


Figure 29.1: Oilfilm visualization of 3d rotational separation with point stagnation and parallel flow separation without stagnation.

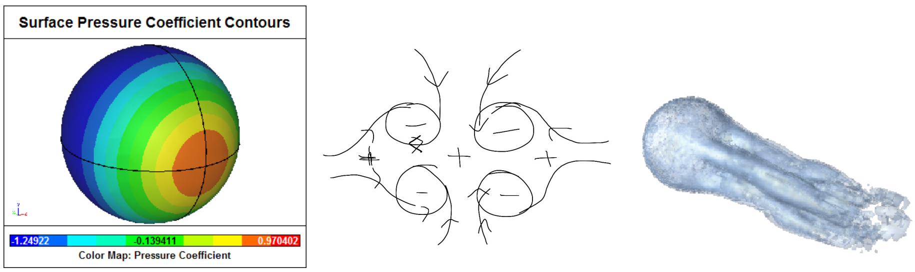


Figure 29.2: Pattern of exponential separation instability of potential flow around a sphere at its single point of stagnation (pressure left), forming four counterrotating rolls of streamwise vorticity (right) attaching around four points of stagnation (middle).

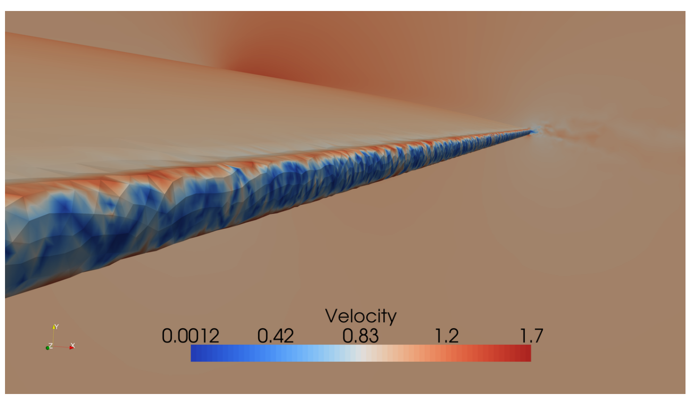


Figure 29.3: Velocity at trailing edge separation for NACA0012 wing showing zig-zag pattern of 3d rotattional separation.

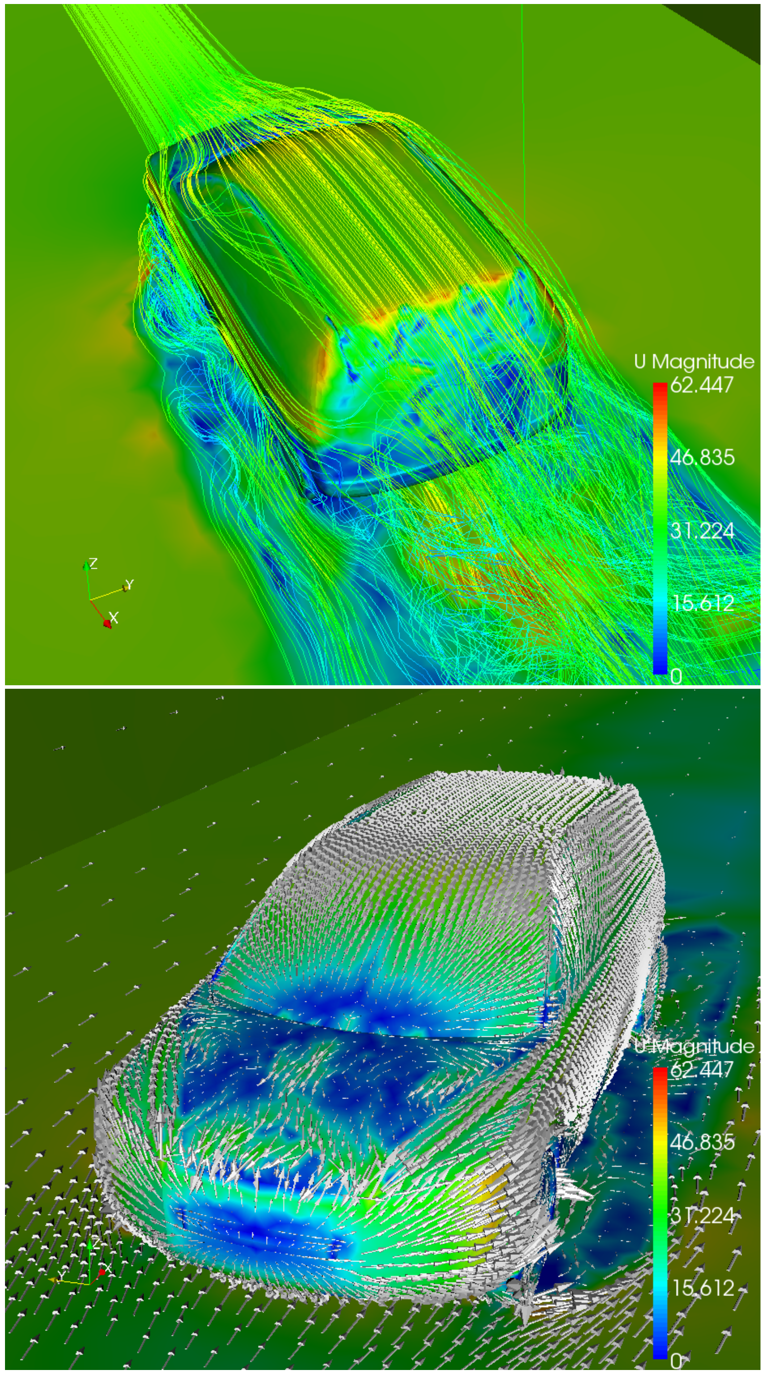


Figure 29.4: Velocity of turbulent flow around a car

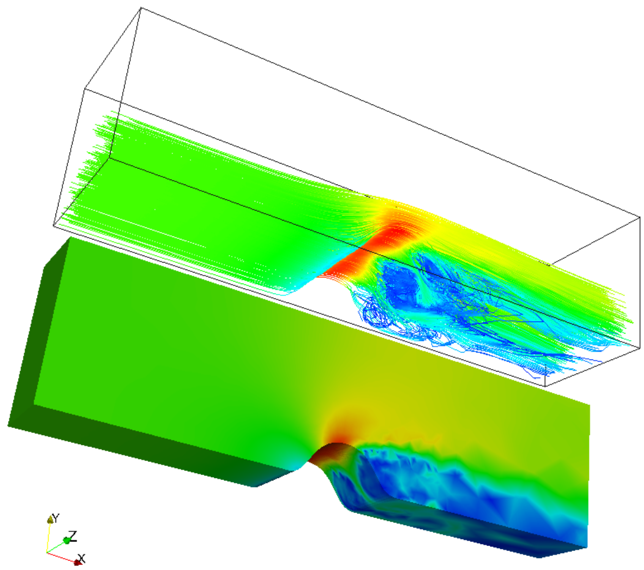


Figure 29.5: Separation after crest of hill.

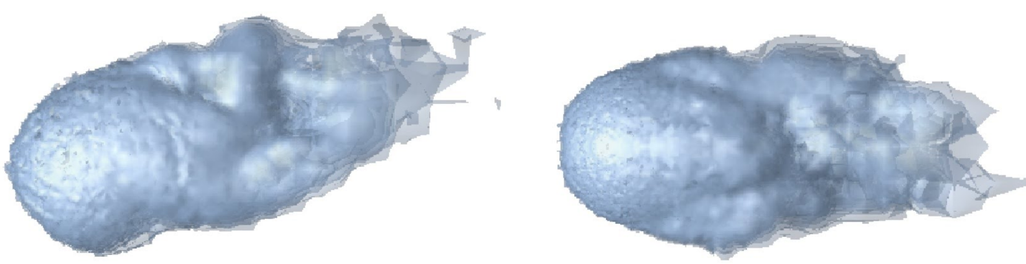

Figure 29.6: Rotating sphere with unsymmetric separation and lift and non-rotating sphere with symmetric separation and no lift.

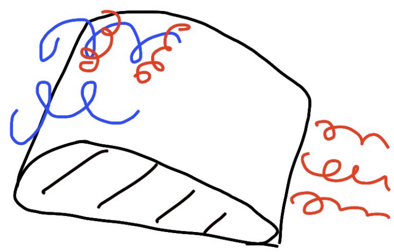

Figure 29.7: Generation of streamwise vorticity from accellerating parallel flow at the leading edge (blue) with secondary transversal vorticity (red), and opposing flow separation at the trailing edge (red) both associated with point stagnation.

# Chapter 30

# Scale Invariance and LES

> Of course, D'Alembert's Paradox is not a paradox at all, for his seemingly paradoxical findings were simply the correct results that naturally followed from the neglect of friction...today we recognize that the neglect of friction was the reason that d'Alembert was not able to calculate drag. (John D. Anderson in *History of Aerodynamics*)

## 30.1 Effect of Shear Layers

Computational simulations of e.g. the flow around a circular cylinder show that the large scale features of the flow (potential flow modified by 3d rotational separation), do not change under mesh refinement, see Fig. 30.1. We thus discover that the Navier-Stokes equations with slip boundary condition and vanishingly small viscosity show a form of *scale invariance* in the sense that the main features of bluff body flow depend on the shape of the body but not on its absolute size. The effect of descreasing viscosity would then be a sharpening of large scale features as in the sharpening of an image by decreasing smoothing effects, which increases contrast while leaving large scales structures invariant, see Fig. 26.1.

To estimate the sharpening effect of a decreasing viscosity $\nu$, we consider a (laminar) shear layer $L$, typically of width $d = \sqrt{\nu}$, connecting two fluid regions with different velocities with velocity gradient $|\nabla u|$ thus scaling with $1/\sqrt{\nu}$, with a total dissipation

$$D_\nu \equiv \int_L \int \nu |\nabla u|^2 ds \sim A\sqrt{\nu}, \tag{30.1}$$

where $A$ is the area of the layer. We thus find that the dissipation $D_\nu$ in a layer with bounded area tends to zero with $\nu$, which indicates that its effect vanishes with vansihing viscosity. We thus expect the effect of a no-slip boundary layer of a bluff body to vanish with vanishing viscosity, contrary to Prandtl's basic postulate. In the case of a turbulent shear layer, experimental data [124] shows $D_\nu \sim A\nu^{0.2}$, and the conclusion is similar.

On the other hand, the area of the interior layers of the rolls the 3d rotational separation increases with the length of the rolls, which allows positive dissipation with major effect on drag and lift. The diameter of the rolls would then be determined by the trailing edge radius, which is what effectively determines the drag and lift of the bluff body in elegant separation, while the length of the rolls behind the body would increase with decreasing viscosity without change of drag and lift. We may thus expect convergence of lift and drag under mesh refinement, and this is also what we observe.

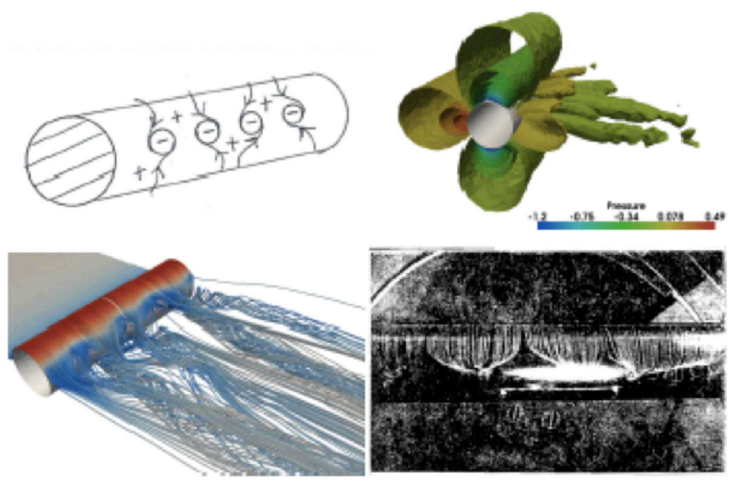

Figure 30.1: Large scale structure invariant under mesh refinment.

## 30.2 Effect of Trailing Edge Diameter

From scale invariance we expect the 3d rotational separation pattern at the trailing edge to remain the same as the radius $d$ of the trailing edge tends to zero. The vorticity and pressure gradient then scale as $\frac{1}{d}$ locally, the pressure locally as 1 with a total effect scaling as $d$, thus tending to zero with $d$. With 3d rotational separation the flow manages to separate without the high pressure of 2d potential flow, which can be seen as the secret of flight. As the radius tends to zero the flow may be viewed to form a type of vortex sheet with a complex pattern of counter rotating streamwise vorticity, corresponding to oscillating high and low pressures which is small in mean by cancellation. By scale similarity we thus expect lift and drag to remain the same as the radius of the trailing edge decreases to zero, which is also observed experimentally, as long as the Reynolds number is large enough.

## 30.3 Large Eddy Simulation (LES)

It is the scale invariance of solutions to the Navier-Stokes equations with slip boundary condition and vanishing viscosity, which make solutions computable on meshes resolving only the large scale features of the flow in what can be referred to as *Large Scale Simulation (LES)*, without any need of user-defined turbulence models. This makes computational solution of slightly viscous flow possible by removing the Prandtl spell of having to resolve thin boundary layers from no-slip boundary conditions, which has paralyzed fluid mechanics. In fact, a no-slip boundary condition is mathematically incompatible with vanishing viscosity and Prandtl's insistence on using no-slip can be seen as a consequence of Prandtl's limitation as mathematician witnessed by e.g. his student von Karman.

If now slightly viscous bluff body flow is computable in general by LES, a fundamentally new capability is brought into fluid mechanics.

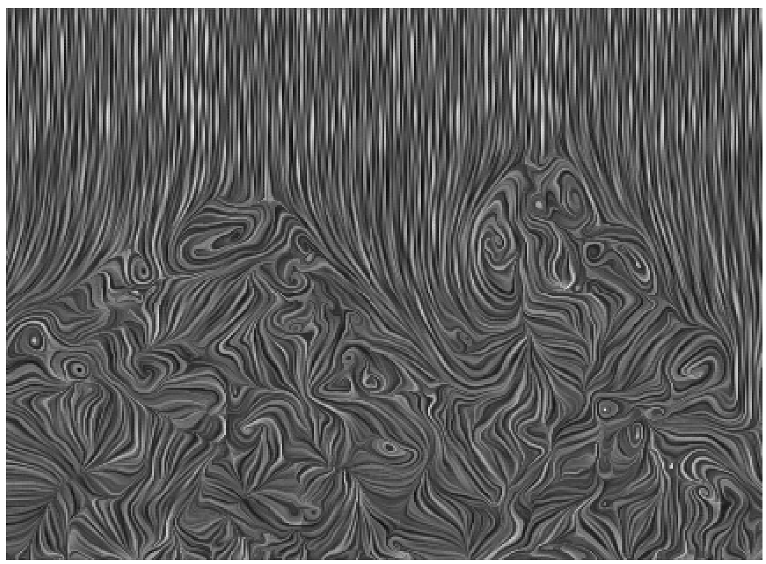

Figure 30.2: Scale invariance in oilfilm representation of separation pattern on top of wing surface at $aoa = 17$.

# Part V

# Incorrect Theory

# Chapter 31

# Incorrect Theories for Uneducated

> It's all one interconnected system. Unless the overall result of that system is for air to end up lower than it was before the plane flew by, there will be no lift. Wings move air downward, and react by being pushed upward. That's what makes lift. All the rest is just interesting details[90].

> The field of hydrodynamic phenomena which can be explored with exact analysis is more and more increasing. (Zhukovsky, 1911)

## 31.1 The Value of Incorrect Theory

To get perspective on the New Theory Flight theory we now check out how flight is explained in standard scientific literature and popular science. We start with some popular theories listed as incorrect by NASA and complete the picture with a theory due to Kutta-Zhukovsky-Prandtl viewed to offer a scientific explanation of both lift and drag, but as we will see is also incorrect.

To properly understand a correct theory, it can be helpful to study an incorrect theory which has gained some prominence, because even an incorrect theory may capture an element of truth, and the correct theory is the one that captures all elements.

However, if there is a correct theory none of the incorrect theories can survive, as illustrated e.g. by the homogeneity of the (correct) homo sapiens without any surviving (incorrect) Neanderthalers.

## 31.2 Incorrect Theories: NASA

You have probably heared some of the explanations offered in popular science, like higher velocity and lower pressure on the upper surface of the wing because it is curved and air there has a longer path to travel than below? Or maybe you are an aeroplane engineer or pilot and know very well why an airplane can fly, because lif is generated by circulation?

In either case, you should get a bit worried by reading that the authority NASA on its website [117] dismissses all popular science theories for lift, including your favorite one, as being incorrect, but then refrains from presenting any theory claimed to be correct! NASA surprisingly ends with an empty out of reach:

- *To truly understand the details of the generation of lift, one has to have a good working knowledge of the Euler Equations*.

This is just a fancy way of expressing that not even NASA understands what keeps an airplane in the air. Of course, it is not possible to find the correct theory by removing all incorrect theories, like forming a correct sculpture out of a block of stone by removing all pieces of stone which are not correct, because there is no list of all theories which will give the correct theory by eliminating all the (infinitely many) incorrect theories.

To present incorrect theories at length is risky pedagogics, since the student can get confused about what is correct and not, but signifies the confusion and misconceptions still surrounding the mechanisms of flight. If a correct theory was available, there would be no reason to present incorrect theories, but the absence of a correct theory is now seemingly covered up by presenting a multitude of incorrect theories.

The following three incorrect theories listed by NASA are commonly presented in text books directed to a general audience. Take a look and check out which you have met and if you they are convincing to you.

## 31.3 Trivial Theory: NASA

The closet NASA comes to a correct theory is the trivial theory depicted in Fig. 31.4: If there is downwash then there is lift. This follows directly by Netwon's 3rd law, but the question is why there is downwash?

> The following results were obtained by attaching wing-like surfaces to a large rotating or whirling machine, having a diameter of 7 metres, the test

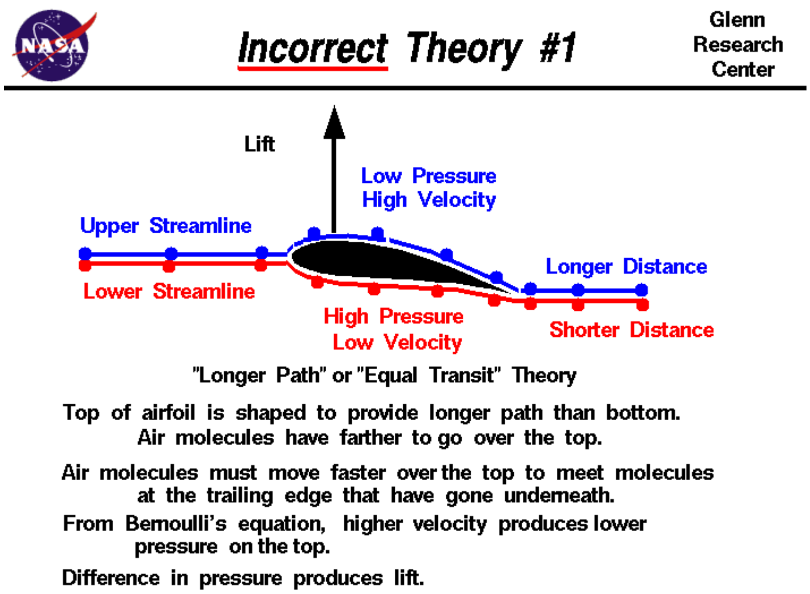


Figure 31.1: The ”Longer Path Theory” is wrong, because an airplane can fly upside down.

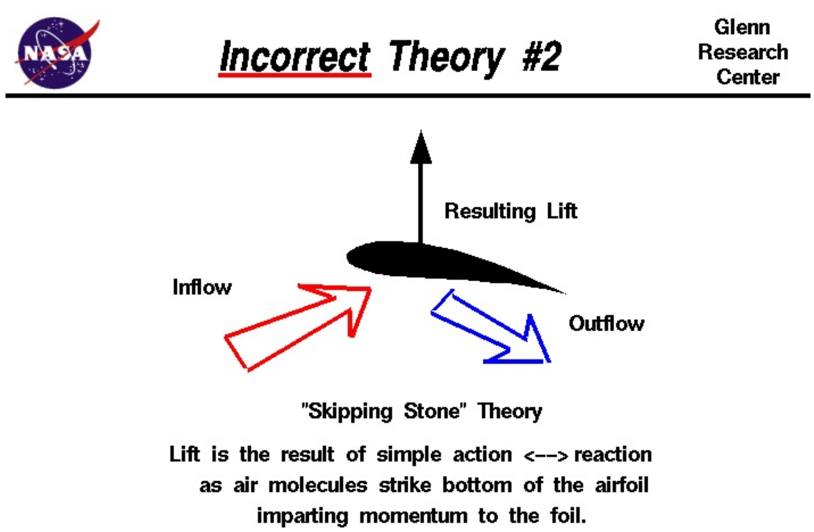


Figure 31.2: The ”Skipping Stone Theory” is Newton’s theory, which gives a way too small lift.

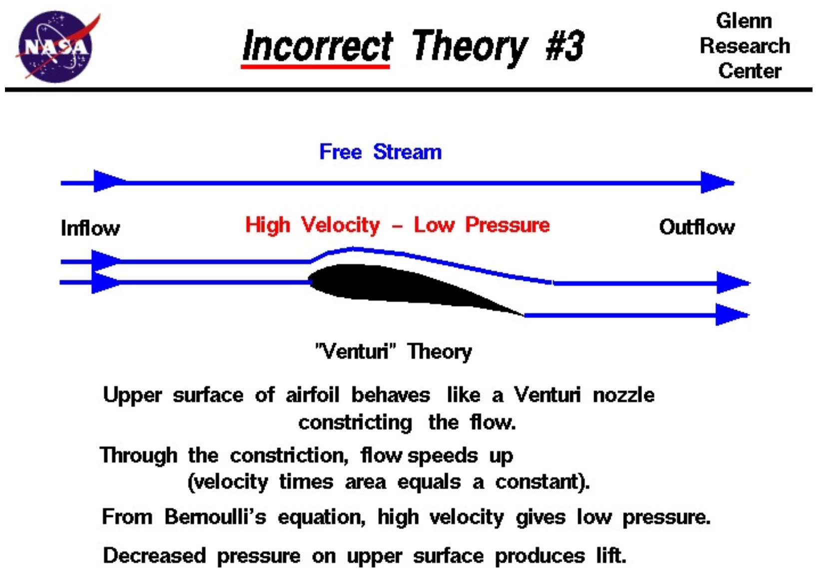


Figure 31.3: The ”Venturi Theory” of higher speed above the wing and thus lower pressure because of the curvature of the upper surface of the wing, is wrong because an airplane can fly upside down.

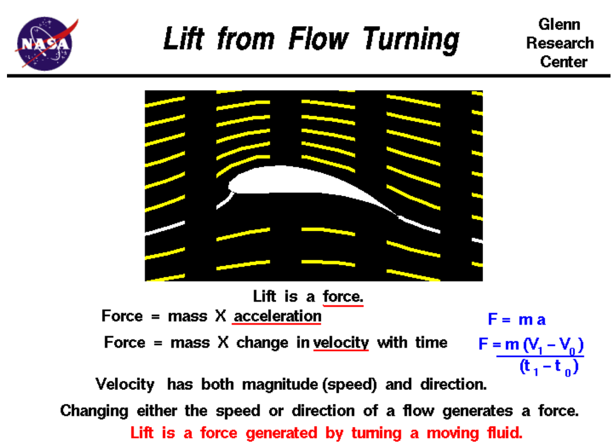


Figure 31.4: Trivial tautological theory of lift presented as correct by NASA.

surfaces being arranged at a height of 4.5 metres above ground. (*Birdflight as the Basis of Aviation* )

# Chapter 32

# Incorrect Theory for Educated

> Lift is a lot trickier. In fact it is very controversial and often poorly explained and, in many textbooks, flat wrong. I know, because some readers informed me that the original version of this story was inaccurate. I've attempted to correct it after researching conflicting "expert" views on all this....If you're about fed up, rest assured that even engineers still argue over the details of how all this works and what terms to use [45].

> If the magnitude of the circulation is known, the Kutta-Joukowsky formula, is of practical value for the calculation of lift. However, it must be clarified as to what way the circulation is related to the geometry of the wing profile, to the velocity of the incident flow, and to the angle of attack. This interrelation cannot be determined uniquely from theoretical considerations, so it is necessary to look for empirical results. (Schlichting-Truckenbrodt in *Aerodynamics of the Airplane*)

## 32.1 Newton, d'Alembert and Wright

When Lilienthal in the 1890s showed that human gliding flight was possible and the Wright brothers in 1903 showed that powered human flight was possible by putting a 12 hp engine on their *Flyer*, this was in direct contradiction to the accepted mathematical theory by Newton, who computed the lift from downwash of air particles hitting the lower part of the wing and found it to be so small that human flight was unthinkable.

Newton was supported by the mathematician d'Almembert who in 1755 proved that both lift and drag was zero for for *potential flow*, which seemed to describe the flow of air around a wing.

## 32.2 Kutta-Zhukovsky: Circulation: Lift

To save theoretical aerodynamics from complete collapse, a theory showing substantial lift had to be invented, and such a theory was delivered in the 1902 doctoral thesis by the young Martin Kutta (1866 - 1944) in Germany and independently in 1906 by the older more experienced Nikolaj Zhukovsky (1847 - 1921) in Russia, who both modified zero lift potential flow by adding a large scale *circulation* around the wing, as suggested already in 1892 by Frederick Lanchester (1868 - 1946) in England. Kutta-Zhukovsky showed that if there is circulation then there is lift, but could not explain from where the circulation came.

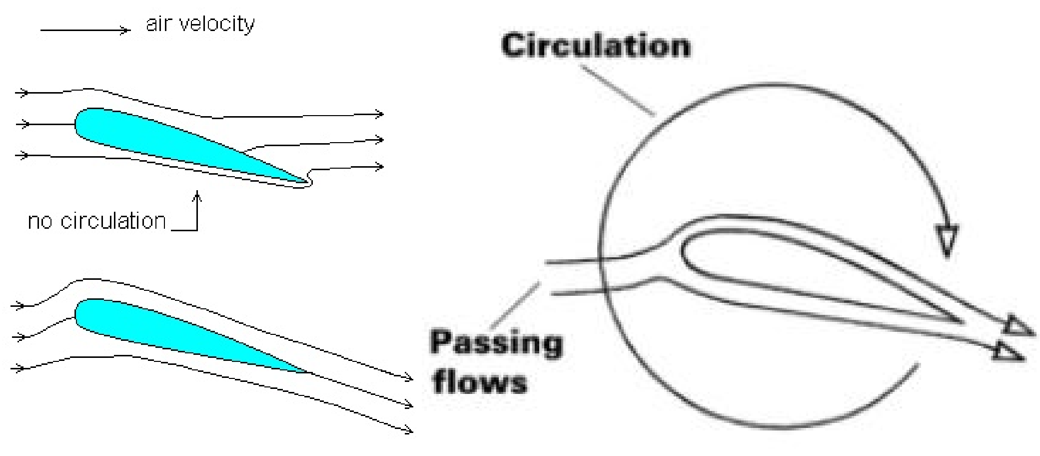


Figure 32.1: Top left figure shows potential flow without downwash (the incoming flow is not redirected) and thus no lift. The bottom left shows a flow with circulation resulting from adding a large scale rotational flow around the wing as shown in the right figure.

## 32.3 Magnus Effect by Circulation

The incorrect circulation theory by Kutta-Zhukovsky originates from an incorrect explanation of the Magnus Effect of a rotating ball moving through air based on large scale circulation around the ball supposedly being created somehow by viscous forces from the rotation, as illustrated in Fig. **??**. The true explanation of the Magnus effect is unsymmetric separation with delayed separation where the relative velocity is smaller and the skin friction smaller, see Fig. **??**.

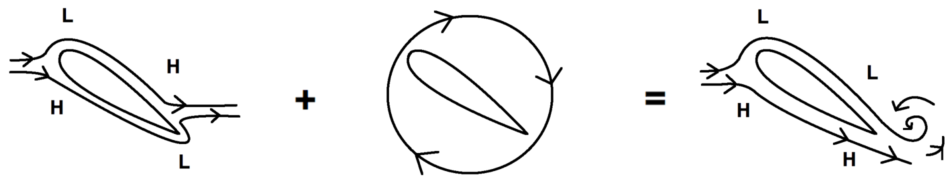


Figure 32.2: Kutta-Zhukovsky theory of lift combining potential flow (left) with large scale circulation, thus changing the zero lift pressure distribution of potential flow to lifting flow by shifting the high (H) and low (L) pressure zones at the trailing edge of the flow by unphysical circulation around the section (middle) resulting in flow with downwash/lift and starting vortex (right).

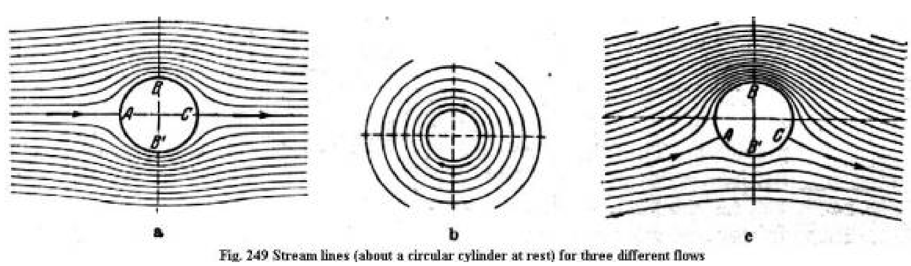


Figure 32.3: Potential flow augmented by circulation creating unsymmetric flow with lift as incorrect explanation of the Magnus Effect.

## 32.4 Prandtl: Boundary Layer: Drag King

This was timely done by the German physicists Ludwig Prandtl, who in a short note in 1904 saved aerodynamics by opening the possibility that both circulation with lift and drag somehow could originate from a thin *boundary layer*.

The theory by Kutta-Zhukovsky-Prandtl has become the theory for the educated specialists of aerodynamics, who very well understand that the theories for uneducated are incorrect. Today 100 years later, this is still the theory of flight presented in text books: Lift comes from circulation, and circulation and drag comes from a thin boundary layer.

The original 2d circulation theory by Kutta-Zhukovsky was extended by Prandtl using ideas put forward by Lanchester into the 3d Lifting Line Theory. Both theories are unphysical being based on unstable mathematic, as discussed in more detail in the two following sections.

There is an alternative to circulation as generation of lift, referred to as the *Coanda effect* stating that upper surface suction is an effect of viscosity causing the flow to stick to the surface, but the support of this theory in the literature is weak.

State-of-the-art thus tells you that to compute drag and lift of an airplane you need to resolve the boundary layer, which however requires 10 quadrillion ($10^{16}$ mesh points, which will require 50 years of Moore's law improving the computational power by a factor of $10^{10}$.

But is the theory of Kutta-Zhukovsky-Prandtl for educated correct? Do we have to wait 50 years to compute lift and drag of an Airbus? The answer in this book is No: Lift and drag and more generally any flow quantity of interest in subsonic flight, can be determined by computationally solving the incompressible Navier-Stokes equations with a slip boundary condition using millions of mesh-points without resolving thin boundary layers.

## 32.5 Vortex Stretching: Kelvin's Theorem Illposed

Formally applying the curl operator $\nabla\times$ to the momentum equation of (21.1), with $\nu = \beta = 0$ for simplicity, we obtain the *vorticity equation*

$$\dot{\omega} + (u \cdot \nabla)\omega - (\omega \cdot \nabla)u = \nabla \times f \quad \text{in } \Omega, \tag{32.1}$$

which is a convection-reaction equation in the vorticity $\omega = \nabla \times u$ with coefficients depending on $u$, of the same form as the linearized equation (26.5), with

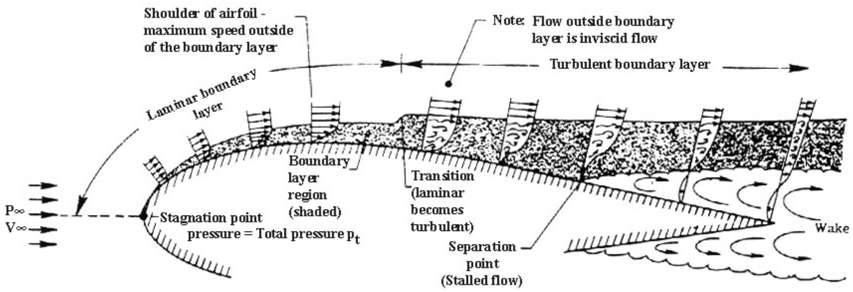


Figure 32.4: Prandtl's boundary layer theory in pictures.

similar properties of exponential perturbation growth $\exp(|\nabla u|t)$ referred to as *vortex stretching*.

*Kelvin's theorem* formally follows from this equation assuming the initial vorticity is zero and $\nabla \times f = 0$ (and $g = 0$), but exponential perturbation growth makes this conclusion physically incorrect: We hace seen that substantial vorticity can develop from irrotational potential flow even with slip boundary conditions. Kelvin's theorem is thus illposed and thus unphysical.

The corner stone of circulation theory is thus unphysical, because stability aspects must be taken into account in a physically meaningful statement, also referred to as wellposedness/illposedness in mathematical literature. This is one of several arguments showing that circulation theory is unphysical; a corner stone of the criticism of this theory in this book.

## 32.6 Lifting Line Theory Illposed

Circulation theory in the form of the Prandtl/Lanchester Lifting Line Theory relies on three mathematical theorems by Helmholtz concerning "vortex filaments" or lines of concentrated vorticity in inviscid incompressible flow:

1. Theorem 1: *The strength of a vortex filament is constant along its length.*

2. Theorem 2: *A vortex filament cannot end in the fluid; it must extend to the boundaries of the fluid or form a closed path.*

3. Theorem 3: *In the absence of of rotational forces, a fluid that is initially irrotational remains irrotational*. (Kelvin's theorem).

We just learned that Theorem 3 is not wellposed and thus does not describe physics. Nevertheless, Lifting Line Theory is based on Theorem 3 with a closed lifting line of vorticity formally adding to zero formed by (i) transversal vorticity along the wing, (ii) streamwise vorticity extending from the wing tips (iii) transversal starting vortex closing the loop, as illustrated in the left picture of Fig. 32.5. The right picture shows Prandtl's concept of streamwise vorticity issued from the wing compensating variation in transversal vorticity along the wing, again from an idea of closed circuits of vorticity summing to zero.

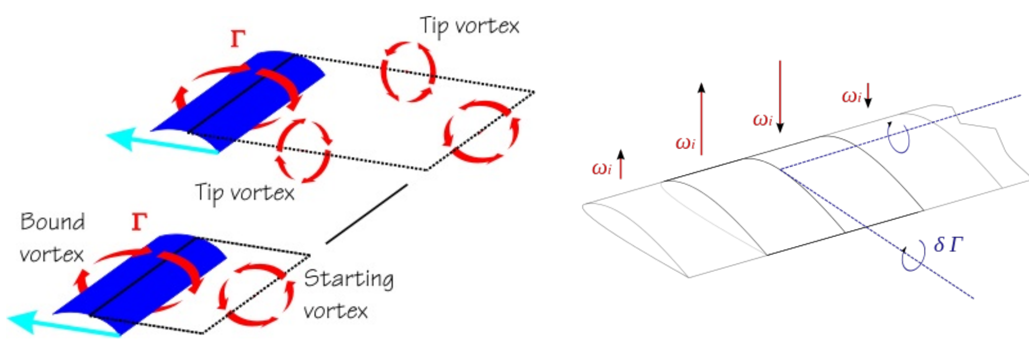


Figure 32.5: Lifting Line Theory: Closed line of vorticity formally adding to zero (left) and streamwise vorticity compensating variation in transversal vorticity again in circuits adding to zero. (right).

The streamwise vorticity emerging from the airfoil compensating variation of transversal vorticity in Lifting Line Theory, superficially connects to the counter-rotating vortex lines attaching to the trailing edge in the 3d rotational slip separation of the New Theory, but the physics is entirely different:

In the New Theory "vortex filaments" are created on the surface from opposing flow instability, while in the Lifting Line Theory they formally appear from variation of transversal vorticity as a consequence to the illposed Theorem 3.

Note that the "vortex filaments" of streamwise vorticity attaching to the trailing edge in the New Theory conforms to Theorem 2 with "vortex filaments" allowed to start from the boundary, as a correct statement. Theorem 1 is questionable, Theorem 2 is ok and Theorem 3 is illposed and not ok. Lifting Line Theory is based on Theorem 3.

One may say that the New Theory is based on unstable physics, while Lifting Line Theory is based on illposed formal mathematics. Unstable physics is real, while unstable formal mathematics is unreal.

## 32.7 More Confusion

Many sources, in addition to NASA, give witness of the lack of convincing scientific answer of how a wing can generate lift with small drag. We give here a sample starting with more from [130]:

- *"Here we are, 100 years after the Wright brothers, and there are people who give different answers to that question," said Dr. John D. Anderson Jr., the curator for aerodynamics at the Smithsonian National Air and Space Museum in Washington. "Some of them get to be religious fervor."*

- *The answer, the debaters agree, is physics, and not a long rope hanging down from space. But they differ sharply over the physics, especially when explaining it to nonscientists. "There is no simple one-liner answer to this," Dr. Anderson said.*

- *The simple Newtonian explanation also glosses over some of the physics, like how does a wing divert air downward? The obvious answer – air molecules bounce off the bottom of the wing – is only partly correct.*

- *If air has to follow the wing surface, that raises one last question. If there were no attractive forces between molecules, would there be no flight? Would a wing passing through a superfluid like ultracold helium, a bizarre fluid that can flow literally without friction, produce no lift at all? That has stumped many flight experts. "I've asked that question to several people that understand superfluidity," Dr. Anderson, the retired physicist, said. "Alas! They don't understand flight."*

- *It is important to realize that, unlike in the two popular explanations described earlier (longer path and skipping stone), lift depends on significant contributions from both the top and bottom wing surfaces. While neither of these explanations is perfect, they both hold some nuggets of validity. Other explanations hold that the unequal pressure distributions cause the flow deflection, and still others state that the exact opposite is true. In either case, it is clear that this is not a subject that can be explained easily using simplified theories. Likewise, predicting the amount of lift created by wings has been an equally challenging task for engineers and designers in the past. In fact, for years, we have relied heavily on experimental data collected 70 to 80 years ago to aid in our initial designs of wing.*[52]

- $http://www.youtube.com/watch?v=uUMlnIwo2Qo$

- $http://www.youtube.com/watch?v = ooQ1F2jb10A$
- $http://www.youtube.com/watch?v = kXBXtaf2TTg$
- $http://www.youtube.com/watch?v = 5wIq75_BzOQ$
- $http://www.youtube.com/watch?v = khca2FvGR - w$

Still more surprising effects are obtained if the curvature is increased to $\frac{1}{12}$ of width; these are recorded in Plate IV (see Fig. **??**). For 90 degrees we again have $L = 0.13AV^2$ that is to say, the same as though the surface were plane. But at other inclinations the air pressures differ most materially from those of a plane surface under similar inclinations and velocities. For the sake of easy comparison, the pressures for a plane surface are shown dotted in Plate IV., Fig. 1, and the advantages of a curved surface over a plane, for flight purposes, are clearly shown. (*Birdflight as the Basis of Aviation*

# Chapter 33

# Summary of State-of-the-Art

> You'd think that after a century of powered flight we'd have this lift thing figured out. Unfortunately, it's not as clear as we'd like. A lot of half-baked theories attempt to explain why airplanes fly. All try to take the mysterious world of aerodynamics and distill it into something comprehensible to the lay audience–not an easy task. Nearly all of the common "theories" are misleading at best, and usually flat-out wrong [87].

> A very satisfactory explanation of the physical process in the boundary layer between a fluid and a solid body could be obtained by the hypothesis of an adhesion of the fluid to the walls, that is, by the hypothesis of a zero relative velocity between fluid and wall (no-slip boundary condition). (Prandtl)

> Actually, through proper application of the laws of modern aerodynamics it is possible today to derive a major portion of the aerodynamics of the airplane from purely theoretical considerations. The very comprehensive experimental material, available in the literature, has been included only as far as necessary to create a better physical concept and to check the theory. We wanted to emphasize that decisive progress has been made not through accumulation of large numbers of experimental results, but rather through synthesis of theoretical considerations with a few basic experimental results. Through numerous detailed examples, we have endeavored to enhance the reader's comprehension of the theory. (Schlichting-Truckenbrodt)

## 33.1 Newton

Classical mathematical mechanics could not give an answer to the mystery of gliding flight: Newton computed by elementary mechanics the lift of a tilted flat plate redirecting a horisontal stream of fluid particles, but obtained a disappointingly small value proportional to the square of the angle of attack. To Newton the flight of birds was inexplicable, and human flight certainly impossible.

## 33.2 D'Alembert and Potential Flow

D'Alembert followed up in 1752 by formulating his paradox about zero lift/drag of *inviscid incompressible irrotational steady flow* referred to as *potential flow,* which seemed to describe the airflow around a wing since the viscosity of air is very small so that it can be viewed as being inviscid (with zero viscosity). Mathematically, potential flow is given as the gradient of a *harmonic funtion* satisfying *Laplace's equation*.

At speeds less than say 300 km/h air flow is almost incompressible, and since a wing moves into still air the flow it could be be expected to be irrotational without swirling rotating vortices. D'Alembert's mathematical potential flow thus seemed to capture physics, but nevertheless had neither lift nor drag, against all physical experience. The wonderful mathematics of potential flow and harmonic functions thus showed to be without physical relevance: This is *D'Alembert's Paradox* which came to discredit mathematical fluid mechanics from start [104, 125, 92].

To explain flight D'Alembert's Paradox had to be resolved, but nobody could figure out how and it was still an open problem when Orwille and Wilbur Wright in 1903 showed that heavier-than-air human flight in fact was possible in practice, even if mathematically it was impossible.

## 33.3 Kutta-Zhukovsky-Prandtl

Mathematical fluid mechanics was then saved from complete collapse by the young mathematicians Kutta and Zhukovsky, called the father of Russian aviation, who explained lift as a result of perturbing potential flow by a large-scale circulating flow or *circulation* around the 2d section of a wing, and by the young physicist Prandtl, called the father of modern fluid dynamics, who explained drag as a result of a *viscous boundary layer* [119, 120, 124, 93].

This is the basis of state-of-the-art [21, 72, 74, **?**, 98, 126, 137], which essentially is a simplistic theory for lift without drag at small angles of attack in inviscid flow and for drag without lift in viscous flow. However, state-of-the-art does not supply a theory for lift-and-drag covering the real case of *3d slightly viscous tur-*

*bulent* flow of air around a 3d wing of a jumbojet at the critical phase of take-off at large angle of attack and subsonic speed (270 km/hour), as evidenced in e.g. [47, 89, 90, 91, 130, 7, 52, 113, 116]. The simplistic theory allows an aeroplane engineer to roughly compute the lift of a wing a crusing speed at a small angle of attack but not lift-and-drag in general [59, 131]. The lack of mathematics has to be compensated by experiment and experience. The first take off of the new Airbus 380 must have been a thrilling experience for the design engineers.

The state-of-the-art theory of flight can be summarized as either (i) correct and trivial or (ii) nontrivial and incorrect:

in the following forms:

- Downwash generates lift: trivial without explanation of reason for downwash from suction on upper wing surface.
- Low pressure on upper surface: trivial without explanation why.
- Low pressure on curved upper surface because of higher velocity (by Bernouilli's law), because of longer distance: incorrect.
- Coanda effect: The flow sticks to the upper surface by viscosity: incorrect.
- Kutta-Zhukovsky: Lift comes from circulation: incorrect.
- Prandtl: Drag comes mainly from viscous boundary layer: incorrect.

## 33.4 Why Prandtl Was Wrong

The legacy of Prandtl as the Father of Modern Aerodynamics is that slightly viscous flow around a solid body is determined by a thin boundary layer around the body arising from a no-slip boundary condition. In particular, both drag and lift of a body moving through a fluid are effects of a no-slip boundary condition creating a thin boundary layer.

Computational solution of the Navier-Stokes equations with slip boundary conditions give lift and drag of a wing in correspondence with observation. This shows that lift and drag of a wing do not originate from thin boundary layers associated with no-slip boundary conditions, contrary to the fundamental claim of Prandtl's boundary layer theory.

## 33.5 Why Kutta-Zhukovsky Were Wrong

Lift is generated by unsymmetric 3d rotational slip separation at a rounded trailing edge. Circulation is not generated at a sharp trailing edge. Lift is not generated by circulation generated at a sharp trailing edge. Kutta-Zhukovsky circulation theory of lift is an unphysical theory.

# Chapter 34

# Text Books

> For man flying forward with suitably curved wings, this energy under the most favourable but probably unattainable conditions, would be $0.4\,hp$, but even this amount could only be exerted by man for a short time. (*Birdflight as the Basis of Aviation* )

To give perspective on the New Theory of Flight it is useful to understand how the Old Theory is presented in the literature and why it is unphysical and thus incorrect. The text book theory of flight has been remarkably stable over 100 year with little improvement in accuracy as if the theory once and for all was set by Kutta-Zhukovsky-Prandtl. But science does not work that way: If no progress is made on a complex scientific topic, like flight, this is a strong indication that what was hammered in stone is incorrect science.

The following text books all present versions of the Kutta-Zhukovsky-Prandtl 2d theory as the Old Theory of Flight:

1. Aircraft Flight, RH Barnard and Dr Philpott, Pearson Prentice Hall, 1989,1995, 2004.

2. Prandtls Essentials of Fluid Mechanics, Herbert Oertel (Ed.), Springer, 2004.

3. Aerodynamics of Wings and Bodies, Holt Ashley and Marten Landahl, Dover, 1985 (1965).

4. Introduction to the Aerodynamics of Flight, Theodore A. Talay, Langley Research Center.

5. Aerodynamics of the Airplane, Hermann Schlichting and Erich Truckenbrodt, Mc Graw Hill, 1979.

6. Airplane Aerodynamics and Performance, Jan Roskam and C T Lan, Darcorporation, 1997.

7. Fundamentals of Aerodynamics, John D Anderson, McGraw-Hill, 2001.

8. Fuhrer durch die Stromungslehre, L Prandtl, Hafner Pub, 1952.

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20. Flight Physics by Torenbeek and Wittenberg, Springer, 2009

21. Two-dimensional Problems in Hydrodynamics and Aerodynamics, L. I. Sedov, Interscience, 1965

22. Principles of Ideal-Fluid Aerodynamics, K. Karamcheti, Krieger, 1980

23. Low Speed Aerodynamics: From Wing Theory to Panel Methods, J. Katz and A. Plotkin

24. Basic Wing and Airfoil Theory, A. Pope, McGraw-Hill, 1951

25. Theory of Wing Sections, E. Abbott and A.von Doenhoff, Dover 1959

26. Encyclopedia of Flight, T. Irons-George, Salem Press, 2002.

We now comment shortly on the main message presented in a selection of the above books.

## 34.1 Aircraft Flight by Barnard-Philpott

- *Most of the worlds aircraft are flown by people who have a false idea of what keeps them in the air.*

- *Our objective is to give an accurate description of the principles of flight in simple physical terms. In the process of doing so, we will need to demolish some well-established myths.*

- *One popular and misleading explanation refers to a typical cambered wing section profile. It is argued, that the air that takes the longer upper-surface-route has to travel faster than that which takes the shorter under-surface-route, in order to keep up.*

- *We find that the production of lift depends, rather surprisingly, on the viscosity or stickiness of air*

- *The feature of the flows meeting at the trailing edge is known as the Kutta conditionthe viscosity is thus ultimately responsible for the production of lift.*

- *A major breakthrough came when it was realized that a wing thus behaves rather like rotating vortex placed in an air stream. This apparent odd conceptual jump was important, because it was relatively easy to mathematically analyze the effect of a simple vortex placed in a uniform flow of air.*

- *It is the viscosity working through the mecahsim of boundary layer separation and starting vortex formation, that is ultimately responsible for the generation of lift.*

We see that the authors seek to sell Kutta-Zhukovskys circulation theory, while admitting that they find it odd, but then try to cover up by hinting at boundary layer effects of small viscosity. This is representative of what is considered as the scientific explanation flight presented in the literature.

The New Theory shows that lift does neither come from circulation nor form boundary layers and thus that the authors, together with all other authors explaining flight, belong to the large group of people who have a false idea of what keeps airplanes in the air.

## 34.2 Mechanics of Flight by Kermode

From Preface to 11th edition by Barnard-Philpott:

- *The late A. C. Kermode was a high-ranking Royal Air Force officer responsible for training. He also had a vast accumulation of practice aeronautical experience, both in the air and on the ground. It is this direct knowledge that provided the strength and authority of this book.*

This suggests that the theory is weak. From Chapter 1 Mechanics:

- *The flight of an airplane provide glorious examples of the principles of mechanics.*

The weaker the higher pitch. From Chapter 3 Aerofoils - subsonic speeds:

- *An interesting way of thinking about the airflow over wings is the theory of circulation. The fact that the air is speeded over the upper surface, and slowed down on the under surface of a wing, can be considered as a circulation round the wing superimposed upon the general sped of airflow (this does not mean that particles of air actually travel around the wing). This circulation is , in effect, the cause of lift. But this is not all. When the wing starts to move, or when the lift is increased, the wing sheds and leaved behind a vortex rotating in the opposite direction to the circulation round the wing – sometimes called the starting vortex – so there is a complete system of vortices, round the wing (then the wing-tip vortices) and finally the starting vortex. The engineer power keeps renewing the circulation round the wing.*

Kermode here struggles to make sense of circulation theory: "this does not mean that particles of air actually travel around the wing": circulation without circulation. Reference to an experiment performed by Prandtl is made:

- *This is not just theory; the flow over the wing can clearly be seen in experiments, while the starting vortex is easily demonstrated by starting to move a model wing, or even one's hand, through water.*

But this is a misleading experiment: The vortices seen by moving a hand trough water (in a bubble bath tub), do not explain the creation of lift of a wing.

## 34.3 Aerodynamics of the Airplane by Schlichting

- *In many technical applications, viscous flow can be neglected in order to simplify the laws of fluid dynamics (inviscid flow). This is down in the theory of lift of airfoils (potential flow).*

- *To determine the drag of bodies, however, the viscosity has to be considered (boundary-layer theory). The theory of inviscid, incompressible flow has been developed mathematically in detail, giving, in many cases, a satisfactory description of the actual flow, for example, in computing airfoil lift at moderate flight velocities. On the other hand, this theory fails completely for the computation of body drag.*

- *Only a very few comprehensive presentations of the scientific fundamentals of the aerodynamics of the airplane have ever been published. The study of the aerodynamics of the airplane requires a thorough knowledge of aerodynamic theory. The lift can be obtained in very good approximation from the theory of inviscid flow.*

- *Lift production on an airfoil is closely related to the circulation of its velocity near-field.*

- *There is higher pressure on the lower surface, lower pressure on the user surface. It follows, from the Bernoulli equation, that the velocities on the lower and upper surfaces are lower or higher, respectively. With these facts in mind it follows that the circulation differs from zero. The velocity field can be thought to have been produced by a clockwise-turning vortex that is located on the airfoil. This vortex, which apparently is of basic importance for the creation of lift, is called the bound vortex of the wing.*

- *If the magnitude of the circulation is known, the Kutta-Zhukovsky formula, is of practical value for the calculation of lift.The circulation cannot be* *determined uniquely from theoretical considerations, so it is necessary to look for empirical results. The magnitude of the circulation can be derived from experience, namely, that there is no flow around the trailing edge. This is the Kutta condition.*

- *It is seen that the viscosity of the fluid, after all, causes the formation of circulation and, therefore, the establishment of lift. Viscosity of the fluid must therefore be taken into consideration temporarily to explain the evolution of lift, that is, the formation of the starting vortex. After the establishment of the starting vortex and the circulation around the wing, the calculation of lift can be done from the laws of frictionless flow using the Kutta-Zhukovsky equation observing the Kutta condition.*

We see that lift is connected to circulation which is determined by the Kutta condition. We see how inviscid theory is combined with viscous effects into a mix generating lift but no drag, which however does not describe actual physics.

## 34.4 Understanding Flight by Anderson-Eberhardt

This book presented as "the simplest, most intuitive book on the toughest lessons of flight–addresses the science of flying in terms, explanations, and illustrations that make sense to those who most need to understand: those who fly", seeks to fill its mission with the following proclamations:

- *There are few physical phenomena so generally studied which are as misunderstood as the phenomenon of flight.*

- *Books written to train engineers often quickly delve into complicated mathematics....the necessary formalism is often achieved at the expense of a fundamental understanding of the principles of flight.*

- *A shortcoming of many books on the topic of aeronautics is that the information is presented in a very complicated manner, often mistaking mathematics for a physical explanation.*

- *In fact, we believe that if something can only be described in complex mathematical terms it is not really understood. To be able to calculate something is not the same as understanding it.*

- *Teachers and students who are looking for a better understanding of flight will find this book useful. Even students of aeronautical engineering will be able to learn from this book, where the physical descriptions presented will supplement the more difficult mathematical descriptions of the profession.*

- *The mathematical description of lift is a general term for the analysis tools of classical aerodynamics and computation aerodynamics. If the objective is to accurately compute the principle or aerodynamics of a wing, these are the tools to use, though the aerodynamic description is mathematical and not physical. This is a point lost on many of its proponents. Fortunately, the physical description of lift, presented here, does not require complicated mathematics.*

- *The physical description of lift is based primarily on Newton's three laws and a phenomenon called the Coanda effect. This description is uniquely useful for understanding the phenomena associated with flight. So why do fluids tend to bend around a solid object? The answer is viscosity, that characteristic that makes a fluid thick and makes it stick to a surface. When a moving fluid comes into contact with a solid object, some of it sticks to the surface.*

Comment: We read that Anderson-Eberhardt informs us that (i) mathematics of flight is one thing, which is difficult to understand, and (ii) physics of flight is another thing, which is easy to understand: The Coanda effect resulting from the viscosity of air.

Yes, (i) is correct, the mathematics of flight of Kutta-Zhukovsky-Prandtl is not understood by any living scientist. But (ii) is plain wrong. Lift is not an effect of viscosity, but results from the fact that air has very small viscosity, as shown by the New Theory of Flight. The book shows that the authors do not understand flight and so the book should more correctly be titled Not Understanding Flight.

Only when mathematics and physics come together is science created. Understanding in physics means understanding of a mathematical model describing real physics. Pseudo-science is characterized by mathematics separated from physics, as in the Kutta-Zhukovsky circulation theory.

## 34.5 Theory of Flight by von Mises

This book was launched as a Dover reprint in 1959 by:

- *Perhaps the most balanced, well-written account of fundamental fluid dynamics ever published. Mises' classic avoids the formidable mathematical structure of fluid dynamics, while conveying by often unorthodox methods a full understanding of the physical phenomena and mathematical concepts of aeronautical engineering. "An outstanding textbook."*

The book presents the Kutta-Zhukovsky theory stating that the lift of an airfoil results from circulation around the airfoil section generated by a sharp trailing edge (p 179):

- *It had been known from the very beginning of flight that wings with a sharp trailing edge must be used in order to obtain a well-defined lift.*

This statement does not describe physical reality, since it has been known from the very beginning that a rounded trailing edge (of radius as large as 10

This was of course known to von Mises when he wrote his book, and so it is natural to ask about the meaning of his statement that a sharp trailing edge "must be used to obtain a well-defined lift".

What von Mises is describing is not real physics, but instead fictional physics in the form of circulation theory stating that lift is proportional to circulation with lift thus determined by circulation, which in general can be of any magnitude, but for an airfoil with sharp trailing edge is determined by the Kutta condition of smooth flow off the trailing edge. Von Mises thus says that in the fictional physics of circulation theory, a sharp trailing edge "must be used" to obtain a well-defined circulation and lift.

Von Mises thus deliberate fools the reader and the world by giving the impression that his "theory of flight" describes real physics, while in fact it only describes mathematical peculiarities of non-real fictional physics. Von Mises thereby sets a standard of modern aerodynamics, which can now be questioned in the light of the New Theory of Flight.

## 34.6 The Simple Science of Flight by Tennekes

Henk Tennekes is Director of Research Emeritus at the Royal Netherlands Meteorological Institute, Emeritus Professor of Meteorology at the Free University (VU) in Amsterdam, and Emeritus Professor of Aerospace Engineering at Pennsylvania State University. He is the coauthor of A First Course in Turbulence (MIT Press, 1972) and The Simple Science of Flight:

- *An investigation into how machines and living creatures fly, and of the similarities between butterflies and Boeings, paper airplanes and plovers. A leisurely introduction to the mechanics of flight and, beyond that, to the scientific attitude that finds wonder in simple calculations, forging connections between, say, the energy efficiency of a peanut butter sandwich and that of the kerosene that fuels a jumbo jet.*

Let us check out how Tennekes describes the "simple science of flight":

- *Unfortunately, most of us learned in high school that one needs the Bernoulli principle to explain the generation of lift. Your science teacher told you that the upper surface of a wing has to have a convex curvature, so that the air over the top has to make a longer journey than that along the bottom of the wing.Polite fiction, indeed. It does not explain how stunt planes can fly upside down....*

- *We will have to do better. I will use a version of Newton's 2nd Law. I will also appeal to Newton's 3rd Law, which says that action and reaction are equal and opposite. Applied to wings these two laws imply that a wing produces an amount of lift that is equal to the downward impulse given to the surrounding air. The lift of a wing is proportional to angle of attack x density x speed squared x wing area.*

This is all Tennekes has to say about the "simple science of flight". Simple indeed: Upward lift on the wing is balanced by a downward force on the air from the wing! And the formula is just trivial similarity and an assumption pulled out of the pocket that lift scales with the angle of attack

What Tennekes offers the general reader in "college-level courses for senior citizens" is non-sensical triviality instead of enlightening simplicity, as if the senior citizens are all Alzheimer patients.

The book, written by an author who describes himself as a turbulence specialist, gives yet another indication of the collapse of theory in the century of aerodynamics.

Tennekes was forced out of office because he expressed skepticism to climate alarmism and so his critique of Bernoulli was not the only case where "he was right after all".

## 34.7 Physics of Flight Reviewed by Weltner

We read in the Introduction:

- *The explanation of the aerodynamic lift has a long history, but there is controversy regarding the fundamental physics and their relation to Newtons mechanics to date. This topic could be one of the most interesting and motivating in physics education. But the physics of flight nearly disappeared from the curriculum in schools and basic physics courses in most European countries. One reason, why teachers despite of students interest neglect this topic might be the fact that the conventional explanation of the aerodynamic lift based on Bernoullis Law has serious drawbacks and is partly erroneous. ... explanations based on Bernoullis law are dominating since the 1920s.*

- *This situation is changing recently. A growing number of authors question the conventional explanation and replace it by an explanation based on fundamental mechanics.*

Weltner eliminates Bernoulli because it requires the velocity field to be known and also circulation theory as a mathematical trick without physics:

- *The concept of circulation is a sophisticated mathematical discription of the velocity distribution but not the cause of the latter.*

Weltner instead advocates a general view - Aerodynamic lifting force as reaction force while air is accelerated downwards by the airplane:

- *The explanation based on the relation between aerodynamic lift and the acceleration of a downward air flow prevailed in textbooks in this simple form until 1920 without having been elaborated further. By approximately the year 1920, when aviation gained much interest in science and public, the explanation based on Bernoullis law appeared and displaced the explanation based on reaction forces. In any case it was necessary that the explanation of lift using Bernoullis law had to be complemented by giving a cause for the higher streaming velocity of the wings upper surface.*

But Weltner's explanation of lift simply as a reaction, is as empty of content today as it was when it was discarded in 1920. Weltner thus effectively reduces the aerodynamics of flight of the 20th century to zero and gives the reason why the physics of flight has disappeared from the curriculum.

## 34.8 Flight Physics by Torenbeek-Wittenberg

This book presents the fundamentals in Chapter 4 Lift and Drag at Low Speeds:

- *An aerofoil is a streamline body designed in such a way that, when set at a suitable angle to the airflow, it produces much more lift than drag.*

- *Similar to the situation with circulation around a cylinder, the flow around an aerofoil can be treated as a combination of two flow types; (a) In a frictionless flow there are two stagnation points: one near the nose point and one above and in front of the tail point. (b) A circulating flow. With airflow from the left and circulation in a clockwise direction, the velocity will increase on the upper aerofoil surface and slow down on the lower one. (c) The result of the superposition is a flow with a higher average velocity and a lower pressure on the upper surface than in case (a), whereas the pressure on the lower surface is higher. The pressure difference between both surfaces is experienced as lift.*

- *However, in contrast with the situation of a rotating cylinder, an aerofoil section is not rotating, which makes it unclear how circulation arises and what determines its value. Although just about every value of circulation seems possible, in reality nature takes care that a certain angle of attack only allows one type of flow. In 1902, W.M. Kutta first proposed for a section with a sharp trailing edge that the circulation adjusts to a value so that no air will flow around the sharp aerofoil tail from the lower to the upper surface, or vice versa... this so-called Kutta condition leads to a correct determination of the circulation and with that the velocity and pressure distribution, in other words: the lift force.*

- *The fundamental question how circulation occurs and remains in existence can be answered in principle by carrying out an experiment such as that done for the first time by Ludwig Prandtl. He placed an aerofoil in a water channel in which the flow was made visible by aluminium particles sprinkled on the surface....the viscous fluid cannot follow the corner at the tail point and will separate while creating a vortex above the trailing edge. At still higher velocity this vortex will move downstream, separate from the wing, and will become a cast-off or starting vortex. This will be quickly left behind and is eventually dissipated through the action of viscosity. Because the original flow did not contain circulation, a reverse circulation will occur around the aerofoil with the opposite direction.Its circulation causes the rear stagnation point to move towards the trailing edge. Since a starting vortex originates from viscosity, there is no circulation and also no lift created in ideal flow. Nevertheless, the lift on a section with a sharp tail* *can be determined by assuming the flow to be ideal and by making use of the Kutta condition. For small angles of attack, viscous effects are manifest only in the boundary layer and the lift is hardly affected by viscosity. The discussed model is therefore a good representation of the real flow.*

We see here the classical Kutta-Zhukovsky 2d circulation theory conceived 100 years ago. There is massive evidence (see e.g. Swedish Aerodynamics Dissident) that the 2d flow thus described is unphysical and thus scientifically incorrect. We can see how the authors struggle to cope with this fact: It is not claimed that 2d potential flow + circulation describes the real physics, only that the model is a "good representation" and the flow can so "be treated" (the accepted way of handling the contradiction between 2d theory and 3d reality).

## 34.9 The Physics of Flight by Lande

This book is presented by:

- *When looking for a textbook on aerodynamics of the airplane for students after their first year of college physics and algebra, the author found a certain gap between elementary introductions and more advanced representations which require a full knowledge of calculus.*

Lande warms up the reader by:

- *The airfoil, with its long span and curved ("cambered") crosssection, its blunt leading edge and sharp trailing edge, is an almost perfect instrument of pure sustentation. The physical explanation and evaluation of the amazing qualities of the transversal wing as means of sustentation consitute the chief topic of this book.*

But Lande then only delivers the classical Kutta-Zhukovsky-Prandtl circulation theory:

- *The aerodynamic process which leads to the production of lift and drag cannot be understood without studying the adhesion of the air to the surface of the wing, and the viscosity or internal friction (stickiness) of the air itself. Lift originates from the downward momentum imparted to the incoming horizontal air current. The downward deflection is the result of wind V and pure circulation w, as shown in the proof of K-J's law....we found that*

> *vorticity superimposed on wind produces lift at right angles to the incident wind in two-dimensional flow, without giving rise to a force of drag parallel to the wind.*

This is the standard presentation going back to Prandtl, which attributes lift to the ”stickiness” of a viscous fluid satisfying a no-slip boundary condition. This is misleading physics since lift in subsonic flight in reality originates from incompressibility and a slip boundary condition modeling the small skin friction of slightly viscous flow, as the main message of this book.

# Chapter 35

# Making of the Prandtl Myth

> But the impression upon my mind is that the motions calculated above for an absolutely inviscid liquid may be found inapplicable to a viscid liquid of vanishing viscosity, and that a more complete treatment might even yet indicate instability, perhaps of a local character, in the immediate neighbourhood of the walls, when the viscosity is very small. (Rayleigh 1892)

## 35.1 By Schlichting: Student

The myth of Prandtl as the Father of Modern Fluid Mechanics was shaped by his former students Theodor von Karman and Hermann Schlichting (photo) serving as aeronautics expert scientists in the US and Germany during the 2nd World War. Upon request from the Allied forces Schlichting documented Prandtl's expertise in the book Boundary Layer Theory, viewed as the bible of modern fluid mechanics. It is possible to argue that the outcome of the war was influenced by incorrect German aeronautics.

Let us analyze how Schlichting builds the Prandtl myth in the Introduction to his book in a sequence of quotes with our comments in parenthesis:

- *The present book is concerned with the branch known as boundary-layer theory. This is the oldest branch of modern fluid dynamics; it was founded by Prandtl in 1904 when he succeeded in showing how flows involving fluids of very small viscosity, in particular water and air, the most important ones from the point of view of applications, can be made amenable to mathematical analysis*. (This sets the scence with boundary-layer theory opening to technological progress by mathematical analysis in the hands of Prandtl).

- *This was achieved by taking the effects of friction into account only in regions where they are essential, namely in the thin boundary layer which exists in the immediate neighbourhood of a solid body*. (This is a clever circular formulation with effects of viscosity taken into account where they are essential and should be taken into account).

- *This concept made it possible to clarify many phenomena which occur in flows and which had previously been incomprehensible*. (Vague. Nothing was clarified, only further mystified).

- *Most important of all, it has become possible to subject problems connected with the occurrence of drag to a theoretical analysis*. (This is the central dogma of Prandtl: Drag originates from boundary layer effects. We show that thus is incorrect by obtaining correct drag without boundary layers).

- *The science of aeronautical engineering was making rapid progress and was soon able to utilize these theoretical results in practical applications. It did, furthermore, pose many problems which could be solved with the aid of the new boundary layer theory. Aeronautical engineers have long since made the concept of a boundary layer one of everyday use and it is now unthinkable to do without it*. (Vague).

- *In other fields of machine design in which problems of flow occur, in particular in the design of turbomachinry, the theory of boundary layers made much slower progress, but in modern times these new concepts have come to the fore in such applications as well*. (This in an admittance that the boundary layer theory is not useful in applications).

- *Towards the end of the 18th century the science of fluid mechanics began to develop in two directions which hadpractically no points in common. On the one side there was the science of theoretical hydrodynamics which was evolved from Euler's equations of motion for a frictionless, non-viscous fluid and which achieved a high degree of completeness. Since, however, the results of this so-called classical science of hydrodynamics stood in glaring contradiction to experimental results in particular as regards the very important problem of pressure losses in pipes and channels, as well as with regard to the drag of a body which moves through a mass of fluid it had little practical importance. For this reason, practical engineers, prompted by the need to solve the important problems arising from the rapid progress* *in technology, developed their own highly empirical science of hydraulics. The science of hydraulics was based on a large number of experimental data and differed greatly in its methods and in its objects from the science of theoretical hydrodynamics*. (This is an admittance that theory and practice do not come together).

- *At the beginning of the present century L. Prandtl distinguished himself by showing how to unify these two divergent branches of fluid dynamics. He achieved a high degree of correlation between theory and experiment and paved the way to the remarkably successful development of fluid mechanics which has taken place over the past seventy years. It had been realized even before Prandtl that the discrepancies between the results of classical hydrodynamics and experiment were, in very many cases, due to the fact that the theory neglected fluid friction. Moreover, the complete equations of motion for flows with friction (the Navier-Stokes equations) had been known for a long time. However, owing to the great mathematical difficulties connected with the solution of these equations (with the exception of a small number of particular cases), the way to a theoretical treatment of viscous fluid motion was barred. Furthermore, in the case of the two most important fluids, namely water and air, the viscosity is very small and, consequently, the forces due to viscous friction are, generally speaking, very small compared with the remaining forces (gravity and pressure forces). For this reason it was very difficult to comprehend that the frictional forces omitted from the classical theory influenced the motion of a fluid to so large an extent*. (The claim that Prandtl unified mathematical theory and practice is without substance. Prandtl was mathematically naive and his dictate that Navier-Stokes equations cannot be combined with force boundary conditions, is incorrect).

- *In a paper on "Fluid Motion with Very Small Friction", read before the Mathematical Congress in Heidelberg in 1004, L. Prandtl — showed how it was possible to analyze viscous flows precisely in cases which had great practical importance*. (The paper is very short (8 sparsely typed pages) and contains no mathematical analysis, only vague speculations, which have showed to be misleading).

- *With the aid of theoretical considerations and several simple experiments, he proved that the flow about a solid body can be divided into two regions: a very thin layer in the neighbourhood of the body (boundary layer) where friction plays an essential part, and the remaining region outside this layer,* *where friction may be neglected.* (This subdivision is mathematically and physically impossible).

- *On the basis of this hypothesis Prandfl succeeded in giving a physically penetrating explanation of the importance of viscous flows, achieving at the same time a maximum degree of simplification of the attendant mathematical difficulties. The theoretical considerations were even (then supported by simple experiments performed in a small water tunnel which Prandtl built with his own hands. He thus took the first step towards a reunification of theory and practice. This boundary-layer theory proved extremely fruitful in that it provided an effective tool for the development of fluid dynamics.* (The claim that Prandtl unifies theory and practice with a maximum of mathematical simplification lacks rationale).

- *Since the beginning of the current century the new theory has been developed at a very fast rate under the additional stimulus obtained from the recently founded science of aerodynamics. In a very short time it became one of the foundation stones of modern Ihiid dynamics together with the other very important developments  the aerofoil theory and the science of gas dynamics.* (This summarizes the myth of Prandtl as the Father of Modern Fluid Mechanics. The truth is that Prandtl misled a whole century of fluid dynamicists in searching for drag and lift in a vanishingly thin boundary layer).

- *The existence of tangential (shearing) stresses and the condition of no slip near solid walls constitute the essential differences between a perfect and a real fluid. Certain fluids which are of great practical importance, such as water and air, have very small coefficients of viscosity. In many instances, the motion of such fluids of small viscosity agrees very well with that of a perfect fluid, because in most eases the shearing stresses are very small.* (This is an admittance that the skin friction forces are small in the boundary layer and thus can be approximated by a slip boundary condition, in direct violation of Prandtl's dictate of no-slip).

- *For this reason the existence of viscosity is completely neglected in the theory of perfect fluids, mainly because this introduces a far-reaching simplification of the. equations of motion, as a result of which an extensive mathematical theory becomes possible. It is, however, important to stress the fact that even in fluids with very small viscosities, unlike in perfect fluids, the* *condition of no slip near a solid boundary prevails. This condition of no slip introduces in many cases very large discrepancies in the laws of motion of perfect and real fluids. In particular, the. very large discrepancy between the value of drag in a real and a perfect fluid has its physical origin in the condition of no slip near a wal*l. (This is Prandtl's dictate of no-slip which has made 20th century fluid mechanics both uncomputable and impossible to rationalize).

- *Foreword by Dryden: Boundary-layer theory is the cornerstone of our knowledge of the flow of air and other fluids of small viscosity under circumstances of interest in many engineering applications. Thus many complex problems in aerodynamics have been clarified by a study of the flow within the boundary layer and its effect on the general flow around the body. Such problems include the variations of minimum drag and maximum lift of airplane wings with Reynolds number, wind-tunnel turbulence, and other parameters. Even in those cases where a complete mathematical analysis is at present impracticable, the boundary-layer concept has been extraordinarily fruitfull and useful*. (Big words without real substance).

## 35.2 By von Karman: Student

The making of the myth of Prandtl as the Father of Modern Fluid mechanics was created also by another student Theodore von Karman in the book Aerodynamics: Selected Topics in the Light of their Historical Development from 1954:

- *At the time of the first human flight, no theory existed that would explain the sustenation obtained by means of a curved surface at zero angle of attack. It seemed that the mathematical theory of fluid motion was unable to explain the fundamental facts revealed by experimental aerodynamics*. (This is correct).

- *The Kutta-Zhukovsky condition seems to be a reasonable hypothesis, both because it is indicated by visual observation and also because the lift calculated by means of this condition seems is in fair accordance with measurements. The usefulness of the theory is restricted to a limited range of angle of attack, comprising relatively small angles. Beyond this range the the measured lift falls far below the values predicted by the theory*. (Von Karman here falls into the logical fallacy of confirming an assumption by observing a consequence: If there is circulation then there is lift, and since lift is observed there must be circulation. This makes the theory fool-proof, and as such pseudo-scientific).

- *The man who gave modern wing theory its practical mathematical form was one of the most prominent representatives of the science of mechanics, and especially fluid mechanics, Ludwig Prandtl. His creates contributions to fluid mechanics were in the field of wing theory and the theory of the boundary layer. His control of mathematical methods and tricks was limited: many of his collaborators and followers surpassed him in solving difficult mathematical problems. but his ability to establish systems of simplified equations which expressed the essential physical relations and dropped the non-essentials, was unique*. (This is the Prandtl myth: Prandtl reveals the mathematical secrets of fluid mechanics without knowing much math).

- *To be sure, Prandtl's theory has limitations, as does every theory. Its first limitation is caused by the phenomenon of stall*. (By admitting limitation the theory is strengthened. The fact that stall is not predicted, should alone be sufficient to eliminate the theory as unphysical, and thus very dangerous to use for the design of real airplanes).

- *Our knowledge of the reasons "why we can fly" and "how we fly" has increased both in scope and depth in a rather impressive way*. (The qualification "rather" means that the knowledge is not convincing).

- *We aerodynamicists were always more modest (than physicists) and did not attempt to change basic beliefs of the human mind or to interfere with the business of the good Lord or divine Providence*. (The theory is strengthened by showing a humble attitude, different from that of physicists).

## 35.3 By Prandtl: Himself

The Father of Modern Aerodynamics inspecting the Ho III 1938 Rhn Contest Challenger: Did Germany lose the war because of incorrect aerodynamics?

On the request by NACA (US National Advisory Committee for Aeronautics) in 1921, Ludwig Prandtl "prepared for the reports of the committee a detailed treatise on the present condition of those applications of hydrodynamics which lead to the calculation of the forces acting on airplane wings and airship bodies" (NACA Report 116 Applications of Modern Hydrodynamics to Aeronautics).

Prandtl "acceded to the request all the more willingly because the theories in question have at this time reached a certain conclusion where it is worth while to show in a comprehensive manner the leading ideas and the results of these theories and to indicate what confirmation the theoretical results have received by tests".

Prandtl states in his report defining the state-of-the-art with our comments in parenthesis:

- *Friction between fluid and solid body never comes into consideration in the fields of application to be treated here, because it is established by reliable experience that fluids like water and air never slide on the surface of the body; what happens is, the final fluid layer immediately in contact with the body is attached to it (is at rest relative to it), and all the friction of fluids with solid bodies is therefore an internal friction with the fluid.* (This is Prandtl's dictate of no-slip boundary condition which made 20th century fluid mechanics into uncomputable magics).

- *In this layer, which we call the boundary layer, the forces due to viscosity are of the same order of magnitude as the forces due to inertia, as may be seen without difficulty.* (This is misleading: both forces due to viscosity and inertia can be small.)

- *Closer investigation concerning this shows that under certain conditions there may occur a reversal flow in the boundary layer which is set in rotation by the viscous forces, so that, further on, the whole flow is changed owing to the formation of vortices.* (This is Prandtl's main thesis: The boundary layer changes the whole flow).

- *In the rear of blunt bodies vortices are formedon the other hand, in the rear of very tapering bodiesthere is no noticeable formation of vortices. The principal successful results of hydrodynamics apply to this casethe theory can be made useful exactly for those bodies which are of most technical interest.* (Prandtl's theory is based on the formation of vortices in the boundary layer, which Prandtl claims are not formed in the cases of interest: Missing logic).

- *On resistance of airships: It is seen that the agreement (pressure distribution) is very complete; at the rear end, however, there appears a characteristic deviation in all cases, since the theoretical pressure distribution reaches the full dynamical pressure at the point where the flow reunites again, while* *actually this rise in pressure, owing to the influence of the layer of air retarded by friction, remains close to the surface*. (Prandtl suggests that the lack of pressure rise at separation in real flow is due to boundary layer friction. We show that this is incorrect: It is instead caused by 3d rotational separation.)

- *As is well known there is no resistance for the theoretical flow in a non-voscpus fluid (potential flow). The actual drag consists of two parts, one resulting from all the noral forces (pressures) acting on the surface of the body, the other from all tangential forces (friction). The pressure resistance, arises in the main from the deviation mentioned at the rear end, and is, as is known, very small*. (This is incorrect: Pressure drag is the main part of drag in slightly viscous flow)

- *We shall concern ourselves in what follows only with non viscous and incompressible fluid, also called "ideal fluid"*. (Confusing since friction forces are supposed to change the flow.)

- *In order that the flow may be like the actual one, the circulation must always be so chosen that the rear rest point coincides with the trailing edge.We are accordingly, by the help of such constructions, in the position of being able to calculate the velocity at every point in the neighborhood of the wing profile. The agreement on the whole is as good as can be expected from a theory which neglects completely the viscosity*. (Empty statement.)

- *That a circulatory motion is essential for the production of lift of an aerofoil is definitely established. The question is then how to reconcile this with the proposition that the circulation around a fluid line in a non viscous fluid remains constant.... There is instantly formed at the trailing edge a vortex of increasing intensity* (This is the mystery of circulation theory: If circulation generates lift, the question is how circulation is generated. The standard answer is by a non-real sharp trailing edge and starting vortex. This is however not the real physics of 3d rotational separation at a real rounded trailing edge.)

## 35.4 By Anderson: Curator of Aerodynamics

We cite from the book *Fundamentals of Aerodynamics* by John D Anderson:

- *The first practical theory for predicting the aerodynamic properties of a finite wing was developed by Ludwig Prandtl and his colleagues at Gottingen, Germany, during the period 1911-1918, spanning World War I. The utility of Prandtl's theory is so great that it is still in use today for preliminary calculations of finite-wing characteristics.*

- *The modern science of aerodynamics rests on a strong fundamental foundation, a large percentage of which was established in one place by one man-at the University of Gottingen by Ludwig Prandtl. Prandtl never received a Noble Prize, although his contributions to aerodynamics and fluid mechanics are felt by many to be of that caliber.* (Anderson captures the essential quality of Prandtl's theory as a **practical method** for preliminary calculation of wing characteristics (lift), and not a physical theory describing the true aerodynamics of the generation of lift of a wing, and thus gives a reason why Prandtl did not get any Nobel Prize).

- *By the 1930s, Prandtl was recognized worldwide as the "elder statesman" of fluid dynamics. Although he continued to do research in various areas, including structural mechanics and meteorology, his "Nobel Prize-level" contributions to fluid dynamics had all been made.*

- *Prandtl remained at Gottingen throughout the turmoil of World War II, engrossed in his work and seemingly insulated from the intense political and physical disruptions brought about by Nazi Germany. In fact, the German Air Ministry provided Prandtl's laboratory with new equipment and financial support.* (Anderson emphasizes the strong connection between Prandtl's work and German war efforts, and thus gives another reason why Prandtl was not near to get a Nobel Prize).

- *Prandtl was considered a tedious lecturer because he could hardly make a statement without qualifying it. However, he attracted excellent students...*

- *Prandtl died in 1953. He was clearly the father of modern aerodynamics-a monumental figure in fluid dynamics. His impact will be felt for centuries to come.*

Concluding analysis: Prandtl's message of 1921 has become the text book canon, which however is non-physical and thus incorrect.

# Chapter 36

# Confessions

We here list some of the many confessions by experts that there is no scientific theory of flight, which is the starting point of the present study.

This is a cumbersome truth for both aerodynamicists as scientists and airlines reassuring their passengers that air transportation is safe. The design of safe airplanes is facilitated by correct theory. If there is no correct theory available now, it is important to find one as soon as possible.

## 36.1 New York Times,

K. Chang in Staying Aloft; What Does Keep Them Up There?, Dec 9, 2003:

- *To those who fear flying, it is probably disconcerting that physicists and aeronautical engineers still passionately debate the fundamental issue underlying this endeavor: what keeps planes in the air?*

- *Here we are, 100 years after the Wright brothers, and there are people who give different answers to that question, said Dr. John D. Anderson Jr.*

## 36.2 AIAA

J Hoffren, Quest for an Improved Explanation of Lift, AIAA, 2001:

- *The basic physical principles tend to be buried and replaced by mystical jargon. Classical explanations for the generation of lift do not make the* *essence of the subject clear, relying heavily on cryptical terminology and theorems from mathematics.*

- *Many classical texts even appear to have a fundamental error in their underlying assumptions.*

- *Although the subject of lift is old, it is felt that a satisfactory general but easily understandable explanation for the phenomenon (of lift), is still lacking, and consequently there is a genuine need for one.*

## 36.3 AVweb

- *Few physical principles have ever been explained as poorly as the mechanism of lift:*

- *Its all one interconnected system. Unless the overall result of that system is for air to end up lower than it was before the plane flew by, there will be no lift. Wings move air downward, and react by being pushed upward. Thats what makes lift. All the rest is just interesting details.*

## 36.4 Airfoil Lifting Force Misconception

W.Beaty:

- *How do airplane wings really work? Amazingly enough, this question is still argued in many places, from elementary school classrooms all the way up to major pilot schools, and even in the engineering departments of major aircraft companies.*

- *This is unexpected, since we would assume that aircraft physics was completely explored early this century. Obviously the answers must be spelled out in detail in numerous old dusty aerodynamics texts.*

- *However, this is not quite the case. Those old texts contain the details of the math, but its the interpretation of the math that causes the controversy.*

- *There is an ongoing Religious War over both the way we should understand the functioning of wings, and over the way we should explain them in childrens textbooks.*

## 36.5 Live Science

Lift is a lot trickier:

- *In fact it is very controversial and often poorly explained and, in many textbooks, flat wrong. I know, because some readers informed me that the original version of this story was inaccurate.*
- *Ive attempted to correct it after researching conflicting expert views on all this.If youre about fed up, rest assured that even engineers still argue over the details of how all this works and what terms to use.*

## 36.6 The Straight Dope

How Do Airplanes Fly, Really?

- *Youd think that after a century of powered flight wed have this lift thing figured out. Unfortunately, its not as clear as wed like. A lot of half-baked theories attempt to explain why airplanes fly.*
- *All try to take the mysterious world of aerodynamics and distill it into something comprehensible to the lay audiencenot an easy task.*
- *Nearly all of the common theories are misleading at best, and usually flat-out wrong. How can aviation be grounded in such a muddy understanding of the un- derlying physics?*
- *As with many other scientific phenomena, its not always necessary to understand why something works to make use of it. We engineers are happy if weve got enough practical knowledge to build flying aircraft.*
- *The rest we chalk up to magic.*

## 36.7 Smithsonian Space Museum

Curator for aerodynamics (From Staying Aloft; What Does Keep Them Up There?):

- *Some of them get to be religious fervor.The answer, the debaters agree, is physics, and not a long rope hanging down from space. But they differ sharply over the physics, especially when explaining it to nonscientists.*

- *There is no simple one-liner answer to this.*
- *The simple Newtonian explanation also glosses over some of the physics, like how does a wing divert air downward? The obvious answer air molecules bounce off the bottom of the wing is only partly correct.*
- *If air has to follow the wing surface, that raises one last question. If there were no attractive forces between molecules, would there be no flight? Would a wing passing through a superfluid like ultracold helium, a bizarre fluid that can flow literally without friction, produce no lift at all? That has stumped many flight experts.*

## 36.8 HowStuffWorks

- *It is important to realize that, unlike in the two popular explanations described earlier (longer path and skipping stone), lift depends on significant contributions from both the top and bottom wing surfaces. While neither of these explanations is perfect, they both hold some nuggets of validity.*
- *Other explanations hold that the unequal pressure distributions cause the flow deflection, and still others state that the exact opposite is true. In either case, it is clear that this is not a subject that can be explained easily using simplified theories.*
- *Likewise, predicting the amount of lift created by wings has been an equally challenging task for engineers and designers in the past. In fact, for years, we have relied heavily on experimental data collected 70 to 80 years ago to aid in our initial designs of wing.*

## 36.9 Wikipedia Lift Force

John D. Anderson, Curator of Aerodynamics at the National Air and Space Museum:

- *It is amazing that today, almost 100 years after the first flight of the Wright Flyer, groups of engineers, scientists, pilots, and others can gather together and have a spirited debate on how an airplane wing generates lift. Various explanations are put forth, and the debate centers on which explanation is the most fundamenta*l.

## 36.10 Desktop Aeronautics

Applied Aeronautics text book Preface:

- *The aerodynamics of bumble bees, disk heads, weather, and many other things is not a solved problem. While it is impressive that the methods in use today do so well, we are still not able to predict many flows.*

# Part VI

# History

# Chapter 37

# Aristotele

> Probable impossibilities are to be preferred to improbable possibilities. (Aristotle)
>
> In all things of nature there is something of the marvelous. (Aristotle)

We have seen that the secret of flying is how to generate large lift by the motion of a wing through air at the expense of small drag. Approaching this problem we face the general problem of *motion*, seriously addressed already by the Greek philosopher *Zeno* in his famous paradox about the arrow, which in every single moment along its path seems to be frozen into immobility, but yet effectively is moving. Today we are familiar with many forms of motion and most people would probably say that Zeno's paradox must have been resolved since long, although they would not be able to account for the details of the resolution.

However, the true nature of e.g. the motion of light through vacuum still seems to be hidden to us, while the motion through a gas/fluid like air or water can be approached following an idea presented already by *Aristotle* in the 4th century BC known as *antiperistasis*. Aristotle states in his *Physics* that that a body in motion through air is pushed from behind by the stream of air around the body contracting in the rear after having been expanded in the front. This is like the *peristaltic* muscle contractions that propels foodstuffs distally through the esophagus and intestines, which is like the squeezing of an object through a lubricated elastic tube by the combined action of the object expanding the tube in the front and the tube contracting in the rear of the object, as expressed in the words of Aristotle:

- *Thirdly, in point of fact things that are thrown move though that which gave them their impulse is not touching them, either by reason of mutual replacement, as some maintain, or because the air that has been pushed pushes* *them with a movement quicker than the natural locomotion of the projectile wherewith it moves to its proper place. But in a void none of these things can take place; the only way anything can move is by riding on something else.*

- *Fourthly, no one could say why a thing once set in motion should stop anywhere; for why should it stop here rather than here? So that a thing will either be at rest or must be moved ad infinitum, unless something stronger than it impedes it.*

- *Fifthly, things are now thought to move into the void because it yields; but in a void this quality is present equally everywhere, so that things should move in all directions.*

Of course, we say today that according to Newton's 2nd law, a body will continue in rectilinear motion at constant speed unless acted upon by some force, while to Aristotle sustained motion would require a force pushing from behind. Nevertheless, we will find that there is something in Aristotle's antiperistasis which correctly describes an important aspect of motion through air, if not through vacuum, but you cannot fly in vaccum...

## 37.1 Liberation from Aristotle

The renewal of learning in Europe, that began with 12th century Scholasticism, came to an end about the time of the Black Death, but the Northern Renaissance (in contrast to th Italian) showed a decisive shift in focus from Aristoteleian natural philosophy to chemistry and the biological sciences. Thus modern science in Europe was resumed in a period of great upheaval: the Protestant Reformation and Catholic Counter-Reformation; the discovery of the Americas by Christopher Columbus; the Fall of Constantinople; but also the re-discovery of Aristotle during the Scholastic period presaged large social and political changes. Thus, a suitable environment was created in which it became possible to question scientific doctrine, in much the same way that Martin Luther and John Calvin questioned religious doctrine. The works of Ptolemy (astronomy) and Galen (medicine) were found not always to match everyday observations.

The willingness to question previously held truths and search for new answers opened the *Scientific Revolution* by Copernicus' *De Revolutionibus* in 1543 stating that the Earth moved around the Sun, followed by Newton's *Principia Mathematica* in 1687.

# Chapter 38

# Medieval Islamic Physics of Motion

> When hearing something unusual, do not preemptively reject it, for that would be folly. Indeed, horrible things may be true, and familiar and praised things may prove to be lies. Truth is truth unto itself, not because [many] people say it is. (Ibn Al-Nafis, 1213-1288 A.D.)

> We start by observing reality  we try to select solid (unchanging) observations that are not affected by how we perceive (measure) them. We then proceed by increasing our research and measurement, subjecting premises to criticism, and being cautious in drawing conclusions In all we do, our purpose should be balanced not arbitrary, the search for truth, not support of opinions...Hopefully, by following this method, this road to the truth that we can be confident in, we shall arrive to our objective, where we feel certain that we have, by criticism and caution, removed discord and suspicion...Yet we are but human, subject to human frailties, against which we must fight with all our human might. God help us in all our endeavors. (Ibn Al-Haytham)

> The knowledge of anything, since all things have causes, is not acquired or complete unless it is known by its causes. (Avicenna)

Medieval Islamic developments in mechanics prepared for the liberation of science from Christian Scholasticism following Aristotle's legacy, through the new mechanics of Galileo and Newton leading into the Enlightment and modern Europe. We recall some early Islamic scientists questioning Aristotle, and preparing for human flight...

## 38.1 Avicenna

*Avicenna* (980-1037) a foremost Persian polymath developed an elaborate theory of motion, in which he made a distinction between the inclination and force of a projectile, and concluded that motion was a result of an inclination (mayl) transferred to the projectile by the thrower, and that projectile motion in a vacuum would not cease. He viewed inclination as a permanent force whose effect is dissipated by external forces such as air resistance. He also developed the concept of momentum, referring to impetus as being proportional to weight times velocity. His theory of motion was also consistent with the concept of inertia in classical mechanics, and later formed the basis of Jean Buridan's theory of impetus and exerted an influence on the work of Galileo Galilei.

## 38.2 Abu'l-Barakat

*Hibat Allah Abu'l-Barakat al-Baghdaadi* (1080-1165) wrote a critique of Aristotelian physics where he was the first to negate Aristotle's idea that a constant force produces uniform motion, as he realized that a force applied continuously produces acceleration as an early foreshadowing of Newton's second law of motion. He described acceleration as the rate of change of velocity and modified Avicenna's view on projectile motion stating that the mover imparts a violent inclination on the moved and that this diminishes as the moving object distances itself from the mover. Abu'l-Barakat also suggested that motion is relative.

## 38.3 Biruni

Another prominent Persian polymath, *Abu Rayan Biruni*, engaged in a written debate with Avicenna, with Biruni criticizing the Peripatetic school for its adherence to Aristotelian physics and natural philosophy preserved in a book entitled *al-As'ila wal-Ajwiba* (Questions and Answers). al-Biruni attacks Aristotle's theories on physics and cosmology, and questions almost all of the fundamental Aristotelian physical axioms. He rejects the notion that heavenly bodies have an inherent nature and asserts that their *"motion could very well be compulsory"* and maintains that *"there is no observable evidence that rules out the possibility of vacuum"*; and states that there is no inherent reason why planetary orbits must be circular and cannot be elliptical. He also argues that *"the metaphysical ax-*

*ioms on which philosophers build their physical theories do not constitute valid evidence for the mathematical astronomer"*, which marks the first real distinction between the vocations of the *philosopher-metaphysician* (like Aristotle and Avicenna) and that of the *mathematician-scientist* (al-Biruni himsel). In contrast to the philosophers, the only evidence that al-Biruni considered reliable were either mathematical or empirical evidence, and his systematic application of rigorous mathematical reasoning later led to the mathematization of Islamic astronomy and the mathematization of nature.

## 38.4 Biruni's Questions

Biruni began the debate by asking Avicenna eighteen questions, ten of which were criticisms of Aristotle's *On the Heavens*, which represents an early example of the scientific method of questioning basic postulates which we seek to use in thi book.

The first question criticized the Aristotelian theory of gravity for denying the existence of levity or gravity in the celestial spheres, and the Aristotelian notion of circular motion being an innate property of the heavenly bodies.

Biruni's second question criticizes Aristotle's over-reliance on more ancient views concerning the heavens, while the third criticizes the Aristotelian view that space has only six directions. The fourth question deals with the continuity and discontinuity of physical bodies, while the fifth criticizes the Peripatetic denial of the possibility of there existing another world completely different from the world known to them. In his sixth question, Biruni rejects Aristotle's view on the celestial spheres having circular orbits rather than elliptic orbits. In his seventh question, he rejects Aristotle's notion that the motion of the heavens begins from the right side and from the east, while his eighth question concerns Aristotle's view on the fire element being spherical.

The ninth question concerns the movement of heat, and the tenth question concerns the transformation of elements. The eleventh question concerns the burning of bodies by radiation reflecting off a flask filled with water, and the twelfth concerns the natural tendency of the classical elements in their upward and downward movements.

The thirteenth question deals with vision, while the fourteenth concerns habitation on different parts of Earth. His fifteenth question asks how two opposite squares in a square divided into four can be tangential, while the sixteenth question concerns vacuum. His seventeenth question asks *"if things expand upon heating and contract upon cooling, why does a flask filled with water break when water* *freezes in it?"* His eighteenth and final question concerns the observable phenomenon of ice floating on water. After Avicenna responded to the questions, Biruni was unsatisfied with some of the answers and wrote back commenting on them.

## 38.5 Ibn al-Haytham

Ibn al-Haytham (965-1039) discussed the theory of attraction between masses, and it seems that he was aware of the magnitude of acceleration due to gravity and he stated that the heavenly bodies *"were accountable to the laws of physics"*. Ibn al-Haytham also enunciated the law of inertia, later known as Newton's first law of motion, when he stated that a body moves perpetually unless an external force stops it or changes its direction of motion. He also developed the concept of momentum, though he did not quantify this concept mathematically. Nobel Prize winning physicist Abdus Salam wrote the following on Ibn al-Haytham: Ibn-al-Haitham was one of the greatest physicists of all time. He made experimental contributions of the highest order in optics. He enunciated that a ray of light, in passing through a medium, takes the path which is the easier and "quicker". In this he was anticipating Fermat's Principle of Least Time by many centuries. He enunciated the law of inertia, later to become Newton's first law of motion. Part V of Roger Bacon's *Opus Majus* is practically an annotation to Ibn al Haitham's Optics.

## 38.6 Others

*Ibn Bajjah* (d. 1138) argued that there is always a reaction force for every force exerted, connecting to Newton's 3rd law, though he did not refer to the reaction force as being equal to the exerted force, which had an important influence on later physicists like Galileo. *Averroes* (1126-1198) defined and measured force as *"the rate at which work is done in changing the kinetic condition of a material body'* and correctly argued *"that the effect and measure of force is change in the kinetic condition of a materially resistant mass."* In the 13th century, *Nasir al-Din al-Tusi* stated an early version of the law of conservation of mass, noting that a body of matter is able to change, but is not able to disappear. In the early 16th century, al-Birjandi developed a hypothesis similar to Galileo's notion of *"circular inertia.*

At night I would return home, set out a lamp before me, and devote myself to reading and writing. Whenever sleep overcame me or I became conscious of weakening, I would turn aside to drink a cup of wine, so that my strength would return to me. Then I would return to reading. And whenever sleep seized me I would see those very problems in my dream; and many questions became clear to me in my sleep. I continued in this until all of the sciences were deeply rooted within me and I understood them as is humanly possible. Everything which I knew at the time is just as I know it now; I have not added anything to it to this day. Thus I mastered the logical, natural, and mathematical sciences, and I had now reached the science. (Avicenna)

# Chapter 39

# Leonardo da Vinci

> The noblest pleasure is the joy of understanding. (da Vinci)
>
> Although nature commences with reason and ends in experience it is necessary for us to do the opposite, that is to commence with experience and from this to proceed to investigate the reason...He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast. (da Vinci)
>
> For once you have tasted flight you will walk the earth with your eyes turned skywards, for there you have been and there you will long to return. (da Vinci)
>
> Life is pretty simple: You do some stuff. Most fails. Some works. You do more of what works. If it works big, others quickly copy it. Then you do something else. The trick is the doing something else. (da Vinci)
>
> Nothing strengthens authority so much as silence...You do ill if you praise, but worse if you censure, what you do not understand...There are three classes of people: those who see, those who see when they are shown, those who do not see...And many have made a trade of delusions and false miracles, deceiving the stupid multitude...Beware of the teaching of these speculators, because their reasoning is not confirmed by experience. (da Vinci)

## 39.1 The Polymath

*Leonardo da Vinci* (1452-1519) is the greatest polymath, universal genious, homo universale or renaissance man all times, with remarkable achievements as a scientist, mathematician, engineer, inventor, anatomist, painter, sculptor, architect, botanist, musician and writer.

Born as the illegitimate son of a notary, Piero da Vinci, and a peasant woman, Caterina, at Vinci in the region of Florence, Leonardo was educated in the studio of the renowned Florentine painter, Verrocchio. Much of his earlier working life was spent in the service of Ludovico il Moro in Milan. He later worked in Rome, Bologna and Venice and spent his last years in France, at the home awarded him by King Francois I.

As an artist Leonardo created the most famous, most reproduced and most parodied portrait and religious painting of all time: Mona Lisa and The Last Supper. As a scientist, he greatly advanced the state of knowledge in the fields of anatomy, civil engineering, optics, and hydrodynamics. As an engineer he conceptualised a helicopter, a tank, concentrated solar power, a calculator, the double hull and outlined a rudimentary theory of plate tectonics.

## 39.2 The Notebooks

Da Vinci recorded his discoveries in journals or *Notebooks* mostly written in mirror-image cursive, probably for practical expediency because Leonardo was left-handed, rather than for reasons of secrecy because it appears they were intended for publication. Although his language was clear and expressive, Leonardo preferred illustration to the written word stating, in the spirit of modern pedagogics:

- *The more detail you write concerning it the more you will confuse the reader*.

It is believed that there were at least 50 notebooks left in the hands of da Vinci's pupil Francesco Melzi at the master's death in 1519, of which 28 remains, but they were virtually unknown during his life-time and remained hidden for over two centuries. His wonderful ideas were forgotten; his inventions were not tested and built for hundreds of years. Dan Brown's best-seller *The Da Vinci Code* have stimulated renewed interest in da Vinci and his complex and inquiring intelligence. Today we can recoognize as an early precursor of an entire lineage oc scientists and philosophers whose central focus was the nature of organic form [4].

Leonardo planned to treat four major themes: the science of painting, architecture, the elements of mechanics, and a general work on human anatomy. To these themes were eventually added notes on his studies of botany, geology, flight, and hydrology. His intention was to combine all his investigations with a unified world view:

- *Plan of Book 15: First write of all water, in each of its motions; then describe all its bottoms and their various materials, always referring to the propositions concerning the said waters; and let the order be good, for otherwise the work will be confused. Describe all the forms taken by water from its greatest to its smallest wave, and their causes.*

Da Vinci made impressive and comprehensive investigations into aerodynamics collected into his *Codex on the Flight of Birds* from 1505, and designed a large variety of *ornithopters* for muscle-powered human flight using flapping wings. After extensive testing da Vinci concluded that even if both arms and legs got involved through elaborate mechanics, human power was insufficient for flapping flight, but during his last years in Florence he began to experiment with designs of flying machines that had fixed wings, not unlike modern hang-gliders.

## 39.3 The Scientist

Da Vinci observes the dynamics of the physical world with mountains, rivers, plants and the human body in ceaseless movement and transformation, according to a basic principle of science:

- *Necessity is the theme and inventor of nature, the curb and the rule.*

Da Vinci recognized the two basic forces of fluid mechanics to be inertial and viscous forces, realized that water is incompressible and though it assumes an infinite number of shapes, its mass and volume is always conserved. Below we will return to the following deep insights expressed by da Vinci:

- *In order to give the true science of the movements of the birds in the air, it is necessary to first give the science of the winds.*

- *As much force is exerted by the object against the air as by the air against the object.*

- *The spiral or rotary movement of every liquid is so much swifter as it is nearer to the center of revolution. What we are here proposing is a fact worthy of admiration, since the circular movement of a wheel is so much slower as it is nearer to the center of the rotating object.*

- *I have found among the excessive and impossible delsusions of men, the search for continuous motion, which is called by some the perpetual wheel.*

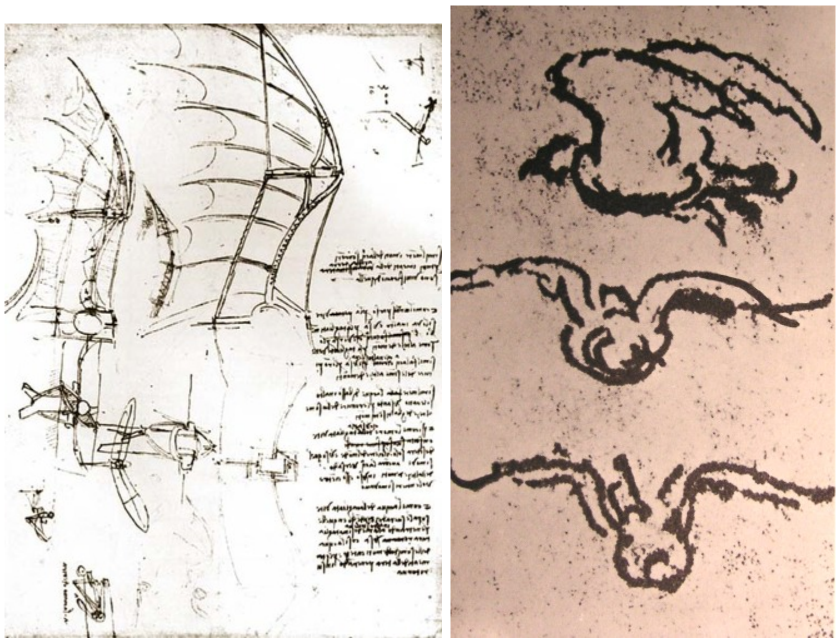

Figure 39.1: Da Vinci studies of bird wings and flight

## 39.4 The Mathematician

Da Vinci had a great admiration for mathematics:

- *A bird is an instrument working according to mathematical law, which is within the capacity of man to reproduce.*
- *There is no certainty, where one cannot apply any of the mathematical sciences, nor those which are connected with the mathematical sciences*.
- *Mechanics are the Paradise of mathematical science, because here we come to the fruits of mathematics.*
- *Let no man who is not a mathematician read my principles*.

Although da Vinci had little technical training in mathematics, he understood basic principles such as the law of free fall motion long before Galileo and conservation of mass:

- *The natural motion of heavy things, at each degree of its descent acquires a degree of velocity.*

- *If the water does not increase, nor diminish, in a river which may be of varying turtuoisities, breadths and dephts, the water will pass in equal quantities in equal times through every degree of the length of the river*

Da Vinci adopted Aristotle's principle of motion:

- *Of everything that moves, the space which it acquires is as great as that which it leaves*.

He also formulated basic priniciples of differential geometry:

- *The line is made with the movement of the point.*

- *The surface is made by the tramsversal movement of the line.*

- *The body is made by the movement of the extension of the surface.*

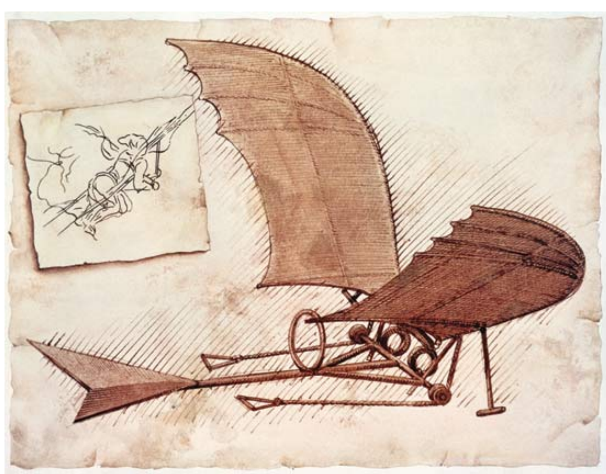

Figure 39.2: Da Vinci design of a glider

## 39.5 The Engineer

Between 1480 and 1505 da Vinci made a series of studies of birds and bats and developed sketches of flying machines, including gliders and more or less impossible devices including a flying machine like a boat. The pilot was intended to lie stretched out and to pull at oars which would propel the craft through air rather than water. Although this does not work for larger devices, this is essentially the mechanism for flight of small insects experiencing a substantial viscosity of air.

The modern helicopter invented by the Ukrainian-American engineer *Igor Sikorsky* in the 1930s, was probably inspired by a design by da Vinci with a helical screw instead of rotor blades (which of course did not work). The most inventive of da Vinci's flying machines was the glider in Fig.39.2 with the following control technique:

- *this [man] will move on the right side if he bends the right arm and extends the left arm; and he will then move from right to left by changing the position of the arms.*

## 39.6 The Philosopher

Da Vinci expressed a view on the interaction of body and soul connecting that of Descartes leading into modern conceptions of mind-brain interaction:

- *It could be said that such an instrument designed by man is lacking only the soul of the bird, which must be counterfeited with the soul of man...However, the soul of the bird will certainly respond better to the needs of its limbs than would the soul of the man, separated from them and especially from their almost imperceptible balancing movements*

- *Spiritual movement flowing through the limbs of sentient animals, broadens their muscles. Thus boadened, these muscles become shortened and draw back the tendons that are connected to them. This is the origin of force in human limbs...Material movement arises from the immaterial.*

# Chapter 40

# Newton's Incorrect Theory

> Dubito ergo cogito; cogito ergo sum. (Descartes)
>
> The foolish dog barks at the flying bird. (Bob Marley)
>
> The man who has no imagination has no wings. (Muhammad Ali)
>
> When a distinguished but elderly scientist states that something is possible, he is almost certainly right. When he states that something is impossible, he is very probably wrong. (Arthur C. Clarke)

We know that a surfing board or water skis can carry the weight of a person, but only in sufficiently rapid motion depending on the weight of the person and the area exposed to the water surface. The vertical force or lift is a reaction to a constant downward push of water as the board meets new water in its horsiontal motion. Without horisontal motion the board with the person will sink into the water. This is illustrated in Fig.**??**.

Newton was the first scientist to seek to develop a theory of lift and drag, and suggested that they should both be proportional to the density of the fluid and the square of the speed, which turns out to be more or less correct. Using a surfing board (or skipping stone) argument, which according to NASA we now know is wrong, Newton derived the above formula

$$L = \sin^2(\alpha)\rho U^2, \tag{40.1}$$

for the lift $L$ of a tilted flat plate of unit area with a quadratic dependence on the angle of attack $\alpha$. This formula follows from the fact that the mass $\rho U \sin(\alpha)$ hits the plate from below per unit time and gets redirected with a downward velocity

$U\sin(\alpha)$ corresponding to a change of momentum equal to $\sin^2(\alpha)\rho U^2$, which equals the lift force $L$ by Newton's 2nd law.

Newton's formula explains the force acting on a surf board on water. The ratio of density of water to that of air is about $1000$ and thus surfing on air requires about 30 times as large speed as surfing on water, because lift scales with the speed squared. Water skiing is possible at a speed of about $20$ knots, which would require a speed about $1000$ km/h, close to the speed of sound, for surfing on air. We understand that Newton's formula grossly under-estimates the lift, at least for subsonic speeds. We understand that flying *in* the air is not at all like surfing *on* water. Newton could thus prove that subsonic flight is impossible in theory, and so must have viewed the the flight of birds with surprise. Apparently birds were not willing to abide by the laws of Newtonian mechanics, but how could they take this liberty?

It is possible that Newton contributed to delaying human flight by making it seem impossible. Only after powered human flight had been demonstrated to be possible by the Wright brothers in 1903, did mathematicians replace Newton's erronous lift formula with a formula compatible with flight, although the derivation of the new formula again turned out to be incorrect, as we will discover below...


Figure 40.1: Incorrect explanation by Newton of lift by surfing.

# Chapter 41

# Robins and the Magnus Effect

> The greates part of military projectiles will at the time of their discharge acquire a whirling motion by rubbing against the inside of their respective pieces; and this whirling motion will cause them to strike the air very differently, from what the would do, had they no other but a progressive motion. By this means it will happen, that the resistance of the air will not always be directly opposed to their flight; but will frequently act in a line oblique to their course, and will thereby force them to deviate from the regular track, they would otherwise describe. (Robins in [65])

The English engineer Benjamin Robins (1707-1751), called the father of ballistics, introduced the concept of rifling the bore of guns to improve the accuracy of projectiles by spinning. In experiments with a whirling arm device he discovered that a spinning projectile experiences a transverse lift force, which he recorded in [65] in 1742. Euler translated Robins' book to German, but added a critical remark stating that on mathematical symmetry grounds the lift must be zero, and thus the measured lift must have been an effect of a non-symmetric projectile resulting from manufacturing irregularities. Recognized as the dominant hydrodynamicist of the eighteenth century, Euler far overshadowed Robins, and thus Robins' finding was not taken seriously for another century. In 1853 Gustav Magnus (1802-1870) in [62] suggested that the lift of a spinning ball, the so-called *Magnus effect*, was real and resulted from a whirlpool of rotating air around a ball creating a non-symmetric flow pattern with lift, an idea which was later taken up by Kutta and Zhukovsky as the decisive feature of their lift theory based on circulation.

Magnus suggested that, because of the whirlpool, for a topspin ball the air velocity would be larger below than above and thus result in a downward lift force because the pressure would be smaller below than above by Bernouilli. Magnus thus claimed to explain why a topspin tennis ball curves down, and a backspin curves up.

We shall see below that this explanation is incorrect: There is no whirlpool of rotating air around a spinning ball, nor is it any circulation around a wing. In both cases the lift has a different origin. We shall see that finding the real cause of the Magnus effect will lead us to an explanation of also the lift of a wing.

In 1749 Robins left the center stage of England, when he was appointed the engineer-general of the East India Company to improve the fortifications at St. David, Madras, where he died of fever at an early age of forty-four.

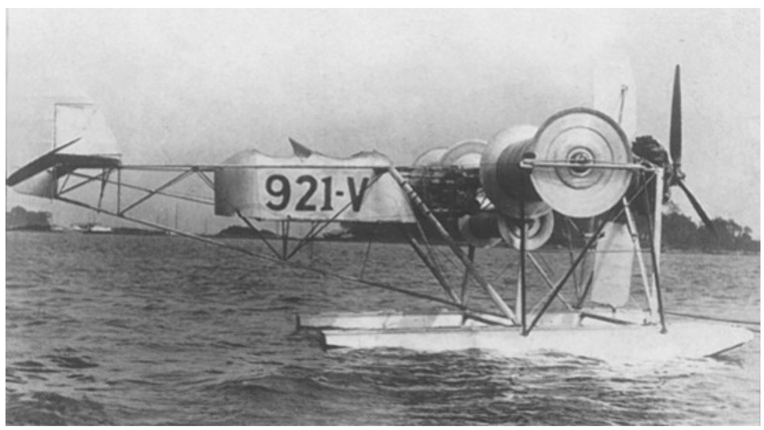


Figure 41.1: Built in 1930 (USA), the 921-V is reported to have been flown at least once - ending it's short carreer with a crash landing. Three cylinders with disks performing as winglets driven by a separate engine. Probably the only aircraft equipped with cylinder wings which made it into the air...

## 41.1 Early Pioneers

The flight of birds has always challenged human curiosity with the dream of human flight described already in the Greek myth about the inventor and master craftsman *Deadalus*, who built wings for himself and his son *Icarus* to escape from imprisonment in the Labyrinth of Knossos on the island of Crete. The leading scholar *Abbas Ibn Firnas* of the Islamic culture in Cordoba in Spain studied the mechanics of flight and in 875 AD survived one successful flight on a pair of wings made of feathers on a wooden frame. Some hundred years later the great Turkish scholar *Al-Djawhari* tied two pieces of wood to his arms and climbed the roof of a tall mosque in Nisabur, Arabia, and announced to a large crowd:

- *O People! No one has made this discovery before. Now I will fly before your very eyes. The most important thing on Earth is to fly to the skies. That I will do now.*

Unfortunately, he did not, but fell straight to the ground and was killed. It would take 900 years before the dream of Al-Djawhari became true, after many unsuccessful attempts. One of the more succesful was made by *Hezarfen Ahmet Celebi*, who in 1638 inspired by work of Leonardo da Vinci, after nine experimental attempts and careful studies of eagles in flight, took off from the 183 foot tall Galata Tower near the Bosphorus in Istanbul and successfully landed on the other side of the Bosphorus. The word Hezarfen means expert in 1000 sciences and a reward of 1000 gold pieces was given to Hezarfen for his achievement.

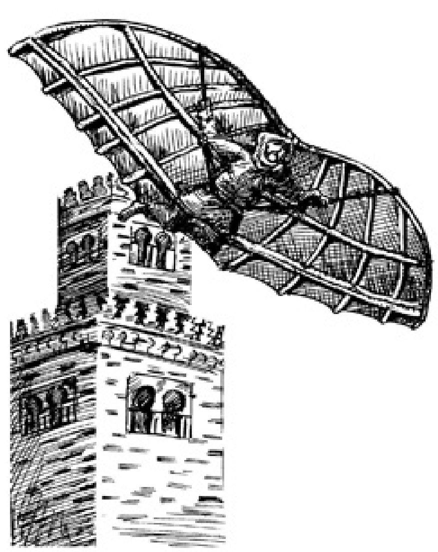

Figure 41.2: Abbas Ibn Firnas flying from the Mosque of Cordoba in 875 AD.

The understanding of *why* it is possible to fly has haunted scientists since the birth of mathematical mechanics in the 17th century. To fly, an upward force on the wing, referred to as *lift* $L$, has to be generated from the flow of air around the wing, while the air resistance to motion or *drag* $D$, is not too big. The mystery is *how* a sufficiently large ratio $\frac{L}{D}$ can be created. In the *gliding flight* of birds and airplanes with fixed wings at subsonic speeds, $\frac{L}{D}$ is typically between 10 and 20, which means that a good glider can glide up to 20 meters upon loosing 1 meter in altitude, or that *Charles Lindberg* could cross the Atlantic in 1927 at a speed of 50 m/s in his 2000 kg *Spirit of St Louis* at an effective engine thrust of 150 kp (with $\frac{L}{D} = 2000/150 \approx 13$) from 100 horse powers (because 1 hp = 75 kpm/s).

By elementary Newtonian mechanics, lift must be accompanied by *downwash* with the wing redirecting air downwards. The enigma of flight is the mechanism generating substantial downwash under small drag, which is also the enigma of sailing against the wind with both sail and keel acting like wings creating lift.

Classical mathematical mechanics could not give an answer. *Newton* computed the lift of a tilted flat plate (of unit area) redirecting a horisontal stream of fluid particles of speed $U$ and density $\rho$, but obtained a disappointingly small value approximately proportional to the square of the tilting angle or *angle of attack* $\alpha$ (in radians with one radian $= \frac{\pi}{180}$ degrees):

$$L = \sin^2(\alpha)\rho U^2, \tag{41.1}$$

since $\sin(\alpha) \approx \alpha$ (for small $\alpha$). The French mathematician *Jean le Rond d'Alembert* (1717-1783) followed up in 1752 with a computation based on *potential flow* (inviscid incompressible irrotational stationary flow), showing that both the drag and lift of a body of any shape (in particular a wing) is zero, referred to as *d'Alembert's paradox*, since it contradicts observations and thus belongs to fiction. To explain flight d'Alembert's paradox had to be resolved.

But the dream was not given up and experiments could not be stopped only because a convincing theory was lacking; undeniably it was possible for birds to fly without any theory, so maybe it could somehow be possible for humans as well.

The first published paper on aviation was *Sketch of a Machine for Flying in the Air* by the Swedish polymath *Emanuel Swedenborg*, published in 1716, describing a flying machine consisting of a light frame covered with strong canvas and provided with two large oars or wings moving on a horizontal axis, arranged so that the upstroke met with no resistance while the downstroke provided lifting power. Swedenborg understood that the machine would not fly, but suggested it as a start and was confident that the problem would be solved:

- *It seems easier to talk of such a machine than to put it into actuality, for it*

> *requires greater force and less weight than exists in a human body. The science of mechanics might perhaps suggest a means, namely, a strong spiral spring. If these advantages and requisites are observed, perhaps in time to come some one might know how better to utilize our sketch and cause some addition to be made so as to accomplish that which we can only suggest. Yet there are sufficient proofs and examples from nature that such flights can take place without danger, although when the first trials are made you may have to pay for the experience, and not mind an arm or leg.*

Swedenborg would prove prescient in his observation that powering the aircraft through the air was the crux of flying.

## 41.2 Cayley

The British engineer *Sir George Cayley* (1773-1857), known as *the father of aerodynamics*, was the first person to identify the lift and drag forces of flight, discovered that a curved lifting surface would generate more lift than a flat surface of equal area. and designed different gliders as shown in Fig.41.3. In 1804 Cayley designed and built a model monoplane glider of strikingly modern appearance. with a cruciform tail, a kite-shaped wing mounted at a high angle of incidence and a moveable weight to alter the center of gravity.

In 1810 Cayley published his now-classic three-part treatise *On Aerial Navigation*, the first to state that lift, propulsion and control were the three requisite elements to successful flight, and that the propulsion system should generate thrust while the wings should be shaped so as to create lift. Cayley observed that birds soared long distances by simply twisting their arched wing surfaces and deduced that fixed-wing machines would fly if the wings were cambered. Thus, one hundred years before the Wright brothers flew their glider, Cayley had established the basic principles and configuration of the modern airplane, complete with fixed wings, fuselage, and a tail unit with elevators and rudder, and had constructed a series of models to demonstrate his ideas. In 1849 Cayley built a large gliding machine, along the lines of his 1799 design, and tested the device with a 10-year old boy aboard. The gliding machine carried the boy aloft on at least one short flight.

Cayley recognized and searched for solutions to the basic problems of flight including the ratio of lift to wing area, determination of the center of wing pressure, the importance of streamlined shapes, the recognition that a tail assembly was essential to stability and control, the concept of a braced biplane structure for strength, the concept of a wheeled undercarriage, and the need for a lightweight source of power. Cayley correctly predicted that sustained flight would not occur until a lightweight engine was developed to provide adequate thrust and lift.

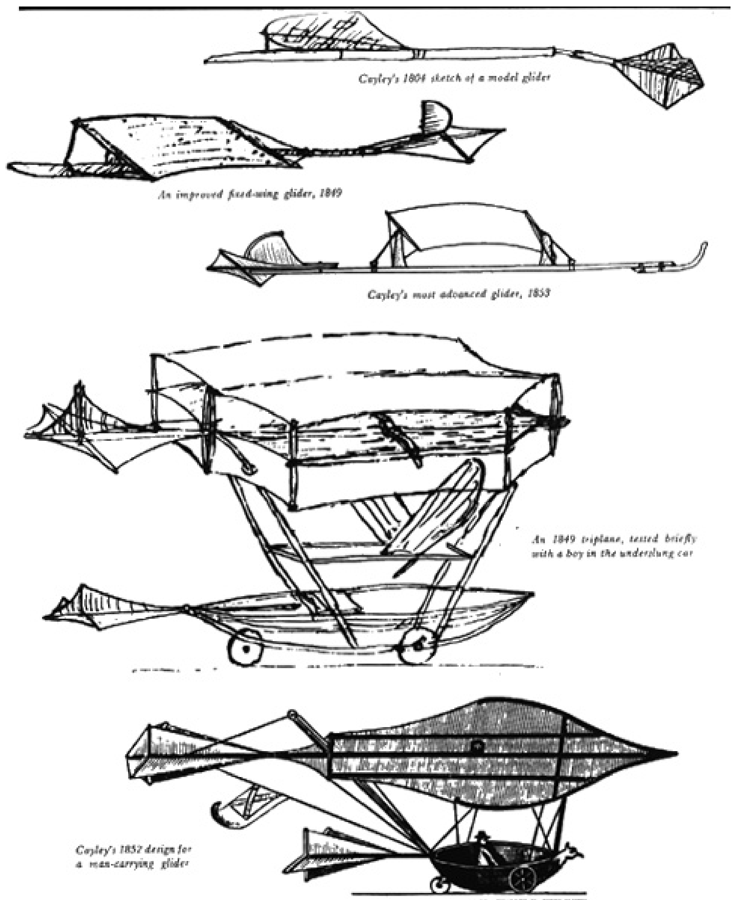


Figure 41.3: Different gliders designed by Cayley.

Cayley's efforts were continued by *William Henson* who designed a large passenger-carrying steam-powered monoplane, with a wing span of 150 feet, named *The Henson Aerial Steam Carriage* for which he received a patent in 1843, but it could not fly. The Aerial Transit Company's publicist, Frederick Marriott, had a number of prints made in 1843 depicting the Aerial Steam Carriage over the pyramids in Egypt, in India, over London, England, and other places, which drew considerable interest from the press.

In 1856, the French aviator *Jean-Marie Le Bris* made the first flight higher than his point of departure, by having his glider *L'Albatros artificiel* pulled by a horse on a beach. He reportedly achieved a height of 100 meters, over a distance of 200 meters. In 1874, *Félix du Temple* built the *Monoplane*, a large plane made of aluminium in Brest, France, with a wingspan of 13 meters and a weight of only 80 kilograms (without pilot). Several trials were made with the plane, and it is generally recognized that it achieved lift off under its own power after a ski-jump run, glided for a short time and returned safely to the ground, making it the first successful powered flight in history, although the flight was only a short distance and a short time.

The British marine engineer *Francis Wenham* (1824-1908) discovered, while unsuccessfully attempting to build a series of unmanned gliders, that the most of the lift from a bird-like wing was generated at the leading edge, and concluded that long, thin wings would be better than the bat-like ones suggested by many, because they would have more leading edge for their weight. He presented a paper on his work to the newly formed Royal Aeronautical Society of Great Britain in 1866, and decided to prove it by building the world's first wind tunnel in 1871. Members of the Society used the tunnel and learned that cambered wings generated considerably more lift than expected by Cayley's Newtonian reasoning, with lift-to-drag ratios of about 5:1 at 15 degrees. This clearly demonstrated the ability to build practical heavier-than-air flying machines; what remained was the problem of controlling the flight and powering them.

In 1866 the Polish illiterate peasant *Jan Wnek* built and flew a controllable glider launching himself from a special ramp on top of the Odporyszow church tower 95 m high above the valley below, especially during religious festivals, carnivals and New Year celebrations.

## 41.3 Lilienthal and Wright

The German engineer *Otto Lilienthal* (1848-1896) expanded Wenham's work, made careful studies of the gliding flight of birds recorded in *Birdflight as the Basis of Aviation* [25] and designed a series of ever-better hang gliders allowing him to make 2000 successful heavier-than-air gliding flights starting from a little artificial hill, before in 1896 he broke his neck falling to the ground after having stalled at 15 meters altitude. Lilienthal rigorously documented his work, including photographs, and for this reason is one of the best known of the early pioneers.

The first sustained powered heavier-than-air flights were performed by the two brothers *Orwille* and *Wilbur Wright*, who on the windy dunes of Kill Devils Hills at Kitty Hawk, North Carolina, on December 17 in 1903, managed to get their 400 kg airplane *Flyer* off ground into sustained flight using a 12 horse power engine. The modern era of aviation had started.

# Chapter 42

# Lilienthal and Bird Flight

> To invent an airplane is nothing. To build one is something. But to fly is everything. (Lilienthal)

> Sacrifices must be made! (Lilienthal near death after breaking his spine in an airplane crash in a glider of his design).

> No one can realize how substantial the air is, until he feels its supporting power beneath him. It inspires confidence at once. (Lilienthal)

> We returned home, after these experiments, with the conviction that sailing flight was not the exclusive prerogative of birds. (Lilienthal, 1874)

Otto Lilienthal gives in *Bird Flight as a Basis of Aviation* [25] the following description of bird flight:

- *From all the foregoing results it appears obvious that in order to discover the principles which facilitate flight, and to eventually enable man to fly, we must take the bird for our model. A specially suitable species of birds to act as our model is the sea-gull.*

- *How does the gull fly? At the very first glance we notice that the slender, slightly curved wings execute a peculiar motion, in so far as only the wing-tips move appreciably up and down, whilst the broader arm-portions near the body take little part in this movement, a condition of things which is illustrated in Fig. 76.*

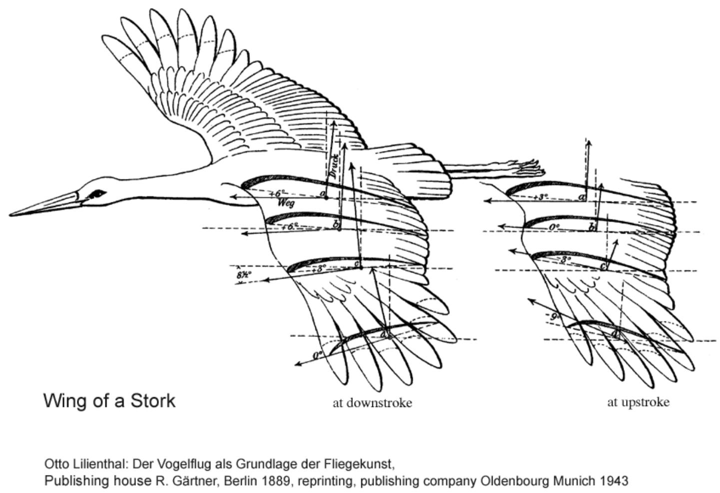


Figure 42.1: Lilienthal's analysis of a stork in flight.

- *May we not assume that the comparatively motionless parts of the wings enable the gull to sail along, whilst the tips, consisting of easily rotating feathers, serve to compensate for the loss of forward velocity ? It is unmistakable that the wide Portion of the wing close to the body, which does little work and has little movement, is intended for sustaining, whilst the narrower tips, with their much greater amplitude of movement, have to furnish the tractive power necessary to compensate for the resistance of the bird's body and for any possible restraining component.*

- *This being conceded, we are forced to consider the flying apparatus of the bird as a most ingenious and perfect mechanism, which has its fulcrum in the shoulder joint, which moves up and down, and by virtue of its articulation permits of increased lift or fall as well as of rotation of the light tips.*

- *The arm portion of the wing is heavy, containing bones, muscles , and tendons, and therefore opposes considerable inertia to any rapid movement. But it is well fitted for supporting, because being close to the body, the air pressure upon it acts on a short lever arm, and the bending strain is therefore less severe on the wing. The tip is very light, consisting of feathers only, and can be lifted and depressed in rapid succession. If the air pressure produced by it increased in proportion to the greater amplitude of movement,* *it would require a large amount of work; and would also unduly strain the wings; we therefore conclude that the real function of the wing-tips is not so much the generation of a great lifting effect, but rather the production of a smaller, but tractive effect directed forward.*

- *In fact, actual observation leaves no doubt on this point. It is only necessary to watch the gull during sunshine, and from the light effects we tan distinctly perceive the changing inclination of the wing-tips, as shown in Figs. 77 and 78, which refer to the upstroke and downstroke of the wings respectively . The gull , flying away from us, presents at the upstroke, Fig. 77, the upper side of its wings strongly illuminated by the sun, whilst during the downstroke (Fig. 78) we have tlie shaded camber presented to us from the back. The tip evidently ascends with the leading edge raised, and descends with the leading edge depressed, both phases resulting in a tractive effect.*

Da Vinci had made similar observations in *Codex on Bird Flight*:

- *Those feathers which are farthest from their fastening will be the most flexible; then the tops of the feathers of the wings will be always higher than their origins, so that we may with reason say, that the bones of the wings will be lower in the lowering of the wings than any other part of the wings, and in the raising these bones of the wing will always be higher than any other part of such a wing. Because the heaviest part always makes itself the guide of the movement.*

- *The kite and other birds which beat their wings little, go seeking the course of the wind, and when the wind prevails on high then they will be seen at a great height, and if it prevails low they will hold themselves low.*

- *When the wind does not prevail in the air, then the kite beats its wings several times in its flight in such a way that it raises itself high and acquires a start, with which start, descending afterwards a little, it goes a long way without beating its wings, and when it is descended it does the same thing over again, and so it does successively, and this descent without flapping the wings serves it as a means of resting itself in the air after the aforesaid beating of the wings.*

- *When a bird which is in equilibrium throws the centre of resistance of the wings behind the centre of gravity, then such a bird will descend with its head down. This bird which finds itself in equilibrium shall have the centre* *of resistance of the wings more forward than the bird's centre of gravity, then such a bird will fall with its tail turned to the earth.*

- *When the bird is in the position and wishes to rise it will raise its shoulders and the air will press between its sides and the point of the wings so that it will be condensed and will give the bird the movement toward the ascent and will produce a momentum in the air, which momentum of the air will by its condensation push the bird up.*

The observations of the wingbeat cycle by da Vinci and Lilienthal da Vinci can be summarized as follows:

- a forward downstroke with the wing increasingly twisted towards the tip with the leading edge down,
- a backward upstroke with the wing twisted the other way with the leading edge up.

Below we shall analyze the lift and drag generated at different moments of the wingbeat cycle, and thus give a scientific explanation of the secret of bird flight. We compare with with the lack of a scientific theory of bird flight according to state-of-the-art [17]:

- *Always there have been several different versions of the flapping flight theory. They all exist in parallel and their specifications are widely distributed. Calculating the balance of forces even of a straight and merely slowly flapping wing remained difficult to the present day. In general, it is only possible in a simplified way. Furthermore, the known drives mechanism and especially wing designs leave a lot to be desired. In every respect ornithopters are still standing at the beginning of their design development.*

Figure 42.2: Lilienthal getting ready to simulate a stork in flight.

# Chapter 43

# Wilbur and Orwille Wright

> It is possible to fly without motors, but not without knowledge and skill. (Wilbur Wright)
>
> The desire to fly is an idea handed down to us by our ancestors who...looked enviously on the birds soaring freely through space...on the infinite highway of the air. (Wilbur Wright)
>
> The natural function of the wing is to soar upwards and carry that which is heavy up to the place where dwells the race of gods. More than any other thing that pertains to the body it partakes of the nature of the divine. (Plato in *Phaedrus*)
>
> Sometimes, flying feels too godlike to be attained by man. Sometimes, the world from above seems too beautiful, too wonderful, too distant for human eyes to see . . . (Charles Lindbergh in *The Spirit of St. Louis*)
>
> More than anything else the sensation is one of perfect peace mingled with an excitement that strains every nerve to the utmost, if you can conceive of such a combination. (Wilbur Wright)
>
> The exhilaration of flying is too keen, the pleasure too great, for it to be neglected as a sport. (Orwille Wright)

The first successful powered piloted controled flight was performed by the brothers Orwille and Wilbur Wright on December 17 1903 on the windy fields of Kitty Hawk, North Carolina, with Orwille winning the bet to be the pilot of the *Flyer* and Wilbur watching on ground, see Fig 43.1. In the words of the Wright brothers from Century Magazine, September 1908:

- *The flight lasted only twelve seconds, a flight very modest compared with that of birds, but it was, nevertheless, the first in the history of the world in which a machine carrying a man had raised itself by its own power into the air in free flight, had sailed forward on a level course without reduction of speed, and had finally landed without being wrecked. The second and third flights were a little longer, and the fourth lasted fifty-nine seconds, covering a distance of 852 feet over the ground against a twenty-mile wind.*

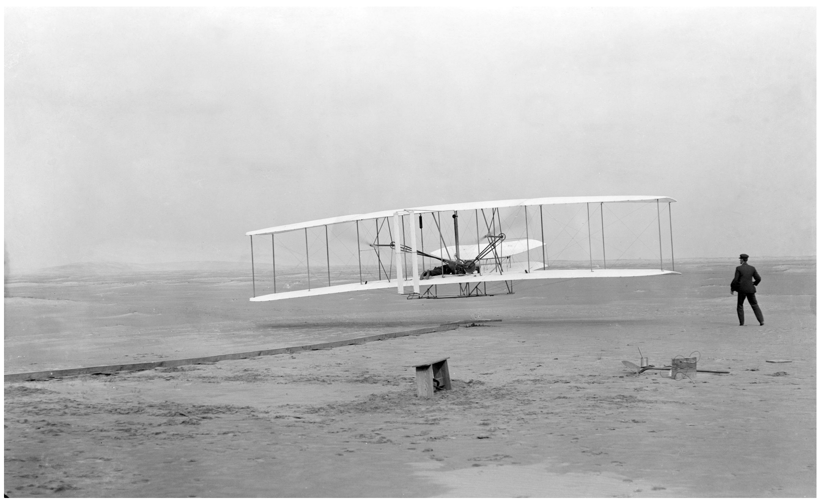

Figure 43.1: Orwille Wright (1871-1948) and Wilbur Wright (1867-1912) and the lift-off at Kitty Hawk, North Carolina, the 17th December 1903.

The work preceeding the success was described by Wilbur Wright in an address to the Western Society of Engineers in 1901 entitled *Some Aeronautical Experiments*:

- *The difficulties which obstruct the pathway to success in flying-machine construction are of three general classes: (1) Those which relate to the construction of the sustaining wings; (2) those which relate to the generation and application of the power required to drive the machine through the air; (3) those relating to the balancing and steering of the machine after it is actually in flight. Of these difficulties two are already to a certain extent solved. Men already know how to construct wings or aeroplanes which, when driven through the air at sufficient speed, will not only sustain the weight of the wings themselves, but also that of the engine and of the engineer as well. Men also know how to build engines and screws of sufficient* *lightness and power to drive these planes at sustaining speed. As long ago as 1884 a machine weighing 8,000 pounds demonstrated its power both to lift itself from the ground and to maintain a speed of from 30 to 40 miles per hour, but failed of success owing to the inability to balance and steer it properly. This inability to balance and steer still confronts students of the flying problem, although nearly eight years have passed. When this one feature has been worked out, the age of flying machines will have arrived, for all other difficulties are of minor importance.*

- *The person who merely watches the flight of a bird gathers the impression that the bird has nothing to think of but the flapping of its wings. As a matter of fact this is a very small part of its mental labor. To even mention all the things the bird must constantly keep in mind in order to fly securely through the air would take a considerable part of the evening. If I take this piece of paper, and after placing it parallel with the ground, quickly let it fall, it will not settle steadily down as a staid, sensible piece of paper ought to do, but it insists on contravening every recognized rule of decorum, turning over and darting hither and thither in the most erratic manner, much after the style of an untrained horse. Yet this is the style of steed that men must learn to manage before flying can become an everyday sport. The bird has learned this art of equilibrium, and learned it so thoroughly that its skill is not apparent to our sight. We only learn to appreciate it when we try to imitate it. Now, there are two ways of learning to ride a fractious horse: One is to get on him and learn by actual practice how each motion and trick may be best met; the other is to sit on a fence and watch the beast a while, and then retire to the house and at leisure figure out the best way of overcoming his jumps and kicks. The latter system is the safest, but the former, on the whole, turns out the larger proportion of good riders. It is very much the same in learning to ride a flying machine; if you are looking for perfect safety, you will do well to sit on a fence and watch the birds; but if you really wish to learn, you must mount a machine and become acquainted with its tricks by actual trial.*

- *Herr Otto Lilienthal seems to have been the first man who really comprehended that balancing was the first instead of the last of the great problems in connection with human flight. He began where others left off, and thus saved the many thousands of dollars that it had theretofore been customary to spend in building and fitting expensive engines to machines which were*

> *uncontrollable when tried. He built a pair of wings of a size suitable to sustain his own weight, and made use of gravity as his motor. This motor not only cost him nothing to begin with, but it required no expensive fuel while in operation, and never had to be sent to the shop for repairs. It had one serious drawback, however, in that it always insisted on fixing the conditions under which it would work. These were, that the man should first betake himself and machine to the top of a hill and fly with a downward as well as a forward motion. Unless these conditions were complied with, gravity served no better than a balky horse – it would not work at all. Although Lilienthal must have thought the conditions were rather hard, he nevertheless accepted them till something better should turn up; and in this manner he made some two thousand flights, in a few cases landing at a point more than 1,000 feet distant from his place of starting. Other men, no doubt, long before had thought of trying such a plan. Lilienthal not only thought, but acted; and in so doing probably made the greatest contribution to the solution of the flying problem that has ever been made by any one man. He demonstrated the feasibility of actual practice in the air, without which success is impossible. Herr Lilienthal was followed by Mr. Pilcher, a young English engineer, and by Mr. Chanute, a distinguished member of the society I now address. A few others have built gliding machines, but nearly all that is of real value is due to the experiments conducted under the direction of the three men just mentioned.*

The Wrights built The Flyer in 1903 using spruce and ash covered with muslin, with wings designed with a 1-in-20 camber. Since they could not find a suitable automobile engine for the task, they commissioned their employee Charlie Taylor to build a new design from scratch. A sprocket chain drive, borrowing from bicycle technology, powered the twin propellers, which were also made by hand. The Flyer was a canard biplane configuration. As with the gliders, the pilot flew lying on his stomach on the lower wing with his head toward the front of the craft in an effort to reduce drag, and steered by moving a cradle attached to his hips. The cradle pulled wires which warped the wings and turned the rudder simultaneously for lateral control, while a forward horisontal stabilizer (forward canard) was controled by the left hand.

To sum up, the Wright brothers were the first to solve the combined problem of (1) generation of lift by (sufficiently large) wings, (2) generation of thrust by a propeller powerd by a (sufficiently light) combustion engine and (3) horisontal control of balance under different speeds and angles of attack as well as lateral control. The data of the Flyer were:

- wingspan: 12.3 m
- wing area: 47 $m^2$
- length: 6.4 m
- height: 2.8 m
- weight (empty): 274 kg
- engine: gasoline 12 hp

At a lift/drag ratio of 10 the drag would be about 35 kp to carry a total weight of 350 kp, which at a speed of 10 m/s would require about 5 effective hp.

The Flyer had a forward canard for horisontal control, like the modern Swedish jet fighter *JAS Gripen*, which is an unstable configuration requiring careful control to fly, but allowing quick turns. The Wrights later replaced the canard with the conventional aft tail to improve stability. The stability of an airplane is similar to that of a boat, with the important design feature being the relative position of the center of gravity and the center of the forces from the fluid (center of buoyancy for a boat), with the center of gravity ahead (below) giving stability.

# Chapter 44

# Lift by Circulation

> Man must rise above the Earthto the top of the atmosphere and beyondfor only thus will he fully understand the world in which he lives. (Socrates)
>
> A single lifetime, even though entirely devoted to the sky, would not be enough for the study of so vast a subject. A time will come when our descendants will be amazed that we did not know things that are so plain to them. (Seneca)
>
> All the perplexities, confusion and distress in America arise, not from defects in their Constitution or Confederation, not from want of honor or virtue, so much as from the downright ignorance of the nature of coin, credit and circulation. (John Adams)
>
> If you would be a real seeker after truth, it is necessary that at least once in your life you doubt, as far as possible, all things. (Descartes)

## 44.1 Lanchester

*Frederick Lanchester*, (1868-1946) was an English polymath and engineer who made important contributions to automotive engineering, aerodynamics and co-invented the field of operations research. He was also a pioneer British motor car builder, a hobby he eventually turned into a successful car company, and is considered one of the *big three* English car engineers, the others being Harry Ricardo and Henry Royce.

Lanchester began to study aeronautics seriously in 1892, eleven years before the first successful powered flight. Whilst crossing the Atlantic on a trip to the

United States, Lanchester studied the flight of herring gulls, seeing how they were able to use motionless wings to catch up-currents of air. He took measurements of various birds to see how the centre of gravity compared with the centre of support. As a result of his deliberations, Lanchester, eventually formulated his circulation theory, which still serves as the basis of modern lift theory. In 1894 he tested his theory on a number of models. In 1897 he presented a paper entitled *The soaring of birds and the possibilities of mechanical flight* to the Physical Society, but it was rejected, being too advanced for its time. Lanchester realised that powered flight required an engine with a far higher power to weight ratio than any existing engine. He proposed to design and build such an engine, but was advised that no one would take him seriously [1].

Stimulated by Lilienthal's successful flights and his widely spread book *Bird Flight as the Basis of Aviation* from 1899, the mathematician *Martin Kutta* (1867-1944) in his thesis presented in 1902 modified the erronous classical potential flow solution by including a new term corresponding to a rotating flow around the wing with the strength of the vortex determined so that the combined flow velocity became zero at the trailing edge of the wing. This *Kutta condition* reflected the observation of Lilienthal that the flow should come off the wing smoothly, at least for small angles of attack. The strength of the vortex was equal to the *circulation* around the wing of the velocity, which was also equal to the lift. Kutta could this way predict the lift of various wings with a precision of practical interest. But the calculation assumed the flow to be fully two-dimensional and the wings to be very long and became inaccurate for shorter wings and large angles of attack.

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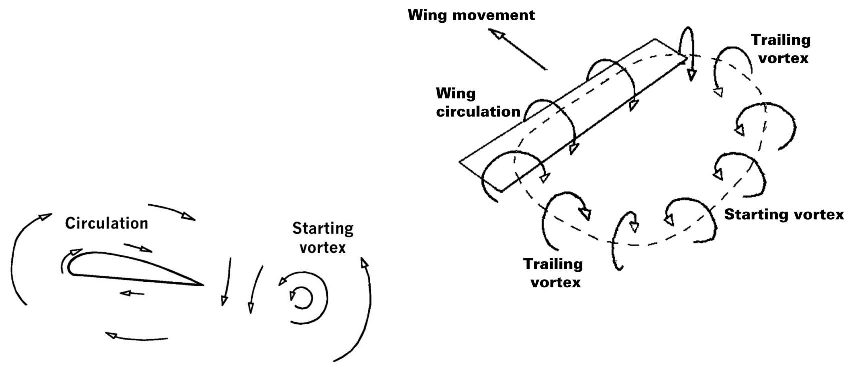


Figure 44.1: Generation of lift according to Kutta-Zhukovsky theory, as extended by Prandtl by connecting the circulation around the wing to the starting vortex by so-called trailing vortices from the wing tips. The circulation around the wing and starting vortices are unphysical, while the trailing vortices from the wing tips are real and often can be observed by condensation in damp weather.

long and became inaccurate for shorter wings and large angles of attack.

## 44.3 Zhukovsky

The mathematician *Nikolai Zhukovsky* (1847-1921), called the *father of Russian aviation*, in 1906 independently derived the same mathematics for computing lift as Kutta, after having observed several of Lilienthal's flights, which he presented before the Society of Friends of the Natural Sciences in Moscow as:

- *The most important invention of recent years in the area of aviation is the flying machine of the German engineer Otto Lilienthal.*

Zhukovsky also purchased one of the eight gliders which Lilienthal sold to members of the public.

Kutta and Zhukovsky thus could modify the mathemathical potential theory of lift of a wing to give reasonable results, but of course could not give anything but a very heuristic justification of their Kutta-Zhukovsky condition for the velocity at the trailing edge of the wing, and could not treat realistic wings in three dimensions. Further, their modified potential solutions are not turbulent, and as we will see below, their calculations were merely happy coincidences (knowing ahead the correct answer to obtain) without connection to the physics of real turbulent flow: There is no circulation around a wing, and connecting lift to circulation is unphysical.

It is remarkable that 400 years passed between Leonardo da Vinci's investigations and the largely similar ones by Lilienthal. Why did it take so long time from almost success to success? What was the role of the misleading mathematics of Newton and d'Alembert, still influencing the judgement of e.g. Lord Kelvin in the late 19th century?

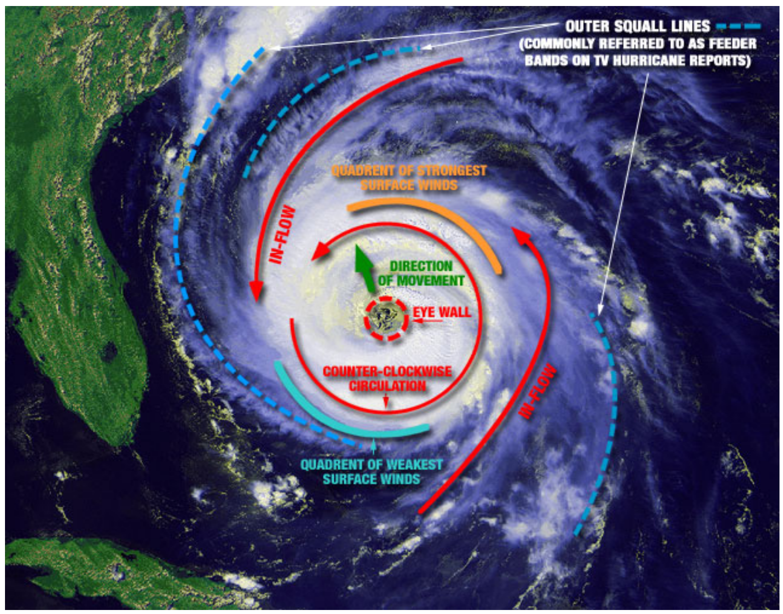


Figure 44.2: Hurricane with physical circulation.

# Chapter 45

# The Disastrous Legacy of Prandtl

## 45.1 Critique by Lancaster and Birkhoff

Prandtl's contribution to fluid mechanics was to explain separation, drag and lift as effects of a very small (vanishingly small) viscosity. This view has been seriously questioned, however with little effect since no alternative to Prandtl's theory has been in sight. Lancaster states already in 1907 in his in *Aerodynamics*[44]:

- *According to the mathematical theory of Euler and Lagrange, all bodies are of streamline form (with zero dragh and lift). This conclusion, which would otherwise constitute a reductio ad absurdum, is usually explained on the gorund the fluid of theory is inviscid, whereas real possess viscosity. It is questionable of this expanlanation alone is adequate.*

Birkhoff follows up in his *Hydromechanics* from 1950 [92]:

- *The art of knowing "how to apply" hydrodynamical theories can be learned even more effectively, in my opinion, by studying the paradoxes I will describe (e.g d'Alemberts paradox). Moreover, I think that to attribute them all to the neglect of viscosity is an oversimplification. The root lies deeper, in lack of precisely that deductive rigor whose importance is so commonly minimized by physicists and engineers.*

However, critique of Prandtl was not well received, as shown in the review of Birkhoff's book by James. J. Stoker [86]. The result is that Prandtl still dominates fluid mechanics today, although the belief in Prandtl's boundary layer theory (BLT) seems to be fading as expressed by Cowley [93]:

- *But is BLT a 20th century paradox? One may argue, yes, since for quantitative agreement with experiment BLT will be outgunned by computational fluid dynmaics in the 21st century.*

The 21st century is now here, and yes, computational fluid mechanics reveals a different scenario than Prandtl's.

But Prandtl's influence is still strong, as evidenced by the common belief that accurate computational simulation requires very thin boundary layers to be resolved. Thus Kim and Moin [43] claim that to correctly predict lift and drag of an aircraft at the relevant Reynolds number of size $10^8$, requires computation on meshes with more than $10^{16}$ mesh points, which is way out of reach for any foreseeable computer. This puts CFD into a deadlock: Either compute at irrelevant too small Reynolds numbers or invent turbulence models, which has shown to be very difficult.

Techniques for preventing laminar separation based on suction and blowing have been suggested. In the recent study [128] computational simulations are presented of synthetic jet control for a NACA 0015 wing at Reynolds number 896.000 (based on the chord length) for different angles of attack. As indicated, the relevant Reynolds number is two orders of magnitude larger, and the relevance of the study can be questioned. The effects of the synthetic jet control may simply be overshadowed by turbulent boundary layers.

## 45.2 Can You Prove that Prandtl Was Incorrect?

Lancaster and Birkhoff did not accept Prandtl's explanation of the generation of drag and lift as an effect of a vanishingly thin boundary layer. We have said that it is difficult to directly prove that an infinitely small cause cannot have a large effect, without access to an infinitely precise mathematical model or laboratory, which are not available. So Prandtl can be pretty safe to direct attacks, but not to indirect: Suppose you eliminate that vanishingly small cause from the consideration altogether, and yet obtain good correspondence between theory and experiment, that is, suppose you observe the effect without the infinitely small cause. Then you can say that the small cause has little to do with the effect.

This is what we do: We compute turbulent solutions of the incompressible Navier-Stokes equations with *slip* boundary conditions, requiring only the normal velocity to vanish letting the tangential velocity be free, and we obtain drag and lift which fit with experiments. We thus obtain the effect (drag and lift) without

Prandtl's cause consisting of a viscous boundary layer with *no-slip* boundary condition requiring also the tangential velocity to vanish. We conclude that the origin of drag and lift in slightly viscous flow, is not viscous boundary layers with no-slip boundary conditions.

We have motivated the use of slip boundary condition by the fact that the *skin friction* of a turbulent boundary layer (the tangential force from a no-slip boundary condition), tends to zero with the viscosity, which is supported by both experiment and computation, also indicating that boundary layers in general are turbulent. More generally, we use a friction-force boundary condition as a model of the skin friction effect of a turbulent boundary layer, with a (small) friction coefficient determined by the *Reynolds number* $Re = \frac{UL}{\nu}$, where $U$ is a representative velocity, $L$ a length scale and $\nu$ the viscosity. The limit case of zero friction with slip then corresponds to vanishing viscosity/very large Reynolds number, while large friction models no-slip of relevance for small to moderately large Reynolds numbers. In mathematical terms we combine the Navier-Stokes equations with a natural (Neumann/Robin type) boundary condition for the tangential stress, instead of an essential (Dirichlet type) condition for the tangential velocity as Prandtl did.

# Chapter 46

# Prandtl and Boundary Layers

> On an increase of pressure, while the free fluid transforms part of its kinetic energy into potential energy, the transition layers instead, having lost a part of their kinetic energy (due to friction), have no longer a sufficient quantity to enable them to enter a field of higher pressure, and therefore turn aside from it. (Prandtl)

> The modern world of aerodynamics and fluid dy- namics is still dominated by Prandtls idea. By every right, his boundary-layer concept was worthy of the Nobel Prize. He never received it, however; some say the Nobel Committee was reluctant to award the prize for accomplish- ments in classical physics...(John D. Anderson in [2])

> No flying machine will ever fly from New York to Paris. (Orville Wright)

## 46.1 Separation

The generation of lift and drag of a wing is closely connected to problem of *separation* in fluid mechanics: As a body moves through a slightly viscous fluid initially at rest, like a car or airplane moving through still air, or equivalently as a fluid flows around a body at rest, fluid particles are deviated by the body in a *contracting flow* switching to an *expanding flow* at a *crest* and eventually *separate* away from the body somewhere in the rear, at or after the crest. In the front there is typically a *stagnation point*, where the fluid velocity vanishes allowing *laminar attachment at stagnation* to the boundary. On the other hand the fluid mechanics of the *turbulent separation* occuring in the rear in slightly viscous flow, which creates *drag* and *lift* forces, appears to be largely unknown, despite its crucial importance in many applications, including flying and sailing. The basic study concerns separation from a convex body like a sphere, circular cylinder, wing, car or boat hull.

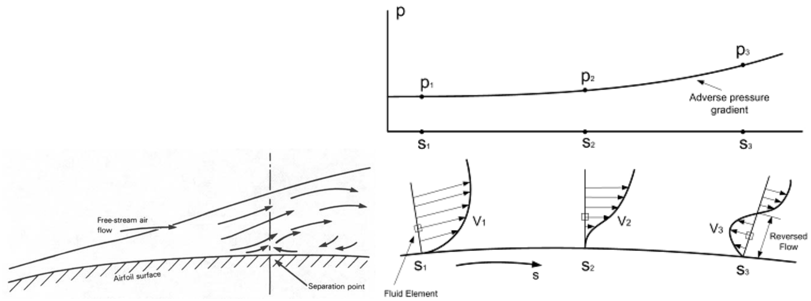


Figure 46.1: Prandtl's idea of laminar viscous separation with no-slip caused by an adverse pressure gradient, which is does not describe the turbulent slightly viscous separation with slip in the flow of air around a wing.

## 46.2 Boundary Layers

In 1904 the young German physicist Ludwig Prandtl (1875-1953) suggested in a 10 page sketchy presentation entitled *Motion of Fluids with Very Little Viscosity* [119] at the Third International Congress of Mathematics in Heidelberg, that the substantial drag of a bluff body moving through a fluid with very small viscosity (such as air or water), possibly could arise from the presence of of a thin *laminar boundary layer*, where the fluid velocity radpidly changes from its free-stream value to zero on the boundary corresponding to a *no-slip boundary condition*, causing the flow to *separate* from the boundary brought to stagnation under an *adverse pressure gradient* (negative pressure gradient in the flow direction), to form a low-pressure wake behind the body. But the acceptance of Prandtl's ideas was slow [2]:

- *Prandtls idea (about the boundary layer) went virtually unnoticed by anybody outside of Göttingen... The fifth and sixth editions of Lambs classic text Hydrodynamics published in 1924, devoted only one paragraph to the boundary-layer concept.*

However, Prandtl had two forceful students, Theodore von Karman (who emigrated to the US in 1930) and Hermann Schlichting (who stayed in Germany), who crowned Prandtl as the father of modern fluid mechanics. Prandtl's main ideas are described as follows in Schlichting's treatise *Boundary Layer Theory* from 1951:

- *Boundary layer flow has the peculiar property that under certain conditions the flow in the immediate neighbourhood of a solid wall becomes reversed causing the boundary layer to separate from it. This is accompanied by a more or less pronounced formation of eddies in the wake of the body. Thus the pressure distribution is changed and differs markedly from that in a frictionless stream. The deviation in pressure distribution from that of the ideal is the cause of form drag, and its calculation is thus made possible with the aid of boundary layer theory.*

- *The first important question to answer is to find when separation of the flow from the wall may occur. When a region with an adverse pressure gradient exists along the wall, the retarded fluid particles cannot, in general, penetrate too far into the region of increased pressure owing to their small kinetic energy. Thus the boundary layer is deflected sideways from the wall, separates from it, and moves into the main stream. In general the fluid particles follow the pressure gradient and move in a direction opposite to the external stream.*

- *In some cases the boundary layer increases its thickness considerably in the downstream direction and the flow in the boundary layer becomes reversed. This causes the decelerated fluid particles to be forced outwards, which means that the boundary layer is separated from the wall. We then speak of boundary layer separation. This phenomenon is always associated with the formation of vortices and with large energy losses in the wake of the body. The large drag can be explained by the existence of large deviation in pressure distribution (from potential flow), which is a consequence of boundary-layer separation.*

- *Downstream the pressure minimum the discrepancies increase very fast on approaching the separation point (for circular cylinder).*

- *The circumstance that real flows can support considerable rates of pressure increase (adverse pressure gradients) in a large number of cases without separation is due to the fact that the flow is mostly turbulent. The best known examples include cases of flow past circular cylinders and spheres, when* *separation occurs much further upstream in laminar than in turbulent flow. It is nevertheless useful to consider laminar flow because it is much more amenable to mathematical treatment than is the case of turbulent flow....At the present time these very complicated phenomena (separation in turbulent flow) are far from being understood completely...*

- *The form drag which does not exist in frictionless subsonic flow, is due to the fact that the presence of the boundary layer modifies the pressure distribution on the body as compared with ideal flow, but its computation is very difficult.*

- *The origin of pressure drag lies in the fact that the boundary layer exerts a displacement action on the external stream. This modifies somewhat the pressure distribution on the body surface. In contrast with potential flow (d'Alembert's paradox), the resultant of this pressure distribution modified by friction no longer vanishes but produces a preessure drag which must be added to skin friction. The two together give form drag.*

- *In the case of the most important fluids, namely water and air, the viscosity is very small and, consequently, the forces due to viscous friction are, generally speaking, very small compared with the remaining forces (gravity and pressure forces). For this resaon it was very difficult to comprehend that very small frictional forces omitted in classical (inviscid) theory influenced the motion of a fluid to so large extent.*

Prandtl described the difficulties himself in *Applied Hydro- and Aeromechanics* from 1934:

- *Only in the case where the "boundary layer" formed under the influence of the viscosity remains in contact with the body, can an approximation of the actual fluid motion by means of a theory in terms of the ideal frictionsless fluid be attempted, whereas in all cases where the boundary leaves the body, a theoretical treatment leads to results which do not coincide at all with experiment. And it had to be confessed that the latter case occurs most frequently.*

In a nutshell, these quotes present much of the essence of modern fluid mechanics propagated in standard books and courses in fluid mechanics: Drag and lift in slightly viscous flow are claimed to arise from separation in a thin viscous laminar boundary layer brought to stagnation with reversed flow due to an adverse pressure gradient. On the other hand, both Prandtl and Schlichting admit that this standard scenario does not describe turbulent flow, always arising in slightly viscous flow, but persists that *"it is nevertheless useful to consider laminar flow because it is much more amenable to mathematical treatment"*. However, turbulent and laminar flow have different properties, and drawing conclusions about turbulent flow from studies of laminar flow can be grossly misleading.

## 46.3 Prandtl's Resolution of d'Alembert's Paradox

The commonly accepted resolution of d'Alembert's Paradox propagated in the fluid dynamics literature is attributed to Prandtl, who in his 1904 article suggested that drag/lift possibly could result from *transversal vorticity* caused by tripping of the flow by a no-slip boundary condition and thereby changing the global flow. Prandtl was inspired by Saint-Venant stating in 1846 [85]:

- *But one finds another result (non-zero drag) if, instead of an inviscid fluid – object of the calculations of the geometers Euler of the last century – one uses a real fluid, composed of a finite number of molecules and exerting in its state of motion unequal pressure forces having components tangential to the surface elements through which they act; components to which we refer as the friction of the fluid, a name which has been given to them since Descartes and Newton until Venturi.*

Saint-Venant and Prandtl thus suggested that drag in a real fluid possibly could result from tangential frictional forces in a thin viscous boundary layer creating transversal vorticity, and accordingly inviscid potential flow could be discarded because it has no boundary layer. These suggestions have over time been transformed to become an accepted fact of modern fluid dynamics, questioned by few. The mathematician *Garret Birkhoff* (1911-1996) conjectured in [92] that drag instead could be the result of an instability of potential flow, but after a devastating review [86], Birkhoff did not pursue this line of thought, and even partly changed position in a second edition of the book.

But we shall see that Birkhoff was correct: Potential flow with zero drag is unstable, and this is the reason it cannot be observed. What can be observed is turbulent flow with substantial drag, and there are aspects or outputs of turbulent flow which are *wellposed* in the sense that they do not change under small perturbations, while potential flow is *illposed* with respect to all outputs of any significance and thus unphysical.

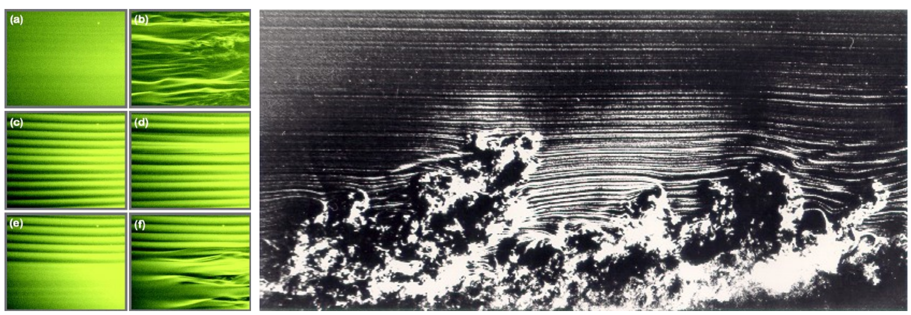


Figure 46.2: Horisontal boundary layers with streaks of streamwise vorticity (top view above), and turbulent boundary layer (side view below).

# Part VII

# AIAA and New Theory

# Chapter 47

# A Kuhnian Case Study

The reception by the scientific community of our New Theory of Flight offers material for a case study in the sociology of science in the spirit of Thomas Kuhn and his influential *The Structure of Scientific Revolutions*. We have here a clear case of a New Theory challenging the Old Theory of Kutta-Zhukovsky-Prandtl, which has dominated the thinking of aerodynamicists over a century with little change.

The New Theory reflects a breakthrough in computational solution of the Naviers-Stokes allowing accurate simulation of slightly viscous turbulent flow with millions of meshpoints feasible on todays supercomputers, by breaking the dictate of Prandtl requiring resolution of thin boundary layers far beyond present computability. In short, the breakthrough boils down to using a slip boundary condition on solid wall, instead of Prandtl's no-slip condition forming thin boundary layers beyond resolution.

The New Theory consists of a mathematical analysis of solutions of the Navier-Stokes equations describing the airflow around a wing in understandable form as potential flow modified by 3d rotational separation. The New Theory illustrates that understanding in physics means understanding essential aspect of a mathematical model.

The New Theory is not a refinement of the Old Theory but a restart from fundamentals. In this sense it similar to the heliocentric vs the geocentric astronomical theory.

As study material we reprint below our article New Theory of Flight submitted to AIAA Journal in March 2012 and the communication with AIAA including referee reports together with our rebuttal and analysis. Notice in particular:

- AIAA is critical to current text-book theory of the aerodynamics of flight, which is described as an incomplete classical theory.
- AIAA seeks in lengthy referee/editor reports to fill in the missing parts of the incomplete classical theory, which cannot be found in text-books.
- AIAA thus defends the Old Theory, while admitting that it is today found incomplete, because the once complete theory has been forgotten.
- AIAA expresses appreciation of essential parts of the article: 3d separation, computational solution of the Navier-Stokes equations with slip boundary condition, and our analysis of computations.
- AIAA does not really question the New Theory, only defends the Old Theory, and then rejects the New Theory without scientific reason.

Of course this recepetion of the New Theory by AIAA is to be expected since AIAA has invested so much in the Old Theory. On the other hand, it is a risky strategy to just swipe the New Theory off the table: There is massive evidence that the New Theory is correct, recorded in this book, and this evidence will not just simply disappear by closing the eyes. In any case, the reception of the New Theory by the establishment of the Old Theory offers material for a Kuhnian study.

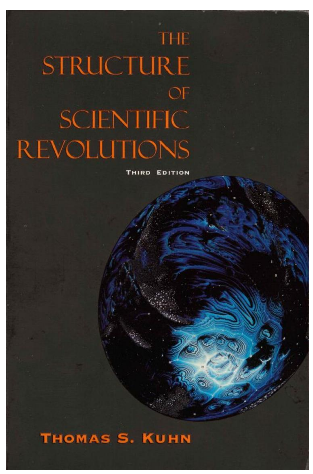


Figure 47.1: Kuhn's Structure of Scientific Revolutions with fluid dynamics front cover.

# Chapter 48

# AIAA Rejection Letter

Dear Drs Hoffman, Jansson and Johnson:

In view of the criticisms of the reviewer(s), your manuscript New Theory of Fligh has been declined for publication in the AIAA Journal.

Please carefully read the attached reviews. Your paper is unusual in that it challenges our existing understanding of aerodynamics. I believe the reviewers have treated your paper fairly and have given thoughtful, well-reasoned critiques of your paper. They have not been simply dismissive in their response. I hope you will follow their suggestions for further reading so that you may better understand the basis of their remarks.

Thank you for considering the AIAA Journal for the publication of your research. I hope the outcome of this specific submission will not discourage you from the submission of future manuscripts.

Sincerely, Gregory Blaisdell AIAA Journal Associate Editor

Figure 48.1: AIAA The World's Forum for Aerospace Leadership.

# Chapter 49

# Referee Report 1

The paper essentially consists of two related parts:

- (a) An attempt to discredit the authors version of the existing circulation theory by Kutta- Zhukovsky-Prandtl formed 100 years ago.
- (b) The presentation of their new theory to calculate and explain the physics of the flow past a lifting wing.

First of all, there is no circulation theory by Kutta-Joukowski-Prandtl that has remained untouched for a century. In fact, the authors appear to misunderstand boundary-layer theory (and the context it provides for potential flow theory) and seem unaware of decades of research of modern modifications to it. The authors provide no documented scientific evidence to discredit the current state of the art.

They claim that a circulation theory of lift (potential flow) and a boundary-layer theory (viscous flow) are unrelated. First of all, the concept of circulation is not necessary for the physical explanation of lift-the physical Kutta condition leads to the correct (as verified by experiment) solution to the potential flow problem. Circulation enters the mathematical problem for the incompressible potential flow past an airfoil since the problem is non-unique without its specification.

More importantly, Prandtls boundary-layer theory is not a viscous theory for drag but an asymptotic theory for the solution to the Navier-Stokes equations at large Reynolds number. Potential flow is not presented as the solution for lift but as the first term in an asymptotic expansion-the potential flow and boundary-layer theories are connected through the matching process. The versions of potential theory and boundary-layer theory the authors present are only the first terms in the expansion. Their claim that the theories of Kutta-Joukowski and Prandtl are both incorrect at separation (undefined by the authors but apparently only considered at the trailing edge) does not take into account the extensive research into the potential flow-boundary-layer coupling. In fact, the inclusion of the effect of the displacement thickness in the second-order potential flow solution renders arguments associated with a trailing-edge stagnation point moot. (The trailing-edge stagnation point does not appear for a cusped trailing edge). In addition, problems arising with the calculation of the boundary layer past the trailing edge or a separation point are addressed with a strong-interaction version of the boundary-layer equations (see the discussion in Chapter 14 of Katz and Plotkin which also includes a detailed discussion of the matching process referred to above).

An example of an approach which blends the potential and viscous flows to provide well- accepted solutions for the lifting flow past an airfoil is given in the XFOIL code of Drela (also described in Chapter 14 of Katz and Plotkin).

The authors new approach is to solve the Navier-Stokes equations numerically with an unphysical slip boundary condition. They state that We will discover that solutions of the Navier-Stokes equations with small viscosity and skin friction can be viewed as modified potential solutions, which are partly turbulent and which arise from the instability of potential flow at separation. The slip boundary condition does not give rise to boundary layers and the real flow may thus stay close to potential flow before separation.

The authors must demonstrate that the slip boundary condition somehow matches the physics of viscous flow near a solid boundary. They do not do this. They therefore create a mathematical problem which isnt based on the physics and use it to claim the absence of boundary layers. They claim that potential flow is a model of the complete flow and that an instability at separation leads to the correct resulting flow. How do the authors even define separation in a potential flow? The authors create their model to explain lift but it must also describe the complete flowfield. The failure of their model to reproduce the flowfield for a well-documented example is demonstrated in their solution for the zero angle-of-attack flow (no lift) past a NACA 0012 airfoil. They state that the drag results from 3d rotational separation at the trailing edge. There is no boundary layer. How would their model behave in the limit of a symmetric airfoil of vanishing thickness at zero angle of attack? Where would the drag come from? The drag is not calculated by an integration of wall shear stress (can the shear stress be found with the authors slip boundary condition?). In addition, they even introduce leading edge suction into the drag discussion when it is only defined with respect to linearized potential flow aerodynamics.

In summary, the authors have not presented us with an aerodynamic theory alternative to the modern boundary-layer theory in the literature which addresses lift and drag. At most, perhaps they present a numerical model of the governing equations which avoids the need to discretize the boundary layer. They would however need to demonstrate how drag is calculated with such a model and that the results match with detailed experiments.

# Chapter 50

# Referee Report 2

The classical theory of flight is one the most beautiful and subtle achievements of applied mathematics. However, it is no longer of interest to mathematicians, because they know that it has been a solved problem for many decades (although they have forgotten the details). Obviously, it remains of interest to engineers on account of its predictive ability, but it can be employed very successfully without knowledge of the subtleties. Aerodynamics today is therefore almost always taught in a truncated version that retains all of the utility, but has lost much of the profundity. Even the truncated version is no longer as highly respected as it used to be, because Computational Fluid Dynamics delivers, with no requirement for deep thought, most of the practical answers that are needed. In consequence, there are many employed today in the aerospace industry, and even in academia, whose grasp of the basic theory of flight contains many gaps. These gaps are apparent to thoughtful students, who frequently attempt to fill them in for themselves, although the remedy is usually worse than the disease. I believe that the authors of the paper under review would have no quarrel with the orthodox theory if they knew all of the details, although they are right to quarrel with the truncated version that they, like others, have apparently received.

The authors citation from Hoffren reveals the unfortunate mathphobia that many critics display. Understanding flight requires intuiting the behavior of an intangible medium for which our evolution has provided no apt language; it is hardly surprising that an exact understanding requires the use of abstract thought, but the gap is not unbridgeable. The response from the New York Times is merely irresponsible journalism, but undoubtedly an air of mystery does pervade flight, and the attempt to dispel it by simplified accounts does as much harm as good. The present authors may be innocent of mathphobia; nevertheless they unfortunately feed the flames of irresponsible journalism. All of the criticisms that comprise Section I of their paper can be answered, and I will try to do this below.

The authors experience great difficulty with the relationship between potential flow and real flow. This is not surprising because it is glossed over in the great majority of contemporary texts. There is a mathematical subtlety involved because the flow of a fluid at infinite Reynolds number (zero viscosity) is not always the same as the limit at very small viscosity (it is a singular perturbation problem), and so the question is what light can be shed by the former on the latter?

Let us deal first with the issue of how circulation arises. Circulation is simply the integral of vorticity, so first we need to ask how voticity arises. The authors have written down the compressible Navier-Stokes equations, and in suitable textbooks they will find, derived from these, the vorticity transport equation. There is only one term in this equation that accounts for the creation of vorticity, and that applies only to compressible flows. There is no way to create vorticity within a viscous incompressible fluid. Vorticity can be created only at a solid boundary. It travels into the interior solely by diffusion, but once there, it can be transported, stretched, and compressed. So the circulation required for Joukowski theory is an integral of the vorticity contained in the boundary layer. In the limit of vanishingly small viscosity, the boundary layer has no thickness, but is still present as an infinitesimal layer of infinite vorticity and hence making a finite contribution to circulation. The flow outside of the boundary layer is not potential flow modified by circulation. It IS potential flow because its vorticity is zero (and the circulation around any contour that does NOT enclose the airfoil is therefore zero) but it obeys boundary conditions that allow for circulation around the airfoil. The citation from [20] is absolutely correct. The above is Prandtls brilliant insight, which explains what Kutta and Joukowsky could only hypothesize. It is of course no criticism whatsoever of any scientific theory that its insights were arrived at gradually.

Many people have difficulty understanding how the apparently local process of vorticity generation can give rise to circulation at infinity. There is a tendency to suppose that the vorticity must be spread by viscosity, which does not seem plausible, and is indeed too slow, by many orders of magnitude. But by definition the circulations around any two contours, both of which surround the airfoil and are therefore separated only by irrotational flow, must be identical. Again, this strikes people as physically implausible. But what happens is that the circulation at infinity is set up by acoustic waves, and, if the flow really were incompressible, these travel infinitely fast. What acoustic waves cannot do is create vorticity.

It still remains to be explained why that particular value of circulation that forces separation to the trailing edge is observed (the Kutta condition). Again, this is glossed over in contemporary texts, but has nothing to do with any instability of potential flow. It is due to instability of the boundary layer (which in real flows is present at any Reynolds number). Suppose the trailing edge T is sharp, and suppose that the rear stagnation point S is somewhere else. The static pressure is maximum at S (by Bernoullis Theorem) and so the flow from T to S will be against an adverse pressure gradient. This is now a problem in boundary layer theory, which tells us that the boundary layer is probably unstable. There is no absolute certainty involved, because details may be important —such as the actual radius of the trailing edge, the structural rigidity, and very importantly the Reynolds number. At the extremely low Reynolds numbers that characterize microbial swimming, the boundary layer is extremely thick and quite stable. At Reynolds numbers that characterize the flight of birds and aircraft, there can be small effects of the radius, as the authors notice. In engineering and in nature, the radius is always made as small as practical, otherwise the stall behavior may be impaired.

No Kutta condition applies at the leading edge L, because the flow from a forward separation point S to L involves a favorable pressure gradient. Leading edges are usually rounded because (a) there is no need to make them sharp, and (b) the flow from L in the direction opposite to S is now in an adverse pressure gradient that needs to be kept small.

The authors greatly underestimate the classical theory, most likely because the usual truncated exposition has not shown it to them in its proper light. If they take time to realize how its parts fit together, they will come to see that is a masterpiece of physical modeling.

Section II describes the computer code that is their basis for disputing the classical theory. This is their area of expertise, and it may be assumed that their description is accurate. However, they state that real flow may thus stay close to potential flow before separation Most emphatically this is not true. The real flow (by which they mean their computed flow) always contains a boundary layer whose influence is not negligible at any Reynolds number. This is characteristic of singular perturbation problems, and is the reason why Prandtls insight was transformative to the theory of flight.

Section III gives the authors intuitive version of their account. Taking their numbered points in order,

1. This is true, although its implications need to be based on a sound understanding of what is, and what is not, potential flow.

2. The authors do not give a mechanism by means of which separation would avoid the building up of pressure. Do they have in mind the separation of a finite boundary layer? In that case, whatever the mechanism, the effect surely involves its thickness, so how do they explain the almost total independence of lift coefficients to several orders of magnitude in Reynolds number? (This is different from the scale invariance of the inviscid flow). If they are thinking of some idealized infinitesimal layer, what do they mean by separation? And by what mechanism is its influence conveyed?

3. This merely states a standard definition, and suggests no consequences.

4. I do not understand this sentence. Would not suction from above and push from below cause Upwash? The truth is that the suction, the push, and the downwash (together with upwash ahead of the wing), are all consequences of circulation. This is because that is how Laplaces equation behaves. It is the inevitable consequence of acoustic disturbances having come into equilibrium.

5. I will leave this to the point where it is developed in more detail.

Section IV states that sharp trailing edges are not necessary. This will not come as a surprise to any practicing aerodynamicist. As CFD practitioners, the authors are familiar with the NACA 0012 airfoil that they employ as a test case, and will know that it represents a standard thickness distribution, empirically derived and algebraically described. If they evaluate the formula for this thickness distribution at x=c, taking the formula from an original source, they will find that the thickness there is (I think I remember) about 0.5

Section V invokes scale invariance to explain how, within their theory, the lift and drag would be independent of trailing edge radius.

Section VI criticizes the classical solution on these grounds:

- There is no mechanism for generating large-scale circulation. Indeed there is; this was discussed earlier.

- The high pressure predicted at the trailing edge is not seen in experiments or computations. This also deserves an answer. The prediction of potential theory is that stagnation pressure will be achieved at the trailing edge if the included angle there is non-zero (for a cusped trailing edge there is no stagnation point). Nevertheless, this pressure decays very rapidly (like some very small negative power of distance) even in ideal flows, provided that the included angle is small. In real (or even computed) flows, the boundary layer absorbs most of the change in slope, even at high Reynolds numbers. The pressure distribution is of course dictated by the displacement surface, which is not singular.

- The classical solution is a mathematical trick to introduce lift. Nothing could be further from the truth. It is in full accordance with physical understanding and experimental observations. The trick, if it deserves to be so called, lies in condensing this to a simple boundary condition, the effect of which is to force the zero-viscosity solution to obey the boundary condition for the small-viscosity solution. The authors are of course familiar with the fact that when one loses the highest order derivatives from a pde, the ability to impose a boundary condition is also lost.

Section VII describes the authors computational experiments, which are three-dimensional as, of course, are real wings. It is well known that it is extremely hard, and probably impossible, to produce two-dimensional flow experimentally. It should be, and usually is, impossible to produce it in a three-dimensional computation. I would have been extremely surprised if the computations had not shown small irregular spanwise variations of the kind presented. In fact, the result is exactly consistent with the expectation that a 3D realization of a 2D flow will behave very similarly to the 2D flow, but with 3D features that are usually small. Although there are examples where the 3D features are not small, a guiding principle of aerodynamic design is to avoid surprises, and this is another reason for designers to prefer sharp trailing edges. There is a computation that the authors should have made, which is to run their code in 2D mode and compare the outcome. I confidently predict that at a low angle of attack there will be almost no difference in the forces.

Section VII fails almost every test for the proper reporting of computational results. The description of the code omits many details that might be important. Additionally, for these particular tests, the following questions should have been answered. What was the radius of the trailing edge as a fraction of the chord? What was the Reynolds number? What was the Mach number? What was the mesh size in the trailing edge region? Have they estimated the errors or run convergence tests? Remarks elsewhere in the text suggest that this may have been run with the dissipative terms turned off to simulate a potential flow. It should be realized that the resulting first-order system is mathematically different from the second-order potential flow equation. This is because the first- order system permits vortical solutions but the second-order equation (by definition) does not. However, In the first-order system, the numerical dissipation will remain, and will serve a function, similar to that of the physical dissipation, of removing energy from the high-frequency modes. Calculations of this kind are often referred to as Implicit Large Eddy Simulation, and are a recognized, but somewhat controversial, approach to modeling some aspects of turbulence. Is that what is being done? In any case, the mere fact of vorticity being observed means that the code did not simulate a potential flow.

If the authors do believe that they are modeling potential flow, it would explain why the observed drag is said to be accounted for by the separation effect. They think that the drag should theoretically be zero and they need to provide an explanation. They need look no further than the numerical dissipation. It is notoriously hard to create an Euler code that does not predict drag at subcritical conditions, especially when compressible codes are run at low Mach numbers. They should make the test suggested above, of running their code in 2D mode, which would eliminate their explanation but leave other explanations in place. What happens to the drag? I very strongly recommend that they do this experiment.

The thrust of the paper so far is that classical explanations of flight, based on two-dimensional potential flow, are wrong because they are thought to be self-evidently inconsistent. Now in section VIII the line is taken that the classical explanations are wrong because the true explanation involves three-dimensional flow. I cannot see how this helps. If the authors had been correct previously–that the classical explanation offers no mechanism to generate circulation–there would still be no such mechanism. If the authors deny that circulation exists, let them calculate it from their simulations; it will be there.

However, the authors are correct that separation might be fundamentally different in 3D than in 2D. One of the best-known examples is the industrial aerodynamics problem of flow past a tall cylindrical chimney. This might naively be supposed to be two-dimensional in planes parallel to the ground, but in practice is always three-dimensional, asymmetric and unsteady, resulting in variable forces perpendicular to the oncoming wind. It is customary to place a spiral band around such chimneys to prevent the flows at different heights from being phase-locked. In the present case, the intuitive expectation would be that such 3D perturbations would appear, but only at the small scale of the trailing edge radius, at least for small incidence. Since the streamwise vorticity must vanish in the mean, the velocities induced by it must substantially cancel. (Incidentally, the matter of induced velocity is also frequently misunderstood. To forestall that possibility, it should be realized that induced velocity does not need mysterious mechanisms to explain it, but is a necessary consequence of vector calculus. The name is unfortunate).

There follows a stability analysis of the linearized Euler equations. This is of doubtful validity because it assumes that a perturbation with non-zero curl can be introduced into an irrotational flow. Physically this cannot be done; I have already explained that there is no mechanism even within the Navier-Stokes equations for vorticity creation, merely the evolution of vorticity already present. Creation must take place at solid surfaces and involve viscosity, or must require external body forces. There is nothing at all wrong with Kelvins Theorem.

Regrettably, it is my conclusion that publication of any of this material, in any form, would be highly retrogressive. The authors have put their fingers accurately on many of the defects in the truncated versions of aerodynamic theory that are now current. However, they have not realized that all these difficult issues were struggled with years ago by the founding fathers of the subject, and resolved in completely satisfactory ways. Sadly, the outcomes of those struggles have since been simplified or discarded in modern presentations to create a pragmatic treatment focusing on utility. Undergraduate textbooks these days all too often simply omit anything that students find difficult.

The authors have then sought their own explanations, stimulated by interesting results from their Navier-Stokes code. However, they have failed to ask questions that would have been suggested by any experienced practical aerodynamicist. Consequently, they have simply added to a proliferating literature of theories of flight that serves only to confuse students and mislead the public. I wish it were possible to retract what has already been written.

This review is very much longer than I would normally write, because I believe that serious issues of substantial public interest are involved. Even so, it may be too brief to carry conviction with the authors, so I am going to recommend some reading books. One is History of Aerodynamics by J D Anderson, and another is An Informal Introduction to Theoretical Fluid Mechanics by M J Lighthill. These are both accounts intended for a non-specialist but technically-literate readership. A book intended for specialists, but that is old enough not to have succumbed to the almost universal dumbing down, is Theory of Flight, by R. von Mises. All of these sources should be consulted before the authors attempt any response to reviews. If there are any statements about two-dimensional flow concerning which they feel skeptical, they have only to run their own code and retrieve the appropriate data. Finally the authors have respected colleagues in the aerospace department at KTH, with whom they should consult freely.

# Chapter 51

# Short Analysis of Referee Reports

The referee reports offer shocking evidence as concerns the state-of-the-art of aerodynamics represented by AIAA, The Worlds Forum for Aerospace Leadership, and give ample evidence that the existing theory is insufficient.

Reviewer 2 starts out with:

- *The classical theory of flight is no longer of interest to mathematicians, because they know that it has been a solved problem for many decades (although they have forgotten the details).*

- *Obviously, it remains of interest to engineers on account of its predictive ability, but it can be employed very successfully without knowledge of the subtleties.*

- *Aerodynamics today is therefore almost always taught in a truncated version that retains all of the utility, but has lost much of the profundity.*

- *Even the truncated version is no longer as highly respected as it used to be, because Computational Fluid Dynamics delivers, with no requirement for deep thought, most of the practical answers that are needed.*

- *In consequence, there are many employed today in the aerospace industry, and even in academia, whose grasp of the basic theory of flight contains many gaps.*

- *These gaps are apparent to thoughtful students, who frequently attempt to fill them in for themselves, although the remedy is usually worse than the disease.*

- *I believe that the authors of the paper under review would have no quarrel with the orthodox theory if they knew all of the details, although they are right to quarrel with the truncated version that they, like others, have apparently received.*

What Reviewer 2 effectively says is that the theory of flight is a mess, abandoned by mathematicians and not understood by engineers designing airplanes. This is serious criticism of the existing theory, by the reviewer.

Reveiwer 2 then continues by dismissing the criticism published in AIAA J and NYT/John D. Anderson (and many other places):

- *The authors citation from Hoffren reveals the unfortunate mathphobia that many critics display.*

- *The response from the New York Times is merely irresponsible journalism, but undoubtedly an air of mystery does pervade flight, and the attempt to dispel it by simplified accounts does as much harm as good.*

Reviewer 2 then pleads for mysticism instead of science:

- *Understanding flight requires intuiting the behavior of an intangible medium for which our evolution has provided no apt language.*

Finally, Reviewer 2 starts out into a lengthy attempt to resurrect the theory that has collapsed:

- *All of the criticisms that comprise Section I of their paper can be answered, and I will try to do this below.*

Reviewer 1 dismisses our article by:

- *The authors provide no documented scientific evidence to discredit the current state of the art.*

Reviewer 1 thereby dismisses all the criticism of the current state of the art, acknowledged by Reviewer 2, and expressed by AIAA/Hoffren among many. Both reviewers thus defend the existing theory without seriously taking our criticism into account.

They do not show that they have read and understood our arguments, which is necessary to give an evaluation. Instead they display a large number of misconceptions as concerns both mathematics and computation underlying the theory of flight. I will return to these misconceptions in an upcoming post.

# Chapter 52

# Analysis of Referee Report 2

The tone is set in the first sentence:

- *The classical theory of flight is one the most beautiful and subtle achievements of applied mathematics*.

This is copied from Stoker's review of Birkhoff's critical book Hydromechanics, which elevates classical theory to a level beyond critique. Nevertheless the reviewer delivers a 6 page defense of classical theory:

- *All of the criticisms that comprise Section I of their paper can be answered, and I will try to do this below*.

The defense contains many remarkable and incorrect statements with our comments in parenthesis:

- *There is a mathematical subtlety involved because the flow of a fluid at infinite Reynolds number (zero viscosity) is not always the same as the limit at very small viscosity (it is a singular perturbation problem), and so the question is what light can be shed by the former on the latter?* (This is so subtle that it is meaningless.)

- *There is no way to create vorticity within a viscous incompressible fluid. Vorticity can be created only at a solid boundary.* (This is incorrect as shown on Kelvin's Theorem Unphysical.)

- *In the limit of vanishingly small viscosity, the boundary layer has no thickness, but is still present as an infinitesimal layer of infinite vorticity and hence making a finite contribution to circulation.* (Meaningless statement without quantification.)

- *Vorticity can be created only at a solid boundary. It travels into the interior solely by diffusion....There is a tendency to suppose that the vorticity must be spread by viscosity, which does not seem plausible...*(Contradiction.)

- *But what happens is that the circulation at infinity is set up by acoustic waves, and, if the flow really were incompressible, these travel infinitely fast. What acoustic waves cannot do is create vorticity.* (Mind boggling.)

- *The authors greatly underestimate the classical theory, most likely because the usual truncated exposition has not shown it to them in its proper light. If they take time to realize how its parts fit together, they will come to see that is a masterpiece of physical modeling.* (The reviewer greatly overestimates classical theory; if it was such a masterpiece the reviewer's rescue operation would not be needed.)

- *The real flow (by which they mean their computed flow) always contains a boundary layer whose influence is not negligible at any Reynolds number. This is characteristic of singular perturbation problems, and is the reason why Prandtls insight was transformative to the theory of flight.* (This is incorrect and is precisely the key element of our criticism of Prandtl which the reviewer does not address.)

- *As described earlier, the desirability of the sharp edge lies in forcing the boundary layer to negotiate an adverse pressure gradient before it could reach any other stagnation point. It is not necessary for the trailing edge to be absolutely sharp to achieve this aim. But the shaper the edge is, the more certain the effect, and the more likely to remain effective at high angles of attack*. (This is incorrect. If it was correct that only a sharp trailing edge would a have a "certain effect", there would be no air transportation.)

- *The trick (Kutta condition) if it deserves to be so called, lies in condensing this to a simple boundary condition, the effect of which is to force the zero-viscosity solution to obey the boundary condition for the small-viscosity solution*. (This is incorrect as shown on The Kutta Trick is Illegal.)

- *It is well known that it is extremely hard, and probably impossible, to produce two-dimensional flow experimentally. It should be, and usually is, impossible to produce it in a three-dimensional computation.* (Confusion about the non-physical 2d problem, which lacks all relevance.)

- *Calculations of this kind (Unicorn) are often referred to as Implicit Large Eddy Simulation, and are a recognized, but somewhat controversial, approach to modeling some aspects of turbulence. Is that what is being done? In any case, the mere fact of vorticity being observed means that the code did not simulate a potential flow*. (Total confusion concerning the computational solution of the Navier-Stokes equations supporting the theory.)

- *There follows a stability analysis of the linearized Euler equations. This is of doubtful validity (But is it valid?) because it assumes that a perturbation with non-zero curl can be introduced into an irrotational flow. Physically this cannot be done; I have already explained that there is no mechanism even within the Navier-Stokes equations for vorticity creation, merely the evolution of vorticity already present. Creation must take place at solid surfaces and involve viscosity, or must require externabody forces. There is nothing at all wrong with Kelvins Theorem*. (Yes, it is, see Kelvin's Theorem Unphysical.)

- *Regrettably, it is my conclusion that publication of any of this material, in any form, would be highly retrogressive*. (Not anything in any form?)

- *The authors have put their fingers accurately on many of the defects in the truncated versions of aerodynamic theory that are now current*. (Compare previous statement)

- *However, they have not realized that all these difficult issues were struggled with years ago by the founding fathers of the subject, and resolved in completely satisfactory ways*. (Incorrect, as shown by the reviewers attempt to rescue the fathers) Sadly, the outcomes of those struggles have since been simplified or discarded in modern presentations to create a pragmatic treatment focusing on utility. (Yes, it is a sad state-of-the-art.)

- *This review is very much longer than I would normally write, because I believe that serious issues of substantial public interest are involved*. (Yes the issues are important and require reviewers with deep insight into both mathematics, computation and fluid mechanics, and Reviewer 2 does not meet these requirements.)

**Summary**: The reviewer has not read and understood the article. The reviewer seeks to stop the article, because it questions the dogmas set by the fathers of aerodynamics 100 years ago. We share the criticism with many, but we are unique by offering a new correct understandable theory of flight backed by solid math, computation, physics and observation. It is not very clever by AIAA to dismiss our work on loose grounds. It will not disappear.

# Chapter 53

# Analysis of Referee Report 1

The punch line is:

- *The authors provide no documented scientific evidence to discredit the current state of the art.*

Yes, we do and we are not alone: Very substantial criticism of the Kutta-Zhukovsky-Prandtl theory of lift has been expressed by many scientists ever since this theory was conceived 100 years ago.

The reviewer then branches out into a sequence of incoherent statements without meaning:

- *First of all, the concept of circulation is not necessary for the physical explanation of lift-the physical Kutta condition leads to the correct (as verified by experiment) solution to the potential flow problem. Circulation enters the mathematical problem for the incompressible potential flow past an airfoil since the problem is non-unique without its specification.*

- *More importantly, Prandtls boundary-layer theory is not a viscous theory for drag but an asymptotic theory for the solution to the Navier-Stokes equations at large Reynolds number.*

- *Potential flow is not presented as the solution for lift but as the first term in an asymptotic expansion - the potential flow and boundary-layer theories are connected through the matching process.*

- *The versions of potential theory and boundary-layer theory the authors present are only the first terms in the expansion.*

- *Their claim that the theories of Kutta-Joukowski and Prandtl are both incorrect at separation (undefined by the authors but apparently only considered at the trailing edge) does not take into account the extensive research into the potential flow-boundary-layer coupling.*

- *In fact, the inclusion of the effect of the displacement thickness in the second-order potential flow solution renders arguments associated with a trailing-edge stagnation point moot. (The trailing edge stagnation point does not appear for a cusped trailing edge).*

- *In addition, problems arising with the calculation of the boundary layer past the trailing edge or a separation point are addressed with a strong-interaction version of the boundary-layer equations (see the discussion in Chapter 14 of Katz and Plotkin which also includes a detailed discussion of the matching process referred to above).*

After this excursion into terra incognita the reviewer returns to our article:

- *The authors new approach is to solve the Navier-Stokes equations numerically with an unphysical slip boundary condition.*

- *The authors must demonstrate that the slip boundary condition somehow matches the physics of viscous flow near a solid boundary. They do not do this.*

Yes, this is precisely what we do. We show that slip models the small skin friction of slightly viscous flow and that solution of Navier-Stokes with slip matches observation. This is a key point of our article.

The reviewer concludes with:

- *In summary, the authors have not presented us with an aerodynamic theory alternative to the modern boundary-layer theory in the literature which addresses lift and drag.*

- *At most, perhaps they present a numerical model of the governing equations which avoids the need to discretize the boundary layer.*

- *They would however need to demonstrate how drag is calculated with such a model and that the results match with detailed experiments.*

The reviewer has understood one of our major points: Discretization of the boundary layer is not necessary, which makes it possible to solve the Navier-Stokes equations with millions of mesh points, instead of the impossible quadrillions required by state-of-the-art. But the reviewer then falls back to misunderstanding: We compute both lift and drag for all angles of attack in close correspondence to observation.

Altogether, this is a very poor report by a reviewer who misrepresents key aspects of the article. The report does not meet the standards of AIAA as The World's Forum for Aerospace Leadership.

# Chapter 54

# Rebuttal by Authors Version 1

## 54.1 Our Criticism of 2d Kutta-Zhukovsky Circulation Theory of Lift

Classical Kutta-Zhukovsky circulation theory of lift, as a 2d theory for a 3d real phenomenon, is unphysical. The Kutta condition of specifying the velocity to be zero at a point on the boundary of inviscid flow where the flow is not entering (including the trailing edge), is mathematically meaningless and physically impossible.

The classical proof of Kelvin's theorem is incorrect from wellposedness, since the vorticity equation can exhibit exponential growth of perturbations, and perturbation is part of wellposedness.

## 54.2 Our Criticism of Prandtl's Boundary Layer Theory of Drag

Prandtl's boundary layer theory attributes drag to the presence of boundary layers. We compute drag in slightly viscous flow accordance with observation by solving the Navier-Stokes equations with slip boundary condition without presence of boundary layers. We conclude that the major part of drag (form drag) in slightly viscous flow does not originate from any boundary layer, and thus that Prandtl's theory does not cover the major part of drag in slightly viscous flow.

## 54.3 Our New Theory of Flight

The reviewers do not question that our New Theory of Flight describes real 3d flow.

What they question is our criticism of classical 2d theory: Instead of frankly admitting that it is unphysical and thus incorrect, as we do, the reviewers want to describe classical theory as correct in principle as a 2d theory, even if this 2d theory does not really describe real 3d flow. This is a common way of handling the unphysical aspect of classical 2d theory; admitting that it is 2d and thus in a sense unphysical as any model (no model is perfect) but insisting that anyway it is correct in some sense as a 2d flow model, which somehow ”represents real 3d flow” without describing the actual 3d physics. Thus correct even if incorrect, as any model (no model is perfect). This is the split between theory and practice which has troubled fluid mechanics starting with d'Alembert's paradox in 1752.

The reviewers claim that a slip boundary condition does not describe the physics of slightly viscous flow. This is not correct because the skin friction of slightly viscous flow is small and slip models small skin friction. Slip is also a mathematical meaningful (and possible) boundary condition. The reviewers are stuck to a Prandtl dictate to use no-slip with lacks both mathematics and physics rationale.

Altogether, the criticism of the New Theory is weak, and the defense of the Old Theory is also weak.

Reviewer 2 offers the following starting point for the continued discussion with AIAA: I believe that serious issues of substantial public interest are involved.

# Chapter 55

# Rebuttal by Authors Version 2

In our paper we question the classical theory for subsonic flight currently being propagated, based on 2d potential flow with the circulation theory for lift with drag resulting from skin friction in the boundary layer.

We present instead a flight theory based directly on the fundamental laws of conservation of mass and momentum, in the form of the 3d Navier- Stokes equations, without any assumption of 2d irrotational flow. Based on the development of theory, computations and experiments over the last decades, we find that it is now possible to describe the basic physics and mathematics of flight in 3d turbulent flow.

Both reviewers defend the classical theory, even if it is clear that the real 3d flow observed in experiments cannot be reproduced with any 2d theory. In particular, we show that potential flow is not stable in 3d, which means that it cannot exist as a stable physical flow, and that the classical proof of Kelvin's Theorem is incorrect from the point of view of wellposedness with respect to small perturbations.

We show this in computational experiments where the flow is initialized as potential, and we show this with a linear stability analysis. Both the computational experiments and the linear stability analysis show that streamwise vorticity develops at the trailing edge, which is also observed in physical experiments with a close match in lift, drag and pressure coefficients.

In this new flow configuration observed in computations and in physical experiments, the main part of the drag results from the change in the pressure distribution at the trailing edge, which dominates any contribution from skin friction in the boundary layer.

et cet with arguments form the above analysis of the referee reports....

# Chapter 56

# Defense by AIAA

Dear Drs Hoffman, Jansson and Johnson:

I have read your response to the reviewers comments and gone back over the reviews and your paper. I agree with the reviewers assessment of your paper. (Note: below I refer to Reviewers 1 and 2 as they are listed in the AIAA manuscript review system; on Professor Johnsons website the two are reversed.) Reviewer 1, especially, has given a thorough, articulate and gentle explanation of the deficiencies of your paper. After reading your paper and your responses to the reviewers comments, I can see several misconceptions you have concerning basic concepts in fluid mechanics. My goal below is not simply to reiterate the comments of the reviewers, but to help you see some of the areas where you need to improve your understanding. I will try to be clear and direct in my comments to avoid any misunderstanding.

First, one of your major objections to the classical theory of the generation of lift is that the potential flow solution has a stagnation point at the (sharp) trailing edge and, therefore, produces a pressure distribution with a high value of pressure at the trailing edge. This high value of pressure is not observed in experimental measurements. Also, the pressure distribution from the potential flow solution does not give rise to any drag, which is incorrect from common experience (dAlemberts paradox). What you seem to be missing, which reviewer 1 alluded to, is that this view of the classical theory is truncated; it is not a correct view.

High Reynolds number flow around an airfoil (or a wing in 3-D) is, as reviewer 1 said, a singular perturbation problem, where a small parameter (1/Re) multiplies the term in the momentum equation with the highest order derivative. The potential flow solution you object to is only the leading order inviscid (outer) solution. The complete theory treats the coupled viscous-inviscid interaction by determining the viscous boundary layer (inner) solution, finding the displacement thickness of the boundary layer, adding that to the starting geometry to find the effective shape of the airfoil (thus accounting for the viscous displacement of the streamlines in the outer inviscid part of the flow field), recomputing the potential flow solution, and then iterating this process until a converged solution is found. This results in a pressure distribution that does not have a high value of pressure at the trailing edge, in agreement with experimental measurements. This iteration process is what is done by the airfoil analysis program XFOIL, which reviewer 2 mentioned. XFOIL is widely used in teaching aerodynamics, and I use it in my classes.

The pressure near the trailing edge differs from that of the leading order inviscid potential flow solution because of the displacement effect of the viscous boundary layer. As a result the pressure does contribute to the drag; this is termed form drag. Both pressure and skin friction contribute to drag. Which one is dominant depends on the airfoil design, angle of attack and Reynolds number. At small angles of attack the skin friction contribution can be much larger than the contribution from the pressure; while pressure or form drag is more important at higher angles of attack where the airfoil acts more like a bluff body. DAlemberts paradox is no longer a paradox. His potential flow solution lacked two effects due to viscosity that create drag  skin friction and form drag due to the displacement effect of the boundary layer. Please note that, contrary to what is sometimes stated, form drag does not only occur due to boundary layer separation. Even if the boundary layer does not separate, the displacement effect of the boundary layer will alter the potential flow solution and result in drag due to pressure.

As reviewer 1 said, it is the truncated form of the classical theory that you object to. Unfortunately, the complete theory is not always taught, with the result that many students have misunderstandings concerning flow over airfoils and wings. In my department we do not require our students to take a course in perturbation methods; as reviewer 1 said, aerospace engineers today use computational fluid dynamics; and, as a result, they lose sight of some of the theoretical underpinnings of what we study.

Many of the concepts associated with viscous boundary layers, viscous-inviscid interactions, circulation, vorticity and the generation of lift, are not intuitively obvious. One of my main jobs as an educator in the area of fluid mechanics is to build up my students intuition by increasing their understanding of basic concepts.

Another set of basic concepts that you misunderstand deal with circulation and vorticity. They are related, but they are not the same, and both are important to understanding the fluid mechanics of airfoils and wings. I do not have the time or space to elaborate in detail; these are concepts that are taught over the course of a full semester. However, there are two points I want to clarify. First Kelvins theorem, that circulation does not change for a closed loop moving as a material curve (moving with the fluid), only depends on the assumptions of (i) inviscid flow, (ii) incompressible (low-speed) flow, and (iii) a conservative body force (e.g., gravity). The flow can be unsteady, 3-D and rotational (nonzero vorticity) and the theorem still holds. It is not invalidated by there being fluctuations in the freestream or instabilities in the flow (more on that below).

One important difference to see between circulation and vorticity is that when a vortex is stretched, as by a strain field discussed in your linear stability analysis, the vorticity is increased as the vortex radius decreases and vortex lines become concentrated, but the circulation remains fixed, in agreement with Kelvins theorem. The vorticity is twice the local rotation rate of fluid elements, and the increase in vorticity is similar to the increase in rotation rate of an ice skater when he spins and then brings his arms in tight. His rotation rate increases, but his angular momentum remains constant (not accounting for friction). Kelvins theorem deals with circulation, and it is in fact the circulation around the airfoil that is important to the lift.

The second point I want to make concerns the generation of vorticity. What reviewer 1 stated about vorticity not being generated in the interior of the flow under the assumptions of (i) incompressible flow and (ii) a conservative body force is correct. This can be proved mathematically from the basic governing equations. In the discussion above about a strained vortex, the vorticity increases locally because the vortex lines become more concentrated; however, no new vorticity is generated. New vorticity is generated on solid surfaces through the action of pressure gradients or unsteady motion of the solid surface. This point is discussed well in the text Incompressible Flow by Panton. The generation of vorticity on solid surfaces is one of the more difficult subjects included in my graduate introductory fluid mechanics course. The fact that some of the concepts are difficult to understand does not make them wrong.

The last technical point I want to make concerns the trailing edge instability and your linear analysis. It is well known that vorticity in a strain field, such as near a stagnation point, results in vortex stretching and an exponential increase in vorticity. The solution for this is worked out in the book The Structure of Turbulent Shear Flow by Townsend and is part of the rapid distortion theory of turbulence. As discussed above, the increase in vorticity magnitude does not mean an increase in circulation. The trailing edge vortices in your simulations form in counter-rotating pairs. Their net circulation is zero, and their presence does not alter the circulation or the lift on the airfoil. I also want to point out a flaw in your analysis. Fluctuations in the flow are not introduced by a non-conservative body force, as you state. For flow over an airfoil or wing, there is no such non-conservative body force. The only body force acting is gravity, and it is conservative and, therefore, does not generate vorticity. Instead fluctuations come from the freestream, as turbulence from upstream is convected toward the airfoil; fluctuations near the trailing edge also come from turbulent flow in the boundary layers or, in the case of a separated boundary layer, turbulence produced in the free shear layer.

Lastly, both reviewer 1 and I had suggested that you contact colleagues at KTH who could help explain some of these concepts to you. In response to one of your previous comments, I want to say that doing so is not a requirement for submitting papers to AIAA. That suggestion was made because there are basic concepts in fluid mechanics that you do not understand, and it is easier to explain those concepts face to face over a period of time, rather than through the limited medium of email. I know some of your colleagues in the Mechanics Department at KTH, and I have a lot of respect for their knowledge of fundamental fluid mechanics and applied mathematics. I strongly suggest you talk with them, or take some of the courses they offer.

Sincerely,

Greg Blaisdell AIAA Journal Associate Editor

# Chapter 57

# Analysis of Defense by AIAA 1

AIAA does not respond to any point in our rebuttal connected to our article. In fact, AIAA does not give any evaluation of the New Theory of Flight at all, and in particular does not claim that it is incorrect. What AIAA does is only to defend the Old Theory according to the following plan:

- *After reading your paper and your responses to the reviewers comments, I can see several misconceptions you have concerning basic concepts in fluid mechanics. My goal below is not simply to reiterate the comments of the reviewers, but to help you see some of the areas where you need to improve your understanding. I will try to be clear and direct in my comments to avoid any misunderstanding.*

Like one of the reviewers, Blaisdell then sets out on a mission to teach us about the Old Theory, which is exactly the theory we have studied very carefully and found to be incorrect. Blaisdell starts out with:

- *High Reynolds number flow around an airfoil (or a wing in 3-D) is, as reviewer 1 said, a singular perturbation problem, where a small parameter (1/Re) multiplies the term in the momentum equation with the highest order derivative. The potential flow solution you object to is only the leading order inviscid (outer) solution. The complete theory treats the coupled viscous-inviscid interaction by determining the viscous boundary layer (inner) solution, finding the displacement thickness of the boundary layer, adding that to the starting geometry to find the effective shape of the airfoil (thus accounting for the viscous displacement of the streamlines in the outer inviscid part of the flow field), recomputing the potential flow solution, and* *then iterating this process until a converged solution is found. This results in a pressure distribution that does not have a high value of pressure at the trailing edge, in agreement with experimental measurements. This iteration process is what is done by the airfoil analysis program XFOIL, which reviewer 2 mentioned. XFOIL is widely used in teaching aerodynamics, and I use it in my classes.*

But XFOIL is a 2d panel code which is unphysical, because the physics is true 3d. The fact that Blaisdell teaches incorrect physics signifies the present sad state of aerodynamics education, acknowledged by Blaisdell himself:

- *Unfortunately, the complete theory is not always taught, with the result that many students have misunderstandings concerning flow over airfoils and wings.*

- *In my department we do not require our students to take a course in perturbation methods; as reviewer 1 said, aerospace engineers today use computational fluid dynamics; and, as a result, they lose sight of some of the theoretical underpinnings of what we study.*

Blaisdell continues with more confession:

- *Many of the concepts associated with viscous boundary layers, viscous-inviscid interactions, circulation, vorticity and the generation of lift, are not intuitively obvious.*

- *One of my main jobs as an educator in the area of fluid mechanics is to build up my students intuition by increasing their understanding of basic concepts.*

- *The generation of vorticity on solid surfaces is one of the more difficult subjects included in my graduate introductory fluid mechanics course. The fact that some of the concepts are difficult to understand does not make them wrong.*

No, it does not necessarily make them wrong, but neither correct. After more defense of exactly what we criticize (Kelvin's theorem, boundary layer generation of vorticity, circulation et cet), without reading our criticism, Blaisdell concludes by renewing the advice to take some fluid mechanics course at KTH:

- *Lastly, both reviewer 1 and I had suggested that you contact colleagues at KTH who could help explain some of these concepts to you. In response to one of your previous comments, I want to say that doing so is not a requirement for submitting papers to AIAA. That suggestion was made because there are basic concepts in fluid mechanics that you do not understand, and it is easier to explain those concepts face to face over a period of time, rather than through the limited medium of email. I know some of your colleagues in the Mechanics Department at KTH, and I have a lot of respect for their knowledge of fundamental fluid mechanics and applied mathematics. I strongly suggest you talk with them, or take some of the courses they offer.*

These are my so respected colleagues at KTH, who are so good friends with Blaisdell, but refuse to speak to CJ. What a world of science.
**Summary**: AIAA does not claim that the New Theory is incorrect, that our 3d Navier-Stokes/slip computations do not describe the real physics of the flow around a wing, or that our analysis of the computations is incorrect. AIAA only defends the Old Theory, while admitting that it is so difficult that it cannot be taught to new generations of engineers responsible for constructing the new airplanes to carry new generations of people safely and efficiently.

AIAA actively suppresses discussion of a very important scientific issue, by intimidating power play. It is not admirable and will not work in the long run.

# Chapter 58

# Analysis of Defense by AIAA 2

Blaisdell formulates his goal to be to re-educate (or brain-wash) us and teach us the Old Theory, which is exactly the theory which we have studied very carefully and found to be incorrect:

- *I can see several misconceptions you have concerning basic concepts in fluid mechanics. My goal below is ... to help you see some of the areas where you need to improve your understanding....there are basic concepts in fluid mechanics that you do not understand.*

- *What you seem to be missing ... is that this (our) view of the classical theory is truncated; it is not a correct view.... it is the truncated form of the classical theory that you object to. Unfortunately, the complete theory is not always taught, with the result that many students have misunderstandings concerning flow over airfoils and wings.*

- *Another set of basic concepts that you misunderstand deal with circulation and vorticity. They are related, but they are not the same, and both are important to understanding the fluid mechanics of airfoils and wings.*

In his re-education mission Blaisdell seeks to force us to accept precisely what we have shown to be incorrect physics:

- *At small angles of attack the skin friction contribution can be much larger than the contribution from the pressure; while pressure or form drag is more important at higher angles of attack where the airfoil acts more like a bluff body. The trailing edge vortices in your simulations form in counter-rotating pairs. Their net circulation is zero, and their presence does not alter the circulation or the lift on the airfoil.*

- *What reviewer 1 stated about vorticity not being generated in the interior of the flow under the assumptions of (i) incompressible flow and (ii) a conservative body force, is correct. Kelvins theorem, that circulation does not change for a closed loop moving as a material curve (moving with the fluid), only depends on the assumptions of (i) inviscid flow, (ii) incompressible (low-speed) flow, and (iii) a conservative body force (e.g., gravity).*

However, Blaisdell is struck by disbelief from the mounting difficulties in his presentation and seeks to encourage himself by twisting lack of evidence into its opposite:

- *Many of the concepts associated with viscous boundary layers, viscous-inviscid interactions, circulation, vorticity and the generation of lift, are not intuitively obvious.*

- *The generation of vorticity on solid surfaces is one of the more difficult subjects included in my graduate introductory fluid mechanics course.*

- *The fact that some of the concepts are difficult to understand does not make them wrong.*

Blaisdell's long lecture is essentially a repetition of the long lecture by Reviewer 2 analyzed in a previous post.

Like Blaisdell, Reviewer 2 not only lectures but also admits that aerodynamics education of today is a mess:

- *Aerodynamics today is therefore almost always taught in a truncated version that retains all of the utility, but has lost much of the profundity.*

- *Even the truncated version is no longer as highly respected as it used to be, because Computational Fluid Dynamics delivers, with no requirement for deep thought, most of the practical answers that are needed. In consequence, there are many employed today in the aerospace industry, and even in academia, whose grasp of the basic theory of flight contains many gaps. These gaps are apparent to thoughtful students, who frequently attempt to fill them in for themselves, although the remedy is usually worse than the disease.*

- *... simplified or discarded in modern presentations to create a pragmatic treatment focusing on utility. Undergraduate textbooks these days all too often simply omit anything that students find difficult.*

- *I believe that the authors ... are right to quarrel with the truncated version that they, like others, have apparently received*

. Reviewer 2 cannot but acknowledge important merits of our article, which Blaisdell ignores:

- *The authors have put their fingers accurately on many of the defects in the truncated versions of aerodynamic theory that are now current.*
- *However, the authors are correct that separation might be fundamentally different in 3D than in 2D.*
- *The authors have then sought their own explanations, stimulated by interesting results from their Navier-Stokes code.*

Reviewer 1 acknowledges one of our key points:

- *At most, perhaps they present a numerical model of the governing equations which avoids the need to discretize the boundary layer.*

But Reviewer 1 does not understand that this suddenly makes the Navier-Stokes equations computable from being uncomputable by Prandtl's dictate of boundary layer resolution, and thus fundamentally changes both theory and practice of aerodynamics.
**Summary**: What is specially remarkable is the fervor of the re-education the editor and reviewers want us to undergo (like e.g. the Emperor in Red China): They want to be sure that we come out of the process fully re-educated to the correct belief of basic concepts. No effort should be spared to reach this goal, as if our ideas somehow are dangerous to AIAA and thus have to be suppressed. But why spend so much energy on cranks?

# Chapter 59

# Letter to Editor-in-Chief AIAA J

To AIAA Journal Editor-in-Chief Peretz P. Friedmann

Concerning New Theory of Flight submitted to AIAA Journal

In March 2012 we submitted the article New Theory of Flight to AIAA Journal. The article presents a new mathematical description of the generation of lift and drag from the flow of air around a wing, which is fundamentally different from the current textbook theory based on the work by Kutta-Zhukovsky-Prandtl. Our new theory is based on a mathematical analysis of computed solutions of the 3d Navier-Stokes equations with lift and drag within experimental error tolerance for all angles of attack including stall and beyond.

The assigned AIAA Journal editor Gregory Blaisdell acknowledges that our article ”is unusual in that it challenges our existing understanding of aerodynamics”. However, the challenge is not really evaluated by the reviewers and the editor, who instead respond by a lengthy defense of exactly the existing theory we question using exactly the same arguments we question, without actually considering our arguments and evidence.

One of the reviewers states: ”The authors provide no documented scientific evidence to discredit the current state of the art, but acknowledges that: At most, perhaps they present a numerical model of the governing equations which avoids the need to discretize the boundary layer.

Our objective is not to ”discredit” classical theory. But we do present a new theory made possible through the recent developments in computational modeling of turbulent flow based on the Navier-Stokes equations, which allows for quantitative prediction of the full 3d flow field including aerodynamic forces, also for separated flow beyond stall. Our evidence consists of accurate solutions of the 3d Navier-Stokes equations in close correspondence with observation, and an analysis of these solutions.

We therefore request a new review process by AIAA of our article, without being simply discarding in a defense of the classical theory.

Sincerely, JH, JJ and CJ

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# Chapter 60

# Redundant Material

By Newton's 3rd law, lift must be accompanied by *downwash* with the wing redirecting air downwards. The enigma of flight is the mechanism of a wing generating substantial downwash at small drag, which is also the enigma of sailing against the wind with both sail and keel acting like wings creating substantial lift.

Flying on wings and sailing against the wind is a miracle, and the challenge from scientific point of view is explain the miracle of $L/D > 10$. NASA pretends to explain the miracle of the flight of the *Flyer* in Fig. 1.4 as a conseqence of Newton's Third Law. Do you get it?

Flying on a barn door tilted at 45 degrees with $L/D \approx 1$ would not be a scientific miracle, but only a rocket can generate a thrust equal to its own weight and then only for a short period of time, and air transportation by rockets is nothing for birds and ordinary people, only for astronauts.

The lift force $L$ increases quadratically with the speed and linearly with the *angle of attack*, that is the tilting of the wing from the direction of flight, until *stall* at about $15$ degrees, when the drag abruptly increases and $L/D$ becomes too small for sustained flight.