Why It Is Possible to Fly

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Abstract

We identify, by computing turbulent solutions of the incompressible Navier-Stokes equations with friction force boundary conditions, the physical mechanisms generating lift and drag of a wing, which make it possible to fly with a lift/drag ratio larger than 10. We discover mechanisms fundamentally different from those of the classical inviscid circulation theory by Kutta-Zhukovsky for lift and the laminar viscous boundary layer theory by Prandtl for drag, which have dominated 20th century flight mechanics. We find that substantial lift originates from turbulent low-pressure rolls of streamwise vorticity generated from a three-dimensional instability mechanism at rear separation, while drag is kept small because of negative drag from the leading edge. We show that computational prediction of flight characteristics of an airplane is possible using millions of meshpoints without resolving thin boundary layers, as compared with the imposssible quadrillions required according to state-of-the-art for boundary layer resolution.

1 Introduction

The problem of explaining *why* it is possible to fly in the air using wings has haunted scientists since the birth of mathematical sciences. To fly, an upward force on the wing, referred to as *lift* L, has to be generated from the flow of air around the wing, while the air resistance to motion or *drag* D, is not too big. The mystery is *how* a sufficiently large ratio $\frac{L}{D}$ can be created. In the *gliding flight* of birds and airplanes with fixed wings at subsonic speeds, $\frac{L}{D}$ is typically between 10 and 20, which means that a good glider can glide up to 20 meters upon loosing 1 meter in altitude, or that Charles Lindberg could cross the Atlantic in 1927 at a speed of 50 m/s in his 2000 kg *Spirit of St Louis* at an effective engine thrust of 150 kp (with $\frac{L}{D} = 2000/150 \approx 13$) from 100 horse powers.

By elementary Newtonian mechanics, lift must be accompanied by *downwash* with the wing redirecting air downwards. The enigma of flight is the mechanism generating substantial downwash under small drag, which is also the enigma of sailing against the wind with both sail and keel acting like wings creating lift.

Classical mathematical mechanics could not give an answer. Newton computed the lift of a tilted flat plate redirecting a horisontal stream of fluid particles, but obtained a disappointingly small value proportional to the square of the tilting angle or *angle of attack*. The French mathematician d'Alembert followed up in 1752 with a computation based on *potential flow* (inviscid incompressible irrotational steady flow), showing that both the drag and lift of a wing is zero, referred to as *d'Alembert's paradox*, since it contradicts observations and thus belongs to fiction. To explain flight d'Alembert's paradox had to be resolved.

It is natural to expect that today the mechanics of gliding flight is well understood, but surprisingly one finds that the authority NASA [17] presents three incorrect mathematical theories for lift and ends with the seemingly out of reach: *"To truly understand the details of the generation of lift, one has to have a good working knowledge of the Euler Equations"*, and the Plane&Pilot Magazine [18] has the same message. In short, state-of-the-art literature [2, 8, 16, 20, 23] presents a theory for *drag without lift* in viscous laminar flow [19] by Prandtl, called the father of modern fluid mechanics [21], and a theory for *lift without drag* at small angles of attack in inviscid potential flow by the mathematicians Kutta and Zhukovsky, called the father of Russian aviation, who augumented inviscid zero-lift potential flow by a large scale *circulation* of air around the wing section causing the velocity to increase above and decrease below the wing, thus generating lift proportional to the angle of attack [16, 23, 1] as illustrated in Fig.1. Kutta-Zhukovsky thus showed that if there is circulation then there is lift, which by a scientific community in search for a theory of lift after the flights by the Wright brothers in 1903, was interpreted as an equivalence: *"If the airfoil experiences lift, a circulation must exist"* ([23], p.94). State-of-the-art is described in [2] as: *"The circulation theory of lift is still alive... still evolving today, 90 years after its introduction"*. However, there is no theory for *lift and drag* in *slightly viscous turbulent incompressible* flow such as the flow of air around a wing of a jumbojet at the critical phase of take-off at large angle of attack (12 degrees) and subsonic speed (270 km/hour).

Figure 1: High (H) and low (L) pressure distributions of potential flow (left) past a wing section with zero lift/drag modified by circulation around the section (middle) to give Kutta-Zhukovsky flow (right) leaving the trailing edge smoothly with downwash/lift and a so-called starting vortex behind.

In this article we present such a theory based on computing turbulent solutions of the incompressible Navier-Stokes equations, which reveals mechanisms of gliding flight fundamentally different from those envisioned by Kutta-Zhukovsky and Prandtl. In particular, we show that lift comes along with drag, in contradiction to a common belief supported by Kutta-Zhukovsky that *"a truly inviscid fluid would exert no drag"* [3]. The new theory of flight comes out of a *new resolution* of d'Alembert's paradox [10, 11, 12], showing that zero-lift/drag potential flow is *unstable* and in both computation and reality is replaced by turbulent flow with lift/drag. The new resolution is fundamentally different from the classical official resolution attributed to Prandtl [20, 22, 16], which disqualifies potential flow because it satisfies a *slip* boundary condition allowing fluid particles to glide along the boundary without friction force, and does not satisfy a *no-slip* boundary condition requiring the fluid particles to stick to the boundary with zero relative velocity and connect to the free-stream flow through a thin *boundary layer*, as demanded by Prandtl.

As an important practical consequence of the new theory, we show that lift and drag of an airplane at subsonic speeds can be accurately predicted by computing turbulent solutions of the incompressible Navier-Stokes equations with millions of mesh points using a slip (small friction force) boundary condition as a model of the small *skin friction* of a turbulent boundary layer of slightly viscous flow. In state-of-the-art dictated by Prandtl this is impossible, because resolving thin no-slip boundary layers for slightly viscous flow requires impossible quadrillions of mesh points [15]. State-of-the-art is decribed in the sequence of *AIAA Drag Prediction Work Shops* [6], focussing on the simpler problem of transonic compressible flow at small angles of attack (2 degrees) of relevance for crusing at high speed, leaving out the more demanding problem of subsonic *incompressible* flow at low speed and large angles of attack at take-off and landing, presumably because a workshop on this topic would not draw any participants.

The new theory is supported by computation using an adaptive stabilized finite element method with duality-based a posteriori error control referred to as *General Galerkin* or *G2* presented in detail in [10] and available in executable open source form from [7]. The stabilization in G2 acts as an automatic turbulence model, and the only input is the geometry of the wing. We find that lift is not connected to circulation in contradiction to Kutta-Zhukovsky's theory and that the curse of Prandtl's laminar boundary layer theory (also questioned in [4, 5, 24]) can be circumvented. Altogether, we show that *ab initio* computational fluid mechanics opens new possibilities of flight simulation ready to be explored.

2 The Secret of Flying

The new resolution of d'Alembert's paradox [10, 11, 12] identifies a basic instability mechanism of potential flow arising from retardation and accelleration at separation, which generates *counter-rotating rolls* or tubes of *streamwise vorticity* forming a *low-pressure wake* effectively generating drag. For a wing this is also an essential mechanism for generating lift by depleting the high pressure before rear separation of potential flow and thereby allowing downwash. This mechanism is illustrated in Fig.2 showing a perturbation (middle) consisting of counter-rotating rolls of low-pressure streamwise vorticity developing at the separation of potential flow (left), which changes potential flow into turbulent flow (right) with a different pressure distribution at the trailing edge generating lift. The rolls of counter-rotating streamwise vorticity appear along the entire trailing edge and have a different origin than the *wing tip vortex*, which adds drag but not lift, which is of minor importance for a long wing [24].

We see that the difference between Kutta-Zhukovsky and the new explantion is the nature of the modification/perturbation of zero-lift potential flow: Kutta and Zhukovsky claim that it consists of a global large scale two-dimensional circulation around the

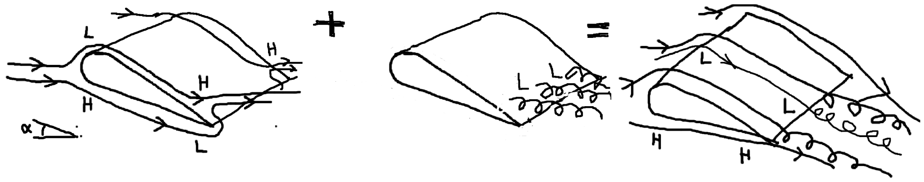

Figure 2: Stable physical 3d turbulent flow (right) with lift/drag, generated from potential flow (left) by a perturbation at separation consisting of counter-rotating tubes of streamwise vorticity (middle), which changes the pressure at the trailing edge generating downwash/lift and drag.

wing section, that is *transversal vorticity* orthogonal to the wing section (combined with a transversal starting vortex), while we find that it is a three-dimensional local turbulent phenomenon of counter-rotating rolls of streamwise vorticity at separation, without starting vortex. Kutta-Zhukovsky thus claim that lift comes from global transversal vorticity without drag, while we give evidence that instead lift is generated by local turbulent streamwise vorticity with drag.

We observe that the real turbulent flow like potential flow adheres to the upper surface beyond the crest and thereby gets redirected, because the real flow is close to potential before separation, and potential flow can only separate at a point of stagnation with opposing flows meeting in the rear, as shown in [11, 12]. On the other hand, a flow with a viscous no-slip boundary layer will (correctly according to Prandtl) separate on the crest, because in a viscous boundary layer the pressure gradient normal to the boundary vanishes and thus cannot contribute the normal acceleration required to keep fluid particles following the curvature of the boundary after the crest [13]. It is thus the slip boundary condition modeling a turbulent boundary layer in slightly viscous flow, which forces the flow to suck to the upper surface and create downwash, as analyzed in detail in [13], and not any Coanda effect [1].

This explains why gliding flight is possible for airplanes and larger birds, because the boundary layer is turbulent and acts like slip preventing early separation, but not for insects because the boundary layer is laminar and acts like no-slip allowing early separation. The *Reynolds number* of a jumbojet at take-off is about 10^8 with turbulent skin friction coefficient < 0.005 contributing less than 5% to drag, while for an insect with a Reynolds number of 10^2 viscous laminar effects dominate.

3 Mechanisms of Lift and Drag

Based on computation and analysis we now identify the basic mechanisms for the generation of lift and drag in incompressible high Reynolds number flow around a wing at different angles of attack α. We find two regimes before stall at $\alpha = 20$ with different, more or less linear growth in α of both lift and drag, a main phase $0 \leq \alpha < 16$ with the slope of the lift (coefficient) curve equal to 0.09 and of the drag curve equal to 0.08 with $L/D \approx 14$, and a final phase $16 \leq \alpha < 20$ with increased slope of both lift and drag. The main phase can be divided into an initial phase $0 \leq \alpha < 4 - 6$ and an intermediate phase $4-6 \leq \alpha < 16$, with somewhat smaller slope of drag in the initial phase. We illustrate below in a sequence of images of velocity, pressure and vorticity, and plots of lift and drag distributions over the upper and lower surfaces of the wing (allowing also pitching moment to be computed), with additional information in the supporting online material.

Figure 3: G2 computation of velocity magnitude (upper), pressure (middle), and non-transversal vorticity (lower), for angles of attack 4, 10, and 18° (from left to right). Notice in particular the rolls of streamwise vorticity at separation.

Phase 1: $0 \leq \alpha \leq 4-6$: At zero angle of attack with zero lift there is high pressure at the leading edge and equal low pressures on the upper and lower crests of the wing because the flow is essentially potential and thus satisfies Bernouilli's law of high/low pressure where velocity is low/high. The drag is about 0.01 and results from rolls of low-pressure streamwise vorticity attaching to the trailing edge. As α increases the low pressure below gets depleted as the incoming flow becomes parallel to the lower surface at the trailing edge for $\alpha = 6$, while the low pressure above intenisfies and moves towards the leading edge. The streamwise vortices at the trailing edge essentially stay constant in strength but gradually shift attachement towards the upper surface. The high pressure at the leading edge moves somewhat down, but contributes little to lift. Drag increases only slowly because of negative drag at the leading edge.

Phase 2: $4-6 \leq \alpha \leq 16$: The low pressure on top of the leading edge intensifies to create a normal gradient preventing separation, and thus creates lift by suction peaking on top of the leading edge. The slip boundary condition prevents separation and downwash is created with the help of the low-pressure wake of streamwise vorticity at rear separation. The high pressure at the leading edge moves further down and the pressure below increases slowly, contributing to the main lift coming from suction above. The net drag from the upper surface is close to zero because of the negative drag at the leading edge, while the drag from the lower surface increases (linearly) with the angle of the incoming flow, with somewhat increased but still small drag slope. This explains why the line to a flying kite can be almost vertical even in strong wind, and that a thick wing can have less drag than a thin.

Phase 3: $16 \leq \alpha \leq 20$: This is the phase creating maximal lift just before stall in which the wing partly acts as a bluff body with a turbulent low-pressure wake attaching at the rear upper surface, which contributes extra drag and lift, doubling the slope of the lift curve to give maximal lift ≈ 2.5 at $\alpha = 20$ with rapid loss of lift after stall.

We understand that this scenario of the action of a wing for different angles of attack is fundamentally different from that of Kutta-Zhukovsky, although for lift there is a superficial similarity because both scenarios involve modified potential flow. The slope of the lift curve according to Kutta-Zhukovsky is $2\pi^2/180 \approx 0.10$ as compared to the computed 0.09.

The lift generation in Phase 1 and 3 can rather easily be envisioned, while both the lift and drag in Phase 2 results from a (fortunate) intricate interplay of stability and instability of potential flow: The main lift comes from upper surface suction arising from a turbulent boundary layer with small skin friction combined with rear separation instability generating low-pressure streamwise vorticity, while the drag is kept small by negative drag from the leading edge. We conclude that preventing transition to turbulence at the leading edge can lead to both decreased lift and increased drag.

4 Comparing Computation with Experiment

Comparing G2 computations with about 150 000 mesh points with experiments [9, 14], we find good agreement with the main difference that the boost of the lift coefficient in phase 3 is lacking in experiments. This is probably an effect of smaller Reynolds numbers in experiments, with a separation bubble forming on the leading edge reducing lift at high angles of attack. The oil-film pictures in [9] show surface vorticity generating streamwise vorticity at separation as observed also in [12, 13].

A jumbojet can only be tested in a wind tunnel as a smaller scale model, and upscaling test results is cumbersome because boundary layers do not scale. This means that

Figure 4: G2 computation of normalized local lift force (upper) and drag force (lower) contributions acting along the lower and upper parts of the wing, for angles of attack 0, 2 ,4 ,10 and 18°, each curve translated 0.2 to the right and 1.0 up, with the zero force level indicated for each curve.

Figure 5: G2 computation of lift coefficient C_L and circulation (upper), and drag coefficient C_D (lower), as functions of the angle of attack.

computations can be closer to reality than wind tunnel experiments. Of particular importance is the maximal lift coefficient, which cannot be predicted by Kutta-Zhukovsky nor in model experiments, which for Boeing 737 is reported to be 2.73 in landing in correspondence with the computation. In take-off the maximal lift is reported to be 1.75, reflected by the rapidly increasing drag beyond $\alpha = 16$ in computation.

5 Kutta-Zhukovsky's Lift Theory is Non-Physical

Fig.5 shows that the circulation is small without any increase up to $\alpha = 10$, which gives evidence that Kutta-Zhukovsky's circulation theory coupling lift to circulation does not describe real flow. Apparently Kutta-Zhukovsky manage to capture some physics using fully incorrect physics, which is not science.

Kutta-Zhukovsky's explanation of lift is analogous to an outdated explanation of the Robin-Magnus effect causing a top-spin tennis ball to curve down as an effect of circulation, which in modern fluid mechanics is instead understood as an effect of non-symmetric different separation in laminar and turbulent boundary layers [13]. Our results show that Kutta-Zhukovsky's lift theory for a wing also needs to be replaced.

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